Growth Uncertainty and Risksharing *

Stefano Athanasoulis
Iowa State University

Eric van Wincoop Federal Reserve Bank of New York

July 24, 1997

*We would like to thank seminar participants at Columbia, Wharton, the Federal Reserve Bank of New York, Yale, Berkeley, UCLA, UBC, the 1997 North American Econometric Society meetings, the Midwest Macro conference, the 1997 AEA meetings, and Iowa State. We thank Andrew Abel, Franklin Allen, Leonardo Bartolini, Guillermo Calvo, Roger Craine, Michael Devereux, Paul Evans, James Harrigan, Urbann Jermann, Aubik Kahn, Amartya Lahiri, Philip Lane, Giuseppe Moscarini, James Nason, Maurice Obstfeld, Robert Shiller, Nicholas Souleses, and Kei-mu Yi for useful comments and suggestions. We also would like to thank Richard Easterlin for his help in obtaining the fertility data. The views expressed in the paper are those of the authors and do not necessarily reflect the position of the Federal Reserve Bank of New York or the Federal Reserve System.

Abstract

We propose a new methodology to evaluate the gains from global risksharing that is closely connected to the empirical growth literature. We obtain estimates of residual risk (growth uncertainty) at various horizons from regressions of country-specific deviations from world growth on a wide set of variables in the information set. Since this residual risk can be entirely hedged, we use it to obtain a measure of the welfare gain that can be achieved by a representative country. We find that nations can reap very large benefits from engaging in such risksharing arrangements. Using post-war data, the gain for a 35-year horizon, corresponding to an equivalent permanent increase in consumption, is 6.6% when based on a set of 49 countries, and 1.5% when based on 21 OECD countries. Using historical data from 1870 to 1990, we find that the potential gain for a 120-year horizon ranges from 4.9% for a small set of rich countries to 16.5% for a broad set of 24 countries.

1 Introduction

How rich will the Russians be compared to the British in twenty, thirty or fifty years? Will India still be poor relative to the rest of the world in one century, and will the US still be the leader? No one can answer these questions. There are many stunning examples in history of countries which were once rich but are now poor, and vice versa. Although one can attempt to make predictions about growth, there will remain significant uncertainty about relative growth rates at long horizons. We show empirically that over a period of 35 years one country's per capita GDP can, purely by chance, easily double relative to that of another. The growth literature so far is primarily concerned with explaining these differences in long term growth rates, but has paid little attention to long term growth uncertainty. ¹ Our goal in this paper is to measure growth uncertainty at various horizons and compute the welfare gain from sharing this macroeconomic risk among nations.

In international macroeconomics a large literature has developed in recent years evaluating the gains from risksharing among countries. This interest stems from two sources. First, risksharing associated with intratemporal asset trade is one of the three key roles of international financial markets. The other two are consumption smoothing through intertemporal asset trade and allocating resources to their most productive uses. The benchmark model in international macroeconomics is one of complete markets, where all three roles are performed perfectly (e.g. Backus, Kehoe and Kydland (1992)).

Second, the evidence indicates that the real world is far removed from the perfect risksharing paradigm that is the centerpiece of our models. The perfect risksharing hypothesis is generally rejected in formal regression-based tests (Canova and Ravn [1994], Lewis [1996] and Obstfeld [1994c]). A serious problem with perfect risksharing models is that the implied correlation of consumption across countries is much larger than the income correlation, while the opposite is observed in the data. Backus, Kehoe and Kydland [1992] have coined this the "quantity anomaly". Direct evidence on international portfolio diversification, e.g. by French and Poterba [1991] and Tesar and Werner [1994], shows that there is a strong home bias in equity portfolios. Trade in claims on a broad measure of national outputs currently does not even exist. Shiller

¹An exception is Easterly, et al. [1993], who conclude that "much variation in growth rates is due to random shocks". They find that the low persistence of decade average growth rates is associated with random shocks since "country-characteristics" are highly persistent.

[1993] has made a case for the introduction of such markets. ²

A lack of risksharing among nations is no surprise if the potential welfare payoff is small. Results reported in the literature vary enormously, ranging from less than 0.1% (welfare equivalent permanent increase in expected consumption) to over 100%. This has led to considerable confusion concerning the true magnitude of these gains. ³

van Wincoop [1994, 1996b] shows that the welfare gain from risksharing depends on four factors ⁴: (i) the risk-free interest rate, (ii) the risk-adjusted growth rate, (iii) the rate of relative risk-aversion, and (iv) uncertainty about the endowment. The first two are relatively easy to measure.⁵ After choosing a consensus estimate for the rate of relative risk-aversion, the main obstacle that remains is measuring endowment uncertainty. ⁶ The approach that is taken throughout the literature is to estimate a specific process for the endowment. The problem with this is that the results are very sensitive to both the type of process assumed and the parameters of a specific process. As an example, van Wincoop [1996b] considers different endowment processes for a set of 21 OECD countries, setting the rate of relative risk-aversion at three. The welfare gain for a representative country at a 100-year horizon is 5.6% when the endowment is assumed to follow an AR in growth rates, 0.2% when a global cointegration term is added to the AR in growth rates, and 0.1% when the process is stationary (approximately an AR(1) with a deterministic trend). ⁷

Rather than assuming a particular endowment process, here we adopt an alternative

²Shiller and Athanasoulis [1995] and Athanasoulis [1996] show how such markets may be constructed.
³See van Wincoop [1996b] for a discussion of this literature and explanations for the widely diverging welfare gain measures. Recent papers that have computed welfare gains from risksharing across countries include Athanasoulis [1996], Backus, Kehoe and Kydland [1992], Cole and Obstfeld [1991], Kose [1995], Lewis [1996a], Mendoza [1994,1995], Obstfeld [1994a,b,1995], Shiller and Athanasoulis [1995], Tesar [1995] and van Wincoop [1994, 1996a,b].

⁴van Wincoop [1994] shows that, given these four factors, the welfare gain does not depend on the type of preferences. Three different types are considered: standard von Neuman Morgenstern expected utility, non-expected utility (Kreps-Porteus preferences), and habit formation preferences.

⁵Preferences are often parameterized in the literature in a way that implies excessively high risk-free interest rates, which reduces the welfare gain measure. For example, the implicit risk-free rate in Cole and Obstfeld (1991) ranges from 5.6% to 56%.

⁶Most of this literature assumes endowment economies rather than production economies since it is concerned with the risksharing role of international capital markets and not the optimal resource allocation role.

⁷The low welfare gain for a stationary process is consistent with the finding by Lucas [1987] that the cost of aggregate business cycle fluctuations is negligible. At long horizons business cycle uncertainty should be overshadowed by long term growth uncertainty.

approach whereby we only make assumptions about the information set used to predict future growth. Since only uncertainty about deviations from world growth matters for risksharing, for a given horizon we regress deviations from world growth on variables in the information set. This provides us with a measure of growth uncertainty at various horizons, which we use to compute the gain from international risksharing.

This approach has three advantages. First, we don't need to specify an entire model or data generating process. Different processes (many quite complicated) can be consistent with the same information set used to predict future growth. Choosing the wrong process, even if it is consistent with the correct information set, can lead to highly misleading results. ⁸ Second, even if one knows what type of process the endowment follows, it is not advisable to estimate it with annual data and then use this to determine growth uncertainty at long horizons. Imprecision in estimated parameters, e.g. due to small samples or multi-collinearity, lead to imprecise predictions that are amplified significantly when making projections far enough into the future. Finally, and most importantly, we find that the results are robust to the size of the information set. After including the three most important variables, expanding the information set with a large set of additional variables does not improve the explanatory power. ⁹

The remainder of the paper is organized as follows. The next section describes how a measure of welfare gains for a representative country is obtained based on uncertainty about deviations from world growth at various horizons. Section 3 describes the data and the empirical implementation method of the welfare gain measure. Most of the analysis is based on post-World War II data for a set of 49 countries and a smaller set of 21 OECD countries. We also use an historical data set starting in 1870 in order to compute welfare gains over very long horizons. We rely heavily on the empirical growth literature in choosing a wide range of 21 possible variables in the information set. The findings are discussed in section 4. Conclusions and directions for future research are

⁸For example, an AR(1) in $\Delta ln\ y\ (y=\text{per capita endowment})$, and the same process with a global cointegration term $(ln\ y-ln\ y^W)$ added, lead to the same information set to predict the deviation from world growth: the current and one period lagged $ln\ y-ln\ y^W$, where y^W is the world per capita endowment. But, as discussed above, these processes have totally different welfare implications.

⁹While in the empirical growth literature additional right hand side variables are significant, this should be no surprise since these are usually *contemporaneous* variables. Instead, we use variables that are in the information set at the time predictions about growth are made. For example, it should be no surprise that ex-post knowledge of significant political instability (revolutions and coups) reduces the ex-post prediction of growth. But, especially for a long horizon, knowledge about political instability in the past will have much less predictive power for future growth.

contained in the final section.

2 A Measure of Welfare Gains

Assume there are N countries. Letting γ denote the constant rate of relative risk aversion, expected utility of a representative agent of country i is

$$V = E_0 \sum_{t=1}^{T} e^{-\beta t} \frac{(c_{it})^{1-\gamma}}{1-\gamma}$$
 (1)

where E_0 denotes the expectation at time zero.

