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SHORT-TERM SPECULATORS AND THE  
ORIGINS OF NEAR-RANDOM-WALK  
EXCHANGE RATE BEHAVIOR

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# **Short-Term Speculators and the Origins of Near-Random-Walk Exchange Rate Behavior**

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## **Abstract:**

This paper suggests that normal speculative activity could be a source of random-walk exchange rate behavior. Using a noise trader model to analyze very short-term exchange rate behavior, it shows that rational, risk-averse speculators will smooth the impact of shocks to exchange rate fundamentals. With sufficient speculative activity, an exchange rate could become statistically indistinguishable from a random walk, regardless of the generating processes of its fundamental determinants.

This result may help resolve the apparent inconsistency between the observed behavior of floating exchange rates and the behavior predicted by existing theoretical models given the actual behavior of exchange rate fundamentals. The result also suggests that heavy speculative activity could cause exchange rates to be forecast better via random-walk than via structural models--even when structural forces are correctly identified. Finally, the paper provides an explanation for the observed extended response of exchange rates to sterilized intervention.

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It is widely known that dollar exchange rates are indistinguishable from random-walk processes and that many forces driving these exchange rates, such as money supplies and real output, do not follow random-walk processes (Blanchard 1981, and Campbell and Mankiw 1987). Despite the familiarity of these attributes of exchange rates and their fundamentals, we do not fully understand how they fit together. In fact, according to many existing exchange rate models, non-random-walk fundamentals are not likely to be consistent with near-random-walk exchange rates. The monetary model of Frenkel and Mussa (1980), for example, is consistent with random-walk exchange rates only in "the unlikely event that [a certain] linear combination of fundamentals ... happens to follow a random-walk process" (Meese 1990). Adams and Chadha (1991) find that the portfolio balance and purchasing power parity models are not consistent with the behavior of the exchange rates' fundamentals and the random-walk character of nominal and real exchange rates. Likewise, Ahking and Miller (1987) investigate systematically whether the monetary approach is consistent with the random-walk behavior of nominal exchange rates.

This paper shows that random-walk exchange rates may be consistent with many more fundamental generating processes than previously thought, because the normal activity of speculators transforms the effects of shocks to exchange rates. If shocks are stationary, speculators reduce their early effects and magnify those that come later. If shocks are nonstationary, speculators magnify the shocks' early effects and diminish their later ones. In both cases, speculators smooth the effect of shocks, driving a wedge between the behavior of exchange rates and that of their determinants. With sufficient speculative activity, the exchange rate will be statistically indistinguishable from a random walk.

The model presented here also lends insight to the relative forecasting success of the random-walk model when compared with structural models (Meese and Rogoff 1983; Frankel 1984; Boughton 1987; Clarida and Taylor 1992). In the model developed below, heavy speculative activity generates exchange rates that are more accurately forecast by a random walk than by a structural model, even when structural forces are correctly identified. In effect, speculators' activity masks the connections between structural factors and exchange rates.

The model may also contribute to our understanding of additional stylized facts of exchange rate behavior. Dominguez (1990) and Baillie and

Bollerslev (1991) have shown that the response of exchange rates to sterilized intervention extends beyond the day of actual intervention.

This will be true in the model developed here because speculators cause the effects of a one-time shock--such as sterilized intervention--to die out monotonically. Furthermore, Ito and Roley (1986), Goodhart et al. (1991), and Goodhart and Figliuoli (1991) find that large exchange rate jumps are partially reversed within a day, a result that is predicted in this model when large shocks are mean reverting.

To best analyze the very short-term properties of the foreign exchange market, this paper employs a model of rational speculation similar to a variety of models used widely in finance literature (Kyle 1985; Dow and Gorton 1991; De Long, Shleifer, Summers, and Waldmann 1989, 1990). The "noise" or "liquidity" traders generally included in these models, as well as rational speculators, can be immediately identified with major players in the foreign exchange market. For instance, speculators have many of the properties of interbank traders and foreign exchange fund managers: They do not risk their own wealth, they do not take positions that are infinitely large, and they have a short horizon. "Liquidity

traders" share many properties of importers and exporters of goods and services: They determine their currency trades according to the level of the exchange rate, rather than its expected change, and their trades arrive somewhat randomly throughout the day.

Noise trader models have an advantage in that they assign a central role to short-term speculators, the same agents responsible for the vast bulk of daily foreign exchange transactions (Goodhart 1988). Although interbank traders historically have dominated this group, their pre-eminence has been challenged in recent years with the emergence of mutual funds dedicated to short-term foreign exchange speculation. However, whether short-term speculators are fund managers or interbank traders, they are frequently relegated to the background of exchange rate models, or assigned unrealistic behaviors, such as risk neutrality.

The specific structure of the model draws on elements found in many noise trader models. The fully rational speculators maximize a standard, constant absolute risk aversion utility function, a form used in models such as those presented in Bhattacharya and Weller (1993) and Balduzzi et al. (1992) and which, under standard assumptions, leads to simple linear demands like those of the speculators in Kyle (1985). The current account traders of the

present model are identical in structure to the noise traders of models such as the one used in Dow and Gorton (1991). Their currency demand is linearly related to both the asset price and a mean-zero disturbance.

The function of the current account/noise traders in the present model is notably different from that of analogous agents in other noise trader models. Through their dependence on exchange rates, the current account traders of the present model determine the exchange rate's ultimate value. This contrasts with some noise trader models where noise traders' activity is simply an independent and identically distributed (i.i.d.) shock and the asset's ultimate value is determined exogenously (for example, Kyle 1985, and Dow and Gorton 1991). The shocks to current account traders in the present model are common knowledge—a trait that the model shares, for example, with the one in DeLong et al. (1989, 1990), but differs from the models used in papers such as Kyle (1985) and Battacharya and Weller (1993), where noise trader shocks cannot be distinguished from other market information and the asymmetric availability of these two information sources is crucial to the results.

The market equilibrium condition of the model presented here, also adopted from the noise trader literature, is straightforward: Currency



supplied to the market during a given period must equal currency demand.

This flow equilibrium, concept differs importantly from the concept of stock equilibrium, which has long been influential in exchange rate modeling.

Nonetheless, it was advocated forcefully by Kouri as long ago as 1981:

In the absence of intervention, the exchange rate is determined so as to equilibrate the demand for foreign exchange with the supply of foreign exchange ... yet, the modern theory of flexible exchange rates appears to have no connection with these market processes. There is no explicit treatment of the sources of supply and demand in the foreign exchange market and no explicit analysis of how supply and demand interact in that market to determine the exchange rate.

More recent observers of the foreign exchange market, including Lyons (1991) and Goodhart (1988), have also noted the importance of currency flows in day-to-day foreign exchange trading, citing as evidence the value attached to information received from brokers (via "squawk boxes") and customers.

The first and possibly only other work to directly investigate the sources of random-walk exchange rate behavior is Rogoff (1992), which suggests sensibly that consumption smoothing may provide a mechanism by which the real exchange rate is smoothed over time. However, the Rogoff paper and this one actually investigate different issues: Rogoff investigates the real exchange rate over horizons from a few quarters to a few years, and

the present paper investigates nominal exchange rates over horizons from a few minutes to a few days, a difference that is reflected in modeling structures.

The next section of this paper, Section I, develops the basic exchange rate model, beginning with a no-speculator equilibrium and then adding speculators. Because the model of this section is extremely general, Section II develops a more specific model and identifies the exact circumstances under which speculators will exert the greatest influence. Section III uses simulations of the model to examine the sets of parameters in which an econometrician is most likely to conclude that an exchange rate follows a random walk, or to find that a random-walk model forecasts better than a structural one. Section IV extends the model to include speculation costs and multiple disturbances. The final section summarizes the results and discusses in greater depth the relationship between this and other exchange rate models.

### **I. The Smoothing Effect of Speculative Activity**

This section develops a simple model of exchange rate behavior, drawn directly from the noise trader models of finance literature. After describing

the speculators and the goods-and-services traders, it examines how exchange rate behavior is affected by the introduction of speculators.