We assume an endowment economy, where y_{it} is the per capita country i endowment at time t. The per capita world endowment is $y_t^W = \sum_{i=1}^N \frac{n_{it}y_{it}}{n_t}$, where n_{it} is population in country i and n_t is world population. Before risksharing $c_{it} = y_{it}$. Next we allow for trade in claims on the endowments. ¹⁰ As is shown in van Wincoop [1994], for a specific country the welfare gain from risksharing depends both on the reduction in risk and the price of a claim on its endowment stream relative to that of a claim on the world endowment stream. Relative prices of endowment claims depend on the expected levels of endowments as well as the stochastic properties of the endowments. However, if one for example has thirty years of data, using our approach it is hard to identify the endowment uncertainty of individual countries over a thirty year horizon. There is only one observation per country. We will therefore avoid the pricing problem by focusing on the welfare gain of a 'representative' country, called rep. 11 For this country it is assumed that (i) the price of a claim on its per capita endowment stream relative to that of a claim on the per capita world endowment stream is one, and (ii) the variance of the innovation of endowment growth minus world endowment growth (defined below) is equal to the average over all countries. The first assumption implies that $c_{rep,t} = y_t^W$ after risksharing.

We impose two conditions which ensure that the relative price of a claim on the representative country's per capita endowment stream is one. First, at all horizons rep's expected per capita endowment is equal to the expected per capita world endowment:

¹⁰We assume that all goods are tradable. Optimal risksharing arrangements in the presence of non-traded goods have been analyzed by Lewis [1996b], Tesar[1993, 1995] and van Wincoop [1996b]. van Wincoop [1996b] shows that taking into account non-separability between traded and non-traded goods in utility generally raises the potential welfare gain from risksharing.

¹¹This 'representative country' is an artificial construction, also used in van Wincoop [1996b]. It does not actually have to exist.

 $E_0 \ y_{rep,t} = E_0 \ y_t^W \ (t = 0, ..., T)$. This assumption has the additional advantage that there are no gains from intertemporal asset trade (consumption smoothing), so that we can focus exclusively on the risksharing role of international financial markets. ¹² Second, the covariance between the growth innovation of rep with the world growth innovation is the same as the variance of the world growth innovation. Using terminology usually used in reference to asset returns one might say that 'the beta of country rep's growth rate' is one. Appendix A shows that these two conditions guarantee that the relative price of the claim on rep's endowment stream is one.

The variables in the information set at time t used to predict future country i per capita endowment level are denoted by the vector z_{it} of z country specific variables, and a vector w_t of w global variables. It is assumed that:

$$y_{i,t+s} = y_{it} e^{\lambda_s' z_{it} + \eta_s' w_t + \epsilon_{i,t,t+s}}$$
(2)

where the innovation $\epsilon_{i,t,t+s}$ over the period [t,t+s] has a normal distribution with expectation zero and variance σ_{is}^2 . The variance depends on the country and the horizon, s.

After taking logs, (2) implies

$$ln y_{i,t+s} - ln y_{it} = K_{st} + \lambda_s' z_{it} + \epsilon_{i,t,t+s}$$
(3)

where $K_{st} = \eta'_s w_t$ is constant across countries. Eqn. (3) is familiar from the empirical growth literature if we interpret y as output.

Define $\theta_{i,t,t+s} = \frac{n_{it+s}E_t \ y_{it+s}}{(n_{t+s}E_t \ y_{t+s}^W)}$. It is the expected country i endowment at tme t+s relative to the expected world endowment.¹³ Using these weights to subtract the global average of variables in (3), we get

$$\ln y_{i,t+s} - \ln y_{it} - \sum_{i=1}^{N} \theta_{i,t,t+s} [\ln y_{i,t+s} - \ln y_{it}] =$$

$$\lambda'_{s} [z_{it} - \sum_{i=1}^{N} \theta_{i,t,t+s} z_{it}] + u_{i,t,t+s}$$

$$(4)$$

where $u_{i,t,t+s} = \epsilon_{i,t,t+s} - \epsilon_{t,t+s}^W$, which subtracts the global growth innovation from the country innovation. The global innovation is defined as $\epsilon_{t,t+s}^W = \sum_{i=1}^N \theta_{i,t,t+s} \epsilon_{i,t,t+s}$.

 $^{^{12}}$ We already excluded the resource allocation role of international capital markets by assuming an endowment economy.

¹³We abstract from uncertainty about future population.

As discussed above, for the representative country: $cov(\epsilon_{rep,t,t+s} - \epsilon_{t,t+s}^W, \epsilon_{t,t+s}^W) = 0$. It says that the 'beta of its growth rate', defined as $\frac{cov(\epsilon_{rep,t,t+s},\epsilon_{t,t+s}^W)}{var(\epsilon_{t,t+s}^W)}$, is one. ¹⁴ It follows that $var(u_{rep,t,t+s}) = \sigma_{rep,s}^2 - \sigma_{Ws}^2$, where σ_{Ws}^2 is the variance of the global growth innovation $\epsilon_{t,t+s}^W$. As we will now show, it is this difference between the variance of country specific and global growth innovation that determines the gain from risksharing. Estimation of eqn. (4) in the next section provides us with an estimate of the average variance of $u_{i,t,t+s}$ across countries, which (according to (ii)) gives us $var(u_{rep,t,t+s})$.

From (2), in the absence of risksharing expected utility of the representative country is

$$E_0 \sum_{t=1}^{T} e^{-\beta t} \frac{(y_{rep,t})^{1-\gamma}}{1-\gamma} = \sum_{t=1}^{T} e^{-\beta t} \frac{(E_0 \ y_t^W)^{1-\gamma}}{1-\gamma} e^{-0.5\gamma(1-\gamma)\sigma_{rep,t}^2}$$
(5)

Here we used the assumption E_0 $y_{rep,t} = E_0$ y_t^W

After risksharing

$$c_{rep,t} = y_t^W = (E_0 \ y_t^W) \sum_{i=1}^N \theta_{i,0,t} \ e^{-0.5\sigma_{it}^2 + \epsilon_{i,0,t}}$$
(6)

In Appendix B it is shown, based on numerical simulations, that we can make the following very precise approximation:

$$E_0 \left[\sum_{i=1}^{N} \theta_{i,0,t} \ e^{-0.5\sigma_{it}^2 + \epsilon_{i,0,t}} \right]^{1-\gamma} / (1-\gamma) \approx$$

$$E_0 \left[e^{-0.5\sigma_{Wt}^2 + \sum_{i=1}^{N} \theta_{i,0,t} \ \epsilon_{i,0,t}} \right]^{1-\gamma} / (1-\gamma) = e^{-0.5\gamma(1-\gamma)\sigma_{Wt}^2} / (1-\gamma)$$
(7)

A similar approximation is also made in van Wincoop [1994] and Lewis [1996a]. Per capita world output is a weighted average of log-normally distributed variables (y_{it}) , which is not log-normally distributed itself. (7) amounts to assuming an approximate lognormal distribution for per capita world output. ¹⁵ Expected utility of the representative country after risksharing can then be approximated as

$$E_0 \sum_{t=1}^{T} e^{-\beta t} \frac{(y_t^W)^{1-\gamma}}{1-\gamma} = \sum_{t=1}^{T} e^{-\beta t} \frac{(E_0 \ y_t^W)^{1-\gamma}}{1-\gamma} e^{-0.5\gamma(1-\gamma)\sigma_{Wt}^2}$$
(8)

¹⁴This holds by construction for a weighted average of countries, since $\sum_{i=1}^{N} \theta_{i,t,t+s} cov(\epsilon_{i,t,t+s} - \epsilon_{i,t+s})$

 $[\]epsilon^W_{t,t+s}, \epsilon^W_{t,t+s}) = 0.$ ¹⁵Lewis [1996a], in Appendix A, shows through Monte Carlo experiments that the log of world output is arbitrary close to a normal distribution if output of individual countries is log-normally distributed. After 4000 draws of a 22 year period for 7 countries, she cannot reject that the log of world output has no skewness and the kurtosis is 3, as it should be under a normal distribution.

From (5) and (8) we can derive an expression of the welfare gain for the representative country. As is common in the welfare gains literature, the gain is computed as the permanent percentage increase in the endowment that leads to a welfare gain equivalent to what can be achieved through risksharing:

$$E_0 \sum_{t=1}^{T} e^{-\beta t} \frac{(y_{rep,t}(1+gain))^{1-\gamma}}{1-\gamma} = E_0 \sum_{t=1}^{T} e^{-\beta t} \frac{(y_t^W)^{1-\gamma}}{1-\gamma}$$
(9)

Together with (5) and (8), the welfare gain can then be written as:

$$gain = \left[\sum_{t=1}^{T} \Omega_t \ e^{-0.5\gamma(1-\gamma)(\sigma_{Wt}^2 - \sigma_{rep,t}^2)} \right]^{1/(1-\gamma)} - 1$$
 (10)

The weights Ω_t are defined as

$$\Omega_t = \frac{e^{-\beta t} (\bar{y}_{rep,t})^{1-\gamma}}{\sum_{s=1}^T e^{-\beta s} (\bar{y}_{rep,s})^{1-\gamma}}$$
(11)

where $\bar{y}_{rep,t} = (E_0 \ y_t^W) e^{-0.5\gamma \sigma_{rep,t}^2}$ is the certainty equivalent of rep's endowment at time t.

The weights Ω_t can be simplified by introducing the following notation. Let μ_t be the expected growth rate of the world per capita endowment over [0,t]: $e^{\mu_t} = \frac{E_0 y_t^W}{y_0^W}$. The certainty equivalent of rep's growth rate is $\bar{\mu}_t = \mu_t - 0.5\gamma\sigma_{rep,t}^2$. From the Euler equation, the t period real interest rate on a riskfree bond is $r_t = \beta t + \gamma \bar{\mu}_t$. With this notation,

$$\Omega_t = \frac{e^{-(r_t - \bar{\mu}_t)}}{\sum_{s=1}^T e^{-(r_s - \bar{\mu}_s)}} \tag{12}$$

So the appropriate discount factor is the difference between the riskfree real interest rate and the risk adjusted growth rate.

While we use (10) to compute the welfare gain for the representative country, a somewhat more intuitive expression, which numerically is very close to (10), can be obtained using the approximations $e^x \approx 1 + x$ and $(1+x)^a \approx 1 + ax$ for x close to zero. This approximate welfare gain is

$$0.5\gamma \sum_{t=1}^{T} \frac{e^{-(r_t - \bar{\mu}_t)}}{\sum_{s=1}^{T} e^{-(r_s - \bar{\mu}_s)}} (\sigma_{rep,t}^2 - \sigma_{Wt}^2)$$
(13)

which is equal to the rate of relative risk aversion divided by two, times a weighted average of the drop in the variance of endowment growth, whereby the weights depend on the difference between the riskfree rate and the riskadjusted growth rate at various horizons.

3 Empirical Implementation

In order to implement (10) to get an estimate of the welfare gain for a representative country, we need to (i) select a set of countries and sample period, (ii) choose a measure for the endowment y, (iii) choose a set of variables z in the information set, (iv) estimate the regression (4) for horizons s = 1, ..., T, (v) choose an estimate for the riskfree real interest rate and risk-adjusted growth rate at different horizons, and (vi) choose an estimate for the rate of relative risk aversion. In this section we describe how this is done. The next section discusses the findings.

3.1 The Data

For now we limit our attention to post war data. Section 4.4 considers an extension to a 120 year sample of historical data. The main advantage of a post war sample is that far more data are available, so that we can select a much broader set of variables z in the information set. The disadvantage is that we have to limit ourselves to shorter horizons.

The two main data sets are the Penn World Table (Mark 5.6) and the Barro-Lee [1994] data set. The Penn World Table (PWT) has data for 152 countries from 1950 to 1992, although for most countries data are not available for this entire sample. The Barro-Lee (BL) data set covers 138 countries, at five year intervals over the period 1960 to 1985, although there are many missing data for individual countries. The remainder of this section describes what information from these datasets is used.

In order to select variables to be included in the information set, the empirical growth literature provides a useful guideline. The problem however, as mentioned in Sala-i-Martin [1994], is that in this enormous literature over 50 variables have been found to be correlated with growth in at least one regression. This should not be a surprise since there is significant multi-collinearity among the explanatory variables. Nonetheless this multi-collinearity problem is less of a concern to us than it is to those that attempt to identify which variables affect growth. After all, we are only interested in the residual variance, without being concerned which of the right hand side variables really cause growth. ¹⁶

Assuming that a representative agent is no 'smarter' than the collective of individuals that have contributed to the empirical growth literature, we restrict our information set

¹⁶Moreover, it is quite possible that one of the right hand side variables simply proxies for another, non-observed, variable that truely causes growth.

to the main variables that have emerged from this research. For now our measure of the endowment is per capita GDP, as in the growth literature, although other measures will be considered in section 4.3. Variables in the information set refer to the year in which predictions about future growth are made. In our 'base information set' we include the log of initial per capita GDP (log GDP); the one year and 5 year lagged change in the log of per capita GDP ($\Delta_1 log GDP$ and $\Delta_5 log GDP$); the five year lagged population growth rate (GPO); the ratio of private consumption to GDP (C/Y), the ratio of investment to GDP (I/Y); the government consumption to GDP ratio (G/Y); openness as measured by exports plus imports, divided by GDP $(\frac{X+M}{Y})$; the gross enrollment ratios for primary, secondary and higher education (PRIE, SECE, HIGHE); the fertility rate (FERT), and life expectancy at birth (LIFE). In sensitivity analysis we consider a set of eight additional variables: political instability measured as an average of revolutions and assassinations over the past five years (PINSTAB), terms of trade growth over the past five years (TOT), percentage of primary, secondary and higher school attained (PRIA, SECA, HIGHA), one year and five year lagged private consumption growth $(\Delta_1 c, \Delta_5 c)$ and the investment ratio (I/Y) averaged over the past five years. The last five of the variables from the 'base information set' and the first five of the additional variables are from the BL data set; the others are from the PWT. Since these variables have been extensively discussed in the growth literature we refer to Barro [1991], Barro and Lee [1994], Barro and Sala-i-Martin [1995] and Levine and Renelt [1992] for justifications for including them in growth regressions.

We construct a sample of annual data from 1955 to 1990 for a set of 49 countries, listed in Appendix C. ¹⁷ The countries are chosen such that the PWT data are available for each year of the sample, and the BL data (with the exception of *PINSTAB* and *TOT*) for each of the five year intervals from 1960 to 1985. The education variables, as well as the fertility rate and life expectancy, change very gradually over time. In order to exploit the annual data available from the PWT, we interpolate the BL data in between observations at 1960, 1965, 1970, 1975, 1980 and 1985. Since one can draw almost a straight line between these observations for most countries we believe very little information is lost in doing so. Between 1955 and 1960, and between 1985 and 1990, we extrapolate, assuming the same growth rate as in the neighbouring 5 year interval.

For the two remaining BL variables, political instability and terms of trade growth, interpolation and extrapolation are unwarranted. In order to evaluate the importance

¹⁷Since this includes five year lagged growth rates in 1955, the sample really starts in 1950.

of these variables, for the same 49 countries we consider a smaller sample from 1970 to 1990 in the sensitivity analysis, using only data at five year intervals.

We also consider a smaller set of 21 OECD countries, listed in Appendix C. The OECD countries are of particular interest since their financial markets are well developed and most barriers to international capital flows have been eliminated. It is the lack of diversification among these countries that is one of the main puzzles in international macroeconomics.

3.2 Estimation of the Growth Regression

Estimation of Eqn. (4) is different from standard growth regressions in several ways. First, only variables in the initial information set are used as right hand side variables. In contrast, it is common in the empirical growth literature to use contemporaneous variables. This leads to well known causality problems, which our approach naturally avoids. Second, we subtract global averages from both the right and left hand side of Eqn. (3). In standard growth regressions Eqn. (3) is estimated directly, with K_{st} (which includes the global variables) replaced with a constant term. The problem there is that the innovations $\epsilon_{i,t,t+s}$ are positively correlated across countries, which is ignored in the regressions. Since the innovations are not independent, Mankiw [1995] concludes that statistical significance is overstated and reported standard errors associated with parameter estimates cannot be relied upon. Since we subtract global averages, it is much more reasonable to assume that the innovations $u_{i,t,t+s}$ are uncorrelated across countries. By construction this has to be correct on average.

A final difference is that the growth literature usually only considers one cross section, while instead we use panel data to estimate Eqn. (4). We use all non-overlapping intervals of a given horizon, starting with the most recent observation in the sample. For example, over the sample 1955-1990, the intervals with an 8 year horizon are 1982-1990; 1974-1982; 1966-1974; 1958-1966. By using non-overlapping intervals the innovations should be uncorrelated across these observations. We can therefore estimate (4) for a particular horizon with OLS since the innovations are assumed to be uncorrelated both across time and across countries. A longer horizon implies fewer observations per country and therefore less precision. For an horizon over 17 years there is only one observation, so that we are back at the standard cross section.

A final issue that needs to be discussed is the measurement of $\theta_{i,t,t+s} = \frac{n_{it+s}E_ty_{it+s}}{(n_{t+s}E_ty_{t+s}^W)}$. Since we don't know what the expectations are before estimating Eqn.(4), we follow the

following multi-step procedure. First we estimate Eqn.(4) with equal country weights in order to obtain an estimate of λ_s .¹⁸ This is used to obtain $\hat{\theta}_{i,t,t+s} = \frac{n_{it+s}y_{it}e^{\lambda_s z_{it}}}{(\sum_{j=1}^{N} n_{jt+s}y_{jt}e^{\lambda_s z_{jt}})}$.

19 We then re-estimate Eqn.(4), which yields a new estimate of λ_s , and therefore $\theta_{i,t,t+s}$. It turns out that after three iterations (three estimations of Eqn. (4)) the $\theta's$ remain practically unchanged. The results reported in the next section apply to the fourth estimation of (4).

3.3 Estimates of σ_s^2 , γ , r_t and $\bar{\mu}_t$

The purpose of the regressions is to obtain for each horizon an estimate of the average variance of residual risk over all countries. Let u be the stacked vector of residuals for a given horizon. Denote by obs the total number of observations, which is equal to the number of countries times the observations per country. The latter ranges from one for horizons over 17 years to 35 for the one year horizon. For horizon s we estimate the average variance in the standard way as $\hat{\sigma}_s^2 = (\hat{u}'\hat{u})/(obs - z)$, where z is the number of right hand side variables. If conditional on the variables in the information set, the expected variance for a particular country is equal to the average variance over all countries, $\hat{\sigma}_s^2$ is an unbiased estimate of the average variance.