#### A. *The Agents*

*Speculators:* Speculators in this paper are rational and risk-averse. They have complete information regarding current and past shocks, and are not subject to position limits. They decide on their positions without considering interest rate differentials, an assumption based on Goodhart (1988), who interviewed numerous interbank traders and found that an "open spot position is seen generally as a pure currency play, with little attention normally being given to interest rates." This lack of attention to interest rates seems reasonable, since the vast majority of these speculators' trades are reversed within a single day.<sup>1</sup>

In each period, speculators bet in the currency market, choosing the size of their positions to maximize the expected utility of profits. For now we will assume that speculators take a short position if they expect the exchange rate to decline between the current period and the next one, and vice versa. More specifically, each speculator's position (in absolute value) will be a non-decreasing function of the size of the expected exchange rate

change, and the chosen position will be symmetric around zero. Later, we will show that this rule is consistent with the constant, absolute risk aversion utility function common to noise trader models. This rule is also consistent with a widely held view of short-term foreign exchange speculation: Money is made by taking larger bets when the likelihood of being right about the direction of change seems particularly great, rather than by being right more often than everyone else.

Let an individual speculator's current bet be represented by the general function  $B$ :

$$B_t = B(E_t\{e_{t+1}\} - e_t) .$$

Here,  $e_t$  is the (log of the) foreign currency price of domestic currency, and  $E_t$  represents the expectations operator, with expectations conditioned on knowledge of all current and past shocks. The assumptions so far imply that  $B(x)$  is odd ( $B(x) = -B(-x)$ ),  $B'(\cdot) \geq 0$ , and  $B'(0) > 0$ .

*Goods-and-services traders:* The random currency demand/supply of goods-and-services traders depends linearly on the level of the exchange rate. Exporters' demand for domestic currency (in exchange for foreign currency),  $X_t$ , and importers' supply of domestic currency,  $M_t$ , are determined as follows:

$$X_t = C_x - S_x e_t + \epsilon_t,$$

$$M_t = C_m + S_m e_t,$$

where  $C_x$  and  $C_m$  are constants, and  $S_x + S_m$  are semi-elasticities. These price responses can be either positive or negative, but it is assumed without loss of generality that their sum is positive.<sup>2</sup> The variable  $\epsilon_t$  represents a random shock to exporters, the unconditional distribution of which is independent and identical with mean zero, though the conditional distributions may vary across time.<sup>3</sup> Given the (intentionally) limited scope of the model, these shocks must represent a broad variety of forces, including price disturbances, fiscal and monetary policies, barriers to trade, and the weather. In a general equilibrium format some of these forces would be simultaneously determined. However, in a model of exchange rate behavior over horizons as short as a few hours or a few days, such simultaneity is unlikely to be relevant.

This approach to modeling importers and exporters makes them correspond exactly to "liquidity" traders in some noise trader papers (for example, Dow and Gorton 1991). Such a linear demand structure is also supported by the continued success of simple linear models of international trade. Goods-and-services traders are assumed not to speculate on exchange

rates for the simple reason that in reality these two groups of agents generally specialize.<sup>4</sup>

### ***B. Equilibrium without Speculators***

To understand exchange rate dynamics in this model, it is natural to begin by considering the exchange rate's behavior in the absence of speculators. To determine the equilibrium exchange rate in this case we impose the requirement that markets clear, or  $M_t = X_t$ , which implies

$$(1) \quad e_t = \frac{C_x - C_m}{S_x + S_m} + \frac{\epsilon_t}{S_x + S_m}.$$

The first term in equation 1 is the exchange rate's unconditional mean, which is determined by the behavioral parameters governing importers and exporters. As will be shown below, this central tendency will not be affected by the presence of speculators, which is why importers and exporters are considered the exchange rate's "fundamentals."