In general this is not the case however. As an illustration consider a single cross country regression over an horizon s and an information set consisting of one variable. Define $x = (x_1, ..., x_N)'$, where $x_i = z_i - \sum_{j=1}^N \theta_j z_j$. Then it is easy to show that ²⁰

$$E \hat{\sigma}_s^2 = \left[\sum_{i=1}^N \sigma_{is}^2 - \frac{\sum_{i=1}^N x_i^2 \sigma_{is}^2}{sum_{i=1}^N x_i^2} \right] / (N - 1)$$
 (14)

where σ_{is}^2 is the expected variance of the country *i* residual conditional on the information set *x*. If for all countries this expected variance is independent of the information set, and equal to the country average σ_s^2 , then (14) becomes σ_s^2 , so that $\hat{\sigma}_s^2$ is an unbiased estimate of σ_s^2 . However, if the variance is different across countries (14) shows that

¹⁸This is the same as estimating (3) with OLS.

¹⁹The exact expression should be $\theta_{i,t,t+s} = \frac{n_{it+s}y_{it}e^{\lambda_s z_{it}+0.5\sigma_{is}^2}}{(\sum_{j=1}^N n_{jt+s}y_{jt}e^{\lambda_s z_{jt}+0.5\sigma_{js}^2})}$. However, we don't have country specific estimates of the σ_{is}^2 's. Moreover, except at very long horizons differences in the σ_{is}^2 's across countries are unlikely to be large enough to have significant effect on the shares $\theta_{i,t,t+s}$.

²⁰The estimated residual is $\hat{u} = Au$, where $A = I_N - x(x'x)^{-1}x'$. Therefore $E \ \hat{\sigma}^2 = \frac{E \ (\hat{u}'\hat{u}|x)}{(N-1)} = \frac{E \ (u'Au|x)}{(N-1)} = \frac{E \ (u'Au|x)}{(N-1)} = \frac{E \ (u'Au|x)}{(N-1)} = \frac{E \ (u'Au|x)}{(N-1)} = \frac{E \ (u'Au|x)}{(N-1)}$, where Tr is the trace. We can write that out as (14).

less weight is given to the variance of countries that are extreme in the sense that z_i is much larger or smaller than the world average (so that x_i^2 is large). To the extent that countries that are 'extreme' in the above sense have a larger expected variance, $\hat{\sigma}_s^2$ leads to an underestimation of the average expected variance. ²¹ The welfare gains reported below for a representative country should then be considered to be on the conservative side.

Throughout the paper the rate of relative risk-aversion is assumed to be three. This is the average of estimates in Friend and Blume [1975]. We consider it to be a good consensus estimate overall. However, it is easy to obtain the welfare gain for any other rate of relative risk-aversion. From (13) it follows that the welfare gain is approximately proportional to γ . So the gain would be one third of what we report if one believes in log-utility; and three times of what we report if one believes the rate of risk-aversion is nine.

The risk-adjusted growth rate is $\bar{\mu}_t = \mu_t - 0.5\gamma\sigma_t^2$. We find that the annualized risk premium, $0.5\gamma\sigma_t^2/t$, is somewhere between 0.001 (0.1%) and 0.005 (0.5%). A measure for μ_t can be found by taking a simple average growth rate of per capita GDP over the sample. The average annual growth rate from 1950 until 1990 is 2.1% for the 49 countries and 3.0% for OECD countries. The latter is high by historical standards though. For the historical data from 1870 until 1990, discussed in section 4.4, the average growth rate over all 24 countries in that sample is 1.7%. We set the risk-adjusted growth rate at a constant 1.5% annually: $\bar{\mu}_t = 0.015t$.

A measure of the riskfree rate r_t is constructed as follows. Over the period 1889 to 1978 Mehra and Prescott [1985] find an average annual real rate of 0.8% on relatively riskless three month US Treasury securities. In order to obtain an average term structure, we use data from 1947 until 1985 from McCulloch [1990] on the zero-coupon yield curve for Treasury bonds. r_t/t is set equal to 0.008, plus the average difference between the t year yield and the 3-month yield. Since only few observations are available at maturities over 30 years, we assume the yield curve is flat after that. ²² For sensitivity analysis, we will briefly consider a 1% higher and lower annual riskfree rate ($\Delta r_t = \pm 0.01t$).

²¹For example, the poorest countries are extreme in that they have relatively low initial GDP. They might have relatively large uncertainty because there is a large potential upward mobility. When lagged growth is part of the information set it may be reasonable to argue that countries whose past growth has deviated significantly from the world average are on average the more 'risky' ones.

²²In fact, the average yield curve is almost flat after a ten year maturity.

4 Findings

4.1 Regression Results

For three different horizons (1, 10 and 35 years) Tables 1 and 2 show regression results for all 49 countries and the 21 OECD countries respectively. Results are reported for the entire base information set as well as for a much smaller set of the three 'best' variables. These are the ones that lead to the lowest welfare gain at a 35 year horizon. As is well known, growth is hard to predict at very short horizons. For a one year horizon the \bar{R}^2 is 0.06 and 0.19 for the two sets of countries respectively. But it rises to 0.50 and 0.75 for the 35 year horizon. Parameter estimates associated with the entire base information set are imprecise, especially at the 35 year horizon. The only parameter that is significant at all horizons is the log of initial per capita GDP. The imprecision is a result of a high degree of collinearity. Moreover, at the 35 year horizon there is one observation per country, which means that there are only eight degrees of freedom left for the 21 OECD countries.

Parameter estimates are quite precise at all horizons when the information set is limited to the three "best" variables, for which the 35 year welfare gain is smallest. For the set of 49 countries these are the log of initial per capita GDP, the fertility rate and the investment rate. For the OECD countries the investment rate is replaced by enrollment in higher education as the third most important variable.

The fertility rate was also found to be significant in Barro and Sala-i-Martin [1995]. Brander [1992] finds that fertility declines precede rather than lag higher growth rates. One reason it may be important as a predictor of future growth is that it has very direct and unambiguous implications for the future demographic structure of an economy. But there must be more to the fertility rate than demographics alone since it is even significant at the one year horizon. The fertility rate turns out to be a good proxy for a wide variety of variables. The average cross country correlation (over all 35 years of the sample) is 0.86 with population growth, -0.83 with the log of GDP, -0.90 with life expectancy, -0.85 with enrollment in secondary education, and -0.66 with the investment rate.

At a theoretical level the important role of the investment rate is not surprising. At an empirical level Levine and Renelt [1992] find the (contemporaneous) investment rate to be the single most important variable in growth regressions, with its significance being robust to other variables added to the regression. They did not include fertility in their

study. It is not surprising that for the OECD countries enrollment in higher education is an even more important predictor of future growth. Growth in industrialized countries is obviously strongly associated with development of new technologies, which to a great extent is a product of investment in higher education.

Tables 1 and 2 also report the standard deviation of the residual term. Even though a significant part of growth is accounted for by variables in the information set, the standard deviation of remaining risk is nonetheless quite large. At the 35 year horizon, for the base information set, it is 0.33 for the 49 countries and 0.16 for the OECD countries.

In order to get a better sense of the amount of uncertainty involved here, we perform a Monte Carlo experiment, taking 10,000 draws from the distribution of the residual for each country. It is assumed that the draws are independent across countries and that each country's standard deviation of residual risk is the same.²³ For the 49 countries the probabilities that, purely by surprise, per capita GDP of the best performing country doubles, triples, quadruples or quintuples relative to that of the worst performing country over 35 years are, respectively 99.99%, 99.99%, 81% and 41%. So it is likely that, controlling for what had already been expected, per capita GDP of the best performing country rises by a factor 5 relative to the worst performing country. For the OECD the probabilities that per capita GDP of the best performing country rises by 30%, 50%, 70%, and 100% relative to that of the worst performing country over 35 years are, respectively, 99.99%, 99.9%, 61%, and 13%. While less spectacular, these numbers are still significant. It is likely that the best performer's per capita income rises by 70% relative to the worst performer over a period of 35 years.

Figures 1 and 2 show the standard deviation of residual risk as a function of all horizons up to 35 years. In order to illustrate the predictive power of a few variables, the results are also shown when (i) the information set is empty (all deviations from 'world' per capita GDP growth are considered a shock), ²⁴ (ii) only the log of initial GDP is in the information set, (iii-iv) the information set consists of the two or three variables leading to the lowest welfare gain at a 35 year horizon.

Consistent with the findings by Barro and Sala-i-Martin [1995], for the larger group of countries initial GDP by itself does not have much predictive power (the convergence

²³The results are even stronger when the standard deviations of residual risk differ across countries.

²⁴Since in that case we have no information set to form expectations concerning the weights θ , these weights are based on the expectation from the entire base information set. In all other cases the corresponding information set is used to form expectations about the weights.

coefficient has the wrong sign and is often insignificant), while for OECD countries there is strong convergence. At a 35 year horizon the standard deviation of risidual risk for OECD countries drops significantly from 0.33 to 0.22 when only the log of initial per capita GDP enters the information set. Barro [1991] first found that even for a large set of 98 countries there is strong evidence of *conditional* convergence, in that initial GDP enters with a significantly negative sign once we control for some other variables. Consistent with that, Table 1 shows that even for the larger set of 49 countries initial GDP is strongly significant once we control for either two or twelve additional variables.

We can see from Figures 1 and 2 that almost all of the explanatory power comes from three variables. At an horizon up to 18 years the estimated standard deviation is practically identical when two, three or thirteen variables are included. Only for longer horizons does the third variable have some additional explanatory power over the first two variables. The Figures also show that while the estimated standard deviation rises smoothly as a function of the horizon when only two or three variables are included, this curve becomes quite bumpy for the entire information set, particularly for OECD countries. This results from the few degrees of freedom remaining beyond a 17 year horizon for OECD countries. ²⁵ On average though, even for the long horizons, the estimated standard deviation for OECD countries based on the entire information set is approximately the same as that for the three best variables.

For illustrative purposes Figure 3 shows pictures for both sets of countries of the actual and predicted deviation from world growth from 1955 to 1990. The pictures are based on the best three variables in the information set. The advantage of showing the pictures with three rather than 13 variables in the information set is that few degrees of freedom are wasted, so that the variance in the pictures corresponds closely to the estimated variance. It should be stressed that these pictures are illustrative. Adding one more variable can affect the position of a particular country in the graph, even though it hardly affects the overall estimate of the variance.

If we take Figure 3 seriously, the OECD countries with the largest positive deviation from expected growth are Japan and Canada. While the growth rate of Japan is 100% above the total OECD growth rate, 34% of that is unexpected. The Canadian growth rate is 5% less than the OECD average, but was expected to be almost 38% less. ²⁶

²⁵Since $\hat{\sigma}_s^2 = \frac{\hat{u}'\hat{u}}{(obs-z)}$, where obs is the number of observations and z is the number of right hand side variables, it is clear that the estimate becomes unstable when both the numerator and denominator approach zero as $z \to obs$.

²⁶The low expected Canadian growth is to a great extent the result of the highest fertility rate in the

Some of the worst performers relative to expectation, with growth almost 20% below the predicted level, are Greece, New Zealand and the UK. Japan grew about 80% more than Greece, even though in 1955 their expected growth rates were very similar.

While our sample ends in 1990, a very recent big growth surprise comes from Ireland. Ireland's economy stagnated during the first half of the 1980s. In 1987 its per capita GDP was 63% of Britain's. But only nine years later, in 1996, Ireland's per capita GDP surpassed Britain's. The economy expanded by 10% in 1995, and 7% in 1996. From the point of view of expectations around the mid-1980s, this remarkable growth episode was clearly a surprise. Even ex-post it is not completely clear what set off this spectacular peformance. ²⁷

For the larger set of 49 countries, some of the best performers relative to expectation are Japan, Thailand and Mexico. Notice that this does not include a lot of the 'Asian miracles', such as Hong Kong, Singapore, South Korea and Taiwan. These countries are not in the sample due to incomplete data. But it is clear that they would have pushed the estimated variance up even further. Many of the worst performers compared to expectation are African and South American countries. Uruguay was expected to grow slightly more than Thailand in 1955, but Thailand's per capita GDP grew by a factor of 5.1 relative to that of Uruguay.

These results might leave the wrong impression that primarily African, South American and East Asian countries are subject to large income shocks. While those countries experienced the most dramatic changes in per capita GDP during the past several decades, what matters for risksharing today is uncertainty about *future* income. It is quite possible that over the next fifty years the biggest income surprises come from other parts of the world. Large gains from risksharing are therefore not necessarily limited to the set of countries that have faced the biggest shocks in the recent past.

OECD. As discussed above, the fertility rate is closely associated with many important macroeconomic variables. Canada was expected to perform significantly below average based on a wide range of such indicators. Compared to the OECD average, in 1955 it had a higher per capita GDP, lower growth in the past one and five years, a higher population growth rate, a higher consumption rate and lower enrollment in secondary and higher education.

²⁷See The Economist, May 17-23, 1997, pp. 21-24, for a discussion of the recent Irish growth performance.

4.2 Welfare Gains

For the same information sets as in Figures 1 and 2, Figures 4 and 5 show the welfare gain for a representative country as a function of the horizon. Since the gain is based on a weighted average of the variance of residual risk at various horizons, the welfare gain curves are smoother than those for the standard deviations. At a 10, 20 and 35 year horizon welfare gains for these and other information sets are also reported in Table 3.

Using the base information set, at a 35 year horizon the welfare gain is an enormous 6.61% for the 49 countries and a still very large 1.52% for OECD countries. In present discounted value terms this amounts to an increase in resources that is 221% of current output for the 49 countries, and 51% of current output for the OECD countries. These gains are also large when comparing this to the size of the US securities industry, which averaged to only 0.6% of GDP in the 1980s. ²⁸ So the potential costs of risksharing must be far below potential benefits.

For the 49 countries the gains are large even for very short horizons, 2.25% for a 10 year horizon. The gain of 0.45% for OECD countries at a 10 year horizon is relatively small though. At horizons longer than 35 years the welfare gain is of course even larger. It is hard to know how long the horizon of a representative investor might be. In theory it could even be infinite if generations are always connected through operative intergenerational transfers. A conservative estimate for longer horizons can be obtained by assuming that the standard deviation of residual risk does not rise further beyond a 35 year horizon. For a 50 year horizon the welfare gain is then 9.55% and 2.27% for the two sets of countries. For a 100 year horizon these numbers are 13.26% and 3.15%.

29 It is hard to ignore the enormous size of these potential welfare benefits. It certainly does not rationalize the lack of risksharing observed in the data.

If we raise or lower the riskfree rate by 1% annually the results do not change dramatically. For example, for the set of 49 countries the welfare gain at the 35 year horizon drops from 6.61% to 6.24% when $\Delta r_t = 0.01t$, and rises to 6.98% when $\Delta r_t = -0.01t$.

From Figures 4 and 5 we see that for most horizons the welfare gain based on the three best variables is very similar to that based on all 13 variables. This is an important

²⁸See van Wincoop [1994], pp. 187.

²⁹While our method does not rely on an assumed underlying data generating process, it is worth noting that gains of this magnitude for OECD countries are somewhere in between those reported in van Wincoop [1996b] for a random walk process and an AR in growth rates. There the same rate of relative risk aversion was assumed, while the risk-adjusted growth rate and riskfree interest rate are of similar magnitude as in this paper.

finding, which is obviously closely related to that for the standard deviations in Figures 1 and 2. It shows that once the three most important variables are included in the information set, the welfare gain results are robust to adding additional variables. This stands in sharp contrast to computing welfare gains based on endowment processes, where results are very sensitive to changes in the type of process and small parameter changes of a given process.

Table 3 reports results from adding some additional variables beyond the base information set. In row 7 we add the average investment rate over the past five years, which is less sensitive to the up and downs of a particular year. It has practically no effect on the results. In row 8 we add the percentage of primary, secondary and higher school attained. These can be considered stock variables, as opposed to the enrollment variables, which represent flows. Again, adding these variables does not significantly affect on the results. Finally, we add the lagged consumption growth rates over the past one and five years. Since consumption should be based on forward looking behavior, these might be important. But again there are no notable effects on the welfare gain measure.

For the shorter period 1970-1990, without interpolated data, we also consider adding political instability (number of revolutions and assasinations) and terms of trade growth, both over the past five years. These variables also have very little effect on the estimated welfare gain. For the 49 countries, the three 'best variables' (leading to the lowest welfare gain over a twenty year horizon) are now initial GDP, the fertility rate and the terms of trade growth. The latter replaces the investment rate. For that information set the welfare gain is 3.72% over a 20 year horizon. However, the welfare gain is an almost identical 3.75% when the information set consists of the 'old top three' (initial GDP, fertility rate and investment rate). When terms of trade growth is added to 'the old top three', the welfare gain drops negligibly to 3.62%. Terms of trade growth has significant predictive power for short horizons, but is insignificant for the twenty year horizon. Since long term growth uncertainty is the primary source of welfare gains, it is likely that the terms of trade plays even less of a role at horizons longer than 20 years.

Evans and Karras [1996] suggest that the parameters of a given output process may be different across countries and that this could significantly affect convergence estimates.

³⁰Easterly, et al. [1993], Fisher [1993] and Mendoza [1994] all find that terms of trade growth has significant predictive power, but they don't include the investment rate in their growth regressions. This is consistent with our findings. Fisher [1993] and Mendoza [1994] also find that the terms of trade is a much better predictor at short than long horizons.

We have assumed that the vector λ_s , which measures the impact of the variables in the information set on growth expectations, is equal across countries. For long horizons we are unable to estimate λ_s separately for each country since we have only one observation per country. However, differences across countries are related to more fundamental economic differences that should be captured in the information set itself. In order to allow for differences across countries in the relationship between specific variables and growth expectations, we experiment with adding to the regressions various interaction terms involving the three best variables. Our welfare measure is not significantly affected however. ³¹

We also experiment with different functional forms, adding the log of initial endowment squared, or using the log of fertility and investment rates instead of the levels. These experiments also don't have a substantial effect on the welfare gain results.

So far we have ignored potential measurement error in our data. DeLong [1988] shows that the convergence parameter can be significantly affected when there is measurement error in initial income. Measurement error is likely to be more severe for data in 1955 than in 1990. We ask ourselves what happens to the estimate of σ_{35} when we add additional measurement error to the log of 1955 per capita output of each country. This leads to additional random variation of 1955 per capita output around the true level. Assuming that this error has a standard deviation of 0.05, a 95 % confidence interval of the additional random variation is 20% of measured per capita GDP, which is quite significant. We compute $\hat{\sigma}_{35}$ 500 times, each time using different randomly perturbed 1955 output data. When the information set consists of the best three variables, the average $\hat{\sigma}_{35}$ we obtain is 34.77% for the 49 countries. This is only slightly larger than the standard deviation of 34.59% of residual risk based on measured 1955 GDP data. A similar result applies to OECD countries. So long term growth uncertainty, which matters most for the welfare gain measure, is not affected significantly by introducing a reasonable degree of measurement error.

4.3 Alternative Endowment Measures

Table 4 reports welfare gains for two alternative endowment measures, corresponding to two potential problems associated with per capita GDP. The first is best illustrated by writing per capita GDP as pr * e, where pr is productivity per worker and e is per capita

³¹If we add to the three best variables all possible interaction terms, the welfare gain for the 49 countries, based on a 35 year horizon, drops only slightly from 7.35% to 7.27%.

effort. Effort is defined as total hours worked. To some extent individual effort is not really a control variable for that individual. Examples are involuntary unemployment, or national customs with respect to female participation in the labor force, the work week and vacation time. If effort were completely determined by national factors, not under the control of individual agents, and we also ignore the utility cost of effort, the correct endowment measure for this paper is per capita GDP. At the other extreme, one may consider effort to be entirely an individual specific choice variable. In that case it is optimal to engage in risksharing with respect to national productivity pr, which is then the correct endowment measure. While it would be even better to have data on GDP per hour worked, data on hours per week are often not available.

A second problem with per capita GDP as an endowment measure is that it includes both investment and consumption goods. As an alternative we therefore consider per capita private consumption. The issue of investment brings up two questions, one associated with the distinction between production and endowment economies, another with moral hazard. Most of the literature on welfare gains from international risksharing considers endowment economies, as does this paper. This ignores potential gains that are specific to actual production economies. It is possible that there are additional gains from trade in risky productive technologies if it leads to a gradual shift over time towards capital in relatively low risks, or high expected return, countries. ³³ We focus exclusively on the risksharing role of asset trade, even though this is hard to separate from the resource allocation role in actual production economies.

The potential moral hazard problem associated with investment is easily illustrated with the Japanese example. If the OECD countries had engaged in perfect risksharing during our sample, Japan would have had to make significant transfers to the rest of the world because of its unexpected high growth. One might argue that this is unreasonable because they have themselves borne the costs of this growth through high investment in

 $^{3^2}$ Individual effort should be broadly interpreted. We implicitly assume that all individuals have access to the same 'production' with productivity pr. If one individual is less productive than another, it is attributed to less 'effort'.

³³Obstfeld [1994a] considers a world with trade in risky capital, whereby he makes the extreme assumption that these productive assets are in infinite supply. This leads to enormous welfare gains, ranging from 22.6% for East Asia to 478.4% for non-East Asia, since agents can shift their resources to the highest expected return, lowest risk assets. While the assumption that these assets are in infinite supply is rather extreme, it does illustrate a potentially significant additional benefit associated with optimal resource allocation.

machinary, equipment, infrastructure, and human capital. This appears to be a moral hazard problem since it would not be optimal for Japan to have made this investment if it were engaged in perfect risksharing. It is important to realize however that for one individual there is no moral hazard problem associated with aggregate investment in either physical or human capital since the individual does not control the behavior of others. Moral hazard does arise at the government level to the extent that government policy affects long term growth. However, Shiller [1993] argues that governmental moral hazard is present even in existing financial markets and has not prevented them from functioning. ³⁴ Governments also have a strong incentive to cooperate after risksharing in order to stimulate growth at a global level, and can impose heavy penalties on a government that cheats.

Table 4 reports the results for all three endowment measures. For the set of 49 countries the welfare gain based on the 35 year horizon drops a bit from 6.61% to 5.81% when output per worker is used as the endowment. But it rises to 7.26% when the endowment is per capita consumption. This is based on all 13 variables, but a similar result applies to the three best variables (1, 6, and 12) and also to shorter horizons. For OECD countries, using GDP per worker as the endowment instead of per capita GDP has practically no effect on the welfare gain measure for the 10 and 20 year horizons. However, using all 13 variables the welfare gain drops from 1.52% to 1.02% for the 35 year horizon. This result is unreliable though due to the few remaining degrees of freedom. For the three best variables (1, 11, 12) the welfare gain drops only slightly from 1.47% to 1.35%. For this same information set, replacing GDP with private consumption as the endowment measure leads to a slightly higher welfare gains at the 10 year horizon, but somewhat lower at the 35 year horizon. Overall we can conclude that the benchmark endowment measure (per capita GDP) does not provide a significantly biased (either upward or downward) estimate of welfare gains in comparison with the alternative endowment measures. The fact that welfare gains are not systematically lower when using consumption data also indicates that the degree of risksharing already achieved is very limited.

³⁴Shiller also notes that governments can be monitored better than individuals and that explicit contract provisions can rule out settlement based on obvious negative policies. Government policy can of course significantly affect both aggregate and private consumption. For moral hazard reasons trade in claims on GDP might therefore be more easily achievable than trade in a consumption based asset.

4.4 Historical Data

We now apply the methodology to historical data from 1870 to 1990. This allows us to compute welfare gains over longer horizons, but has the disadvantage that the information set is limited to a much smaller set of variables for which historical data are available. We include both initial GDP and the fertility rate in the information set. The results for post war data indicate that most of the explanatory power comes from these two variables. We also experiment with including lagged growth rates of GDP and population in the information set.

GDP and population data are from Maddison [1995]. Fertility rates come from various sources, listed in Appendix D, and are available for 24 countries. ³⁵ GDP data are not available for every year. We only use years for which data are available for all countries in the sample. For all 24 countries GDP data are available in 1870, 1900, 1913, 1929, 1938, and 1947-1990. For smaller sets of countries it is available for more years. Table 5 reports the countries and their per capita GDP (in 1990 Geary-Khamis dollars) at different points in time. ³⁶

One has to be careful in choosing the set of countries to be included in the sample. DeLong [1988] criticized Baumol's [1986] cross sectional growth regressions as being biased towards convergence since the sample includes only countries that have ex-post converged. Instead, DeLong [1988] chooses a set of 22 countries based on a relatively high initial (rather than ex-post) income. ³⁷ We will consider various sets of countries, dependent on the cutoff for 1870 GDP. This ranges from the richest eight in 1870 to the entire set of all 24 countries. Baumol, Blackman and Wolff [1989] show that the cutoff point is important for convergence. They consider the top 8 through the top 14 based on 1870 GDP and find stronger convergence among the richest countries.

Figure 6 shows the welfare gain for a 120 year horizon as a function of the number

³⁵The fertility rate is defined as the crude birth rate. The fertility rate from the Barro Lee dataset that we used for the post-war data is defined as the expected number of babies during the lifetime of a woman, which is based on current information on the number of babies women have at various ages. However, we find that for the post-war period the average time series correlation between these two series (over 49 countries) is 0.997, and the 1960 cross section correlation is 0.9898. We therefore find that our welfare gain results for the post-war period remain substantially unaltered when we use the crude birth rate as the definition of the fertility rate.

³⁶GDP in Geary-Khamis dollars refers to a purchasing power parity based GDP measure, using international average prices of commodities, just as in the Penn World Tables. There is a nice discussion of the Geary-Khamis approach in Maddison [1995], pp. 162-165.

³⁷All countries with 1870 GDP above 300 1975 dollars are included.

of countries in the sample, including successively poorer countries based on 1870 GDP. For the richest eight the welfare gain is 4.88%. Only after inluding the 17th country, Argentina, does the gain rise significantly to 7.75%. Argentina's performance has been far below average during the sample. The other break occurs after including the last country, India. The welfare gain then rises from 9.3% to 16.5%. As shown in Table 5, India's per capita GDP only slightly more than doubled from 1870 to 1990, while for the average country it increased by a factor of 9.

These gains are very substantial, even for the smallest set of rich countries. For the richest 8 countries in 1870, growth performance has varied widely. As shown in Table 5, Australia was 75% richer than Switzerland in 1870, but was 32% poorer than Switzerland in 1990. Similarly, New Zealand was 27% richer than the US in 1870, but was 56% poorer than the US in 1990. Within Europe, the UK was 71% richer than Germany in 1870, but was 15% poorer than Germany in 1990. While these different growth rates were to a limited extent predictable in 1870, the potential welfare gain of 4.88% shows that there is substantial residual risk. We believe that this is indicative of the current OECD countries. Up to the 35 year horizon the estimated standard deviation of residual risk is about the same as that reported for the 21 OECD countries in section 4.2. After that it rises somewhat more, from 0.15 at the 35 year horizon to 0.20 at the 120 year horizon. The welfare gain of 17% for the set of 24 countries is indicative of the gain from risksharing among a broad set of countries. For the 35 year horizon the estimated standard deviation of residual risk is 0.30, similar to that for the set of 49 countries based on post war data. It then rises to 0.40 at the 120 year horizon.

Adding lagged growth rates of per capita GDP and population to the information set has little effect on the results. For a set of 22 countries ³⁸ the welfare gain for a 90 year horizon is 13.54% when the information set consists of the fertility rate and initial per capita GDP, and drops only marginally to 12.96% when 20 year lagged endowment and population growth are added. We find that the welfare gain estimate even rises a bit when 10 or 30 year lagged growth rates are included in the information set.

Since prewar output measurement error is potentially severe, we perform a similar measurement error experiment as at the end of section 4.2. This time we consider what happens when we introduce an additional random disturbance to the log of 1870 per capita GDP with a standard error of 15%. This means that the 95 % confidence interval of the additional random variation is an enormous 60% of measured 1870 per capita

³⁸We take out Switzerland and Hungary, for which data in 1890 are not available

GDP. Nonetheless we find that our average $\hat{\sigma}_{120}$ for the 24 countries, based on 500 different draws of perturbed 1870 per capita GDP in each country, is 41.50%, which is only a little bit higher than the estimate of 40.15% based on measured data. So it is unlikely that our welfare gain measure is significantly affected by pre-war measurement error in GDP.

5 Conclusions

We have developed a new methodology for measuring potential welfare gains from international risksharing, one that is closely connected to the empirical growth literature. In contrast to the latter, we have focused on growth uncertainty rather than factors that might explain growth ex-post. Thus, our regressions are not subject to the standard causality problems of the empirical growth literature. Our results are robust to the size of the information set in that adding additional variables beyond the two or three most important ones (initial GDP, fertility rate and investment) does not change the welfare measure substantially. In contrast, previous work on international risksharing has relied on specific assumptions about the endowment process and found the results to be extremely sensitive to the particular process assumed. Here we have taken a much more direct route in measuring what ultimately matters for risksharing, the variance of residual risk at various horizons.

Using a variety of sources for both post-war and historical data, we conclude that potential welfare gains from global risksharing are very large. For a 35 year horizon, using post war data, the gain corresponds to an equivalent permanent increase in consumption of 6.6% when based on a set of 49 countries, and 1.5% when based on 21 OECD countries. Using historical data from 1870 to 1990, we find that the potential gain for a 120 year horizon ranges from 4.9% for a small set of rich countries to 16.5% for a broad set of 24 countries.

There are three important directions for future research. One question that we have not addressed in this paper is how large potential gains from risksharing are among regions within a country. Since macro markets do not currently exist even within nations, there might be a significant potential to share risk for example among the US states. ³⁹ A second and closely related question that needs to be addressed is whether one really

³⁹van Wincoop [1995] shows that for Japanese prefectures cross region consumption correlations are just as low as cross country consumption correlations and explains this in a model where asset trade among regions is limited to riskfree bonds and equity (no macro markets).

needs macro markets (claims on GDP) in order to achieve risksharing. If for example the return of claims on corporate dividends (stock) is highly correlated with the return of claims on GDP over long periods of time, most of the potential welfare gains can be achieved through the existing stock markets. Finally, even though the focus of this paper is on risksharing, the methodology can also be applied to compute potential gains from consumption smoothing through intertemporal asset trade (a riskfree bond). To that end all we need to know is the expected deviation from world consumption growth at different horizons. This is a byproduct of our analysis.

Appendix A

We will prove that the price of a claim on rep's per capita endowment stream relative to the price of a claim on the world per capita endowment stream is 1 when (i) $E_0 y_{rep,t} = E_0 y_t^W$, (ii) $cov(\epsilon_{rep,0,t}, \epsilon_{0,t}^W) = var(\epsilon_{0,t}^W)$ for t = 1, ..., T. To do this, all we need to show is that for any horizon t the relative price of a claim on rep's per capita endowment stream is one. There are n_{it} claims on the country i per capita endowment in period t, each with a payoff of y_{it} and a price of p_i . Consider an investor in any country, who at time 0 invests a total of Y_t in period t claims ($\sum_{t=1}^T Y_t$ is the investor's period 0 revenue from selling all claims on its own endowment). The investor maximizes $E_0 \left(\sum_{i=1}^N q_i y_{it}\right)^{1-\gamma}/(1-\gamma)$, s.t. $\sum_{i=1}^N q_i p_i = Y_t$. Here q_i is the quantity of country i equity purchased. The first order conditions with respect to the q_i 's are:

$$E_0 \left(\sum_{i=1}^N q_i y_{it}
ight)^{-\gamma} y_{jt} =
u p_j \quad j=1,..,N$$

The price of a claim on the per capita world endowment is $\sum_{i=1}^{N} \frac{p_i n_{it}}{n_t}$. Using the first order conditions, and the fact that in equilibrium $\frac{q_i}{\sum_{j=1}^{N} q_j} = \frac{n_{it}}{n_t}$, the relative price of a claim on rep's per capita endowment is:

$$\frac{p_{rep}}{\sum_{i=1}^{N} p_{i} n_{it} / n_{t}} = \frac{E_{0} (y_{t}^{W})^{-\gamma} y_{rep,t}}{E_{0} (y_{t}^{W})^{1-\gamma}}$$

 $y_{rep,t}$ has a log-normal distribution, and so does the per capita endowment for every other country. While the distribution of y_t^W is not exactly log-normal, Appendix B shows that it is very close to a log-normal distribution of the form $y_t^W = (E_0 \ y_t^W) e^{-0.5\sigma_W^2 + \epsilon_{0,t}^W}$. Using this approximation and $y_{rep,t} = (E_0 \ y_{rep,t}) e^{-0.5var(\epsilon_{rep,0,t}) + \epsilon_{rep,0,t}}$. Substituting these expressions for y_t^W and $y_{rep,t}$ into to formula for the relative price, it follows immediately from the assumptions (i) and (ii) that the relative price is one.

Appendix B Numerical approximation in eqn. (7)

In order to show that the numerical approximation in (7) is very close we draw $\epsilon = (\epsilon_{1,0,t},...,\epsilon_{N,0,t})'$ from a normal distribution with mean zero and variance Σ . We don't have any direct measure of ϵ , as the analysis focuses on the uncertainty about the deviation from world growth, captured by the u's. We know that $var(\epsilon_{rep,0,t}) = var(u_{rep,0,t}) + var(\epsilon_{0,t}^W)$. In order to allow for large uncertainty about global shocks, for this exercise we assume $var(\epsilon_{0,t}^W) = var(u_{rep,0,t})$, so that $var(\epsilon_{rep,0,t}) = 2var(u_{rep,0,t})$. We moreover assume that $var(\epsilon_{i,0,t})$ is the same for all countries, which appears on the diagonal of

 Σ . We consider both a set of 21 countries with $var(u_{rep,0,t}) = 0.35^2$ (the upper bound for OECD countries at the 35 year horizon-see Figure 2) and a set of 49 countries with $var(u_{rep,0,t}) = 0.45^2$ (upper bound 35 year horizon Figure 1). In order to introduce as much risk as possible these numbers are based on the case of empty information sets, which lead to the largest possible estimates of $var(u_{rep,0,t})$. The weights $\theta_{i,0,t}$ are set based on 1990 GDP shares. We assume the same correlation of the $\epsilon_{i,0,t}$'s across sets of countries. This correlation is set such that $\theta'\Sigma\theta = var(\epsilon_{0,t}^W) = var(u_{rep,0,t})$.

We take 100,000 draws of ϵ from the $N(0,\Sigma)$ distribution in order to compute $a1=E_0$ $[\sum_{i=1}^N \theta_{i,0,t} \ e^{-0.5\sigma_{it}^2+\epsilon_{i,0,t}}]^{1-\gamma}$ and $a2=E_0$ $[e^{-0.5\sigma_{Wt}^2+\sum_{i=1}^N \theta_{i,0,t} \epsilon_{i,0,t}}]^{1-\gamma}$. We set $\gamma=3$. For the 49 countries these numbers are a1=1.83655 and a2=1.83605. More meaningful is the corresponding certainty equivalent of $y_t^W/E_0y_t^W=a1^{1/(1-\gamma)}$. The approximation used in the paper is $a2^{1/(1-\gamma)}=0.7379$, which is very close to the true value $a1^{1/(1-\gamma)}=0.7380$. For the OECD countries we have $a2^{1/(1-\gamma)}=0.8314$ and $a1^{1/(1-\gamma)}=0.8318$. These differences are even smaller for more realistic lower values of $var(\epsilon_{0,t}^W)$ and for shorter horizons.

Appendix C Countries post war period

49 Countries: Kenya, Mauritius, Uganda, Canada, Costa Rica, Dominican Republic, El Salvador, Guatemala, Honduras, Mexico, Nicaragua, Panama, Trinidad & Tobago, US, Argentina, Bolivia, Brazil, Chile, Colombia, Ecuador, Paraguay, Peru, Uruguay, Venezuela, India, Japan, Pakistan, Philippines, Sri Lanka, Thailand, Austria, Belgium, Cyprus, Denmark, Finland, France, Germany, Greece, Ireland, Italy, Netherlands, Norway, Portugal, Spain, Sweden, Turkey, UK, Australia, New Zealand.

21 OECD Countries: Canada, US, Japan, Austria, Belgium, Denmark, Finland, France, Germany, Greece, Ireland, Italy, Netherlands, Norway, Portugal, Spain, Sweden, Turkey, UK, Australia, New Zealand.

Appendix D Fertility Data Historical Sample

We use a wide variety of sources to construct an historical sample of fertility rates, from 1870 to 1990, for 24 countries. Most of the sources come from Easterlin [1996]. Sometimes, such as for the US, yearly observations going back to 1870 are available. Other times the data are only available at 5 or 10 year intervals. In that case we interpolate between the observations. Often pre-war fertility rates are reported as averages over 5 or ten year intervals. Since these rates are quite smooth over time, we then assume that

the fertility rate is the same for all years in the interval. For some countries the first observation is somewhat after 1870. In that case we extrapolate the earliest observations back to 1870. The sources we use are:

All countries, 1951-1990: United Nations, 1993, World Population Prospects.

Austria, Belgium, Denmark, Finland, France, Hungary, Ireland, Italy, Netherlands, Norway, Spain, Sweden, Switzerland and the United Kingdom, 1861-1950: Glass, D.V. and E. Grebenik, 1965, World Population, 1800-1950, in H.J. Habakkuk and M. Postan (eds.), the Cambridge Economic History of Europe: Volume IV, Cambridge: Cambridge University Press.

Argentina, 1870-1949: Collver, A.O., 1965, Birth Rates in Latin America: New Estimates of historical Trends and Fluctuations, University of California, Berkeley.

Australia, 1881-1947: Chandrasekhar, S., 1967, Asia's Population Problems with a Discussion of Population and Immigration in Australia, Frederick A. Praeger Publishers.

Canada, 1866-1950: Statistics Canada, 1993, Selected Birth and Fertility Statistics, Canada, Although the data before 1921 are unpublished, we obtained them directly from the Research Department.

Germany, 1861-1939: Knodel, J.E., 1974, The Decline of Fertility in Germany, 1871-1939, Princeton University Press.

India and Pakistan, 1881-1941: Coale, A.J. and E.M. Hoover, 1958, Population Growth and Economic Development in Low-Income Countries, Princeton University Press.

Japan, 1875-1925: Mosk, C., 1983, Patriarchy and Fertility: Japan and Sweden, 1880-1960, Academic Press.

New Zealand, 1888-1938: New Zealand Five Million Club, Birth Rate Committee, 1939, After the First 100 Years; Causes and Consequences of a Declining Population.

Portugal, 1871-1950: Bacci, M.L., 1971, A Century of Portuguese Fertility, Princeton University Press.

United States, 1870-1949: Coale, A.J. and M. Zelnik, 1963, New Estimates of Fertility and Population in the United States, Princeton University Press.

USSR, 1926-1940: Lewis, R.A., R.H. Rowland and R.S. Clem, 1976, Nationality and Population Change in Russia and the USSR: An Evaluation of Census Data, 1897-1970, Praeger Publishers; 1870-1914: Pisarev, I., 1962, The Population of the U.S.S.R.: A Socio-Economic Survey, Progress Publishers.

References

Athanasoulis, S.G., 1996, Optimal Securities Design for International Risk Sharing, working paper, Iaowa State University.

Backus, D.K., Kehoe, P.J. and F.E. Kydland, 1992, International Real Business Cycles. *Journal of Political Economy*, 100, 745-775.

Barro, R.J., 1991, Economic Growth in a Cross Section of Countries, *Quarterly Journal of Economics* 61 (2), 407-444.

Barro, R.J. and X. Sala-I-Martin, 1995, Economic Growth, McGraw-Hill, New York.

Barro, R.J. and J. Lee, 1994, Sources of Economic Growth, Carnegie-Rochester Conference Series on Public Policy 40, 1-46.

Baumol, W.J., 1986, Productivity Growth, Convergence, and Welfare: What the Long-Run Data Show, *American Economic Review*, 76(51), 1072-1085.

Baumol, W.J. and E.N. Wolff, 1988, Productivity growth, convergence and welfare: Reply, *American Economic Review*, 78(5), 1155-1159.

Baumol, W.J., S.A. Blackman and E.N. Wolff, 1989, Productivity and American Leadership: The Long View, MIT Press, Cambridge.

Brander, J.A., 1992, Comparative Economic Growth: Evidence and Interpretation, Canadian Journal of Economics 25(4), 792-816.

Canova, F. and M.O Ravn, 1993, International Consumption Risksharing, working paper, Universitat Pompeu Fabra, Barcelona.

Cole, H.L. and M. Obstfeld, 1991, Commodity Trade and International Risksharing: How much do Financial Markets Matter? *Journal of Monetary Economics* 28, 3-24.

DeLong, J.B., 1988, Productivity Growth, and Welfare: Comment, American Economic Review, 78(5), 1138-1154.

Easterlin, R.A., 1996, Growth Triumphant: The 21st Century in Historical Perspective, University of Michigan Press, Ann Arbor.

Easterly, W., M. Kremer, L. Pritchett and L.H. Summers, 1993, Good Policy or Good Luck? Country Growth Performance and Temporary Shocks, *Journal of Monetary Eco*-

nomics 32, no. 3, 459-484.

Evans, P. and G. Karras, 1996, Convergence Revisited, *Journal of Monetary Economics* 37 (2), 249-265.

Fisher, S., 1993, The Role of Macroeconomic Factors in Growth, *Journal of Monetary Economics* 32, no. 3, 485-512.

French, K.R. and J.M. Poterba, 1991, Investor Diversification and International Equity Markets. *American Economic Review, Papers and Proceedings* 81, 222-26.

Friend, I. and M.E. Blume, 1975, The Demand for Risky Assets. *American Economic Review* 65, 900-922.

Kose, M.A., 1995, Welfare Implications of International Risk Sharing. Working Paper, University of Iowa.

Levine, R. and D. Renelt, 1992, A sensitivity analysis of cross country growth regressions, *American Economic Review*, 82(4), 942-963.

Lewis, K.K., 1996a, Consumption, Stock Returns, and the Gains from International Risksharing. NBER Working Paper no. 5410.

Lewis, K.K., 1996b, What can Explain the Apparant Lack of International Consumption Risksharing? *Journal of Political Economy* 104(2), 267-297.

Lucas, R.E., 1987, Models of Business Cycles. Oxford, Blackwell.

Maddison, A., 1995, Monitoring the World Economy, 1820-1992, OECD.

Mankiw, N.G., 1995, The Growth of Nations, *Brookings Papers on Economic Activity*, 275-310.

McCulloch, J.H., 1990, Appendix B to The Term Structure of Interest Rates, by R. Shiller, in B. Friedman and F. Hahn, eds., The Handbook of Monetary Economics V. I, New York, North-Holland.

Mehra, R. and E.C. Prescott, 1985, The Equity Premium: A Puzzle. *Journal of Monetary Economics* 15, 145-161.

Mendoza, E., 1994, Terms of Trade Uncertainty and Economic Growth: are risk indicators significant in growth regressions, International Fianance Discussions Papers, no.

491, Board of Governors of the Federal Reserve System.

Mendoza, E., 1995, The Terms of Trade, the Real Exchange Rate and Economic Fluctuations. *International Economic Review* 36, 101-137.

Obstfeld, M., 1994a, Risk-Taking, Global Diversification, and Growth. *American Economic Review* 84, 1310-1329.

Obstfeld, M., 1994b, Evaluating Risky Consumption Paths: the Role of Intertemporal Substitutability, *European Economic Review* 38, 1471-86.

Obstfeld, M., 1994c, Are Industrial-Country Consumption Risks Globally Diversified? in: L. Leiderman and A. Razin, eds., *Capital Mobility: The Impact on Consumption, Investment and Growth*, Cambridge University Press, Cambridge.

Obstfeld, M., 1995, International Capital Mobility in the 1990s. in: P.B. Kenen, ed., *Understanding Interdependence*, Princeton, Princeton University Press.

Sala-I-Martin, X., 1994, Economic Growth: Cross-Sectional Regressions and the Empirics of Economic Growth, *European Economic Review* 38(3-4), 739-747.

Shiller, R.J., 1993, Macro Markets: Creating Institutions for Managing Society's Largest Economic Risks, Clarendon, New York.

Shiller, R.J. and S. Athanasoulis, 1995, World Income Components: Measuring and Exploiting International Risk Sharing Opportunities, NBER Working paper No. 5095.

Tesar, L.L., 1993, International Risksharing and Non-Traded Goods, *Journal of International Economics* 35, 69-89.

Tesar, L.L., 1995, Evaluating the Gains from International Risksharing. Carnegie-Rochester Conference Series on Public Policy 42, 95-143.

Tesar, L.L., and I.M. Werner, 1994, International Equity Transactions and US Portfolio Choice, in: J.A. Frankel, ed., *The Internationalization of Equity Markets*, Chicago, University of Chicago Press.

van Wincoop, E., 1994, Welfare Gains from International Risksharing, *Journal of Monetary Economics* 34, 175-200.

van Wincoop, E., 1995, Regional Risksharing. European Economic Review 37, 1545-

1567.

van Wincoop, E., 1996a, A Multi-Country Real Business Cycle Model with Heterogeneous Agents. *Scandinavian Journal of Economics* 98(2), 233-251.

van Wincoop, E., 1996b, How Big are the Potential Welfare Gains from International Risksharing?, unpublished manuscript, Boston University .