The second term in equation 1 captures the effects of the random shocks to goods-and-services traders. Since only the current shock enters the equation, the exchange rate's distribution will be isomorphic to that of the underlying disturbances: i.i.d. disturbances will be associated with i.i.d.

exchange rates; disturbances with autoregression (AR) coefficients  $\{\rho_i\}_{i \geq 1}$  will generate exchange rates with identical AR coefficients, etc. The only source of near-random-walk behavior in the exchange rate would be identical near-random-walk behavior in the fundamentals. Once we bring speculators onto the scene, this restriction will no longer hold.

### C. *Equilibrium with Speculators*

Speculators interact with the rest of the foreign exchange market through their flow demand for currency, which is the difference between their current desired bet and their position at the end of the previous period:

$$(2) \quad B(E_t\{e_{t+1}\} - e_t) - B(E_{t-1}\{e_t\} - e_{t-1}) .$$

Market equilibrium now requires the following:

$$(3) \quad C_x S_x e_t + \epsilon_t + NB_t - NB_{t-1} = C_m + S_m e_t$$

Here  $N$  represents the total number of speculators. Under rational expectations, equilibrium also requires that the speculators' behavioral function  $B$  be consistent with the exchange rate dynamics it generates.

The central intuition of the model is that speculative activity smooths the effects of fundamental shocks. In this way, speculation drives a wedge between the generating processes of fundamentals and those of exchange rates, increasing the resemblance between the exchange rate and a random walk. The intuition behind this result can be most easily clarified by the following example, in which it is assumed that shocks are i.i.d. and that in the speculators' absence a positive shock of one would raise the exchange rate by ten.

Suppose there is one speculator in the market, and a positive shock of one occurs. Seeing the exchange rate rise by ten, the speculator will expect the exchange rate to return to its mean thereafter, and will try to profit by selling foreign exchange currently and buying it back in the next period. The "selling pressure" in the first period will reduce the exchange rate's initial increase from ten to perhaps eight. In the second period, "buying pressure" will make the exchange rate higher by two than it would have been in the speculator's absence. In this way, the speculator's natural endeavors transfer some of the shock's ten-unit initial effect to the subsequent period.

This is not the final equilibrium, however. Noting that the exchange rate will likely decline between the second and third periods, the speculator



will enter the market again, thereby transferring some of the second-period impulse to the third period. The speculator will also adjust its initial bet to accommodate the fact that the exchange rate does not decline by a full ten units between the first and second periods. For a one-unit shock this smaller bet would leave the initial exchange rate at perhaps nine units above its conditional mean, rather than eight, and leave the exchange rate in the next period one unit above its conditional mean, rather than two. In the final expectations equilibrium, the effects of the speculator's actions are fully anticipated and the initial shock impacts all future exchange rates.

Because they smooth the influence of a one time shock, taking some of the initial effect and distributing it across all future exchange rates, speculators increase the resemblance between the exchange rate and a random walk, since the latter implies a constant influence of a shock on all future exchange rates. Speculative activity will also increase the resemblance between exchange rates and a random walk when shocks are non-stationary. Suppose, for concreteness, that the shock in each period is a moving average of a set of i.i.d. disturbances, and that the influence of each i.i.d. disturbance on the shock itself increases continuously while approaching an asymptote. If we define these i.i.d. disturbances as " $\omega_{t,i}$ ," and define by " $\rho_i$ " their

influence on the current trade shock,  $\epsilon_t$ , then we can write the trade shocks as follows:

$$\epsilon_t = \rho(L)\omega_t = \sum \rho_i L^i \omega_t$$

where  $\rho_i < \rho_{i+1}$  and  $\lim_{i \rightarrow \infty} \rho_i = A < \infty$ . These shocks will be referred to as "mean averting."

In the absence of speculators, a current disturbance  $\omega_0$  would raise the current exchange rate by  $\rho_0/(S_x+S_m)$  relative to its unconditional mean, raise the next period's exchange rate by  $\rho_1/(S_x+S_m)$  relative to the same unconditional mean, etc. Since  $\rho_1 > \rho_0$ , the exchange rate would be expected to rise between the current and next periods, and speculators would buy currency. This activity would drive the current exchange rate up by more than  $\rho_0/(S_x+S_m)$ . When speculators ultimately liquidate their positions after one period, they will tend to reduce the exchange rate relative to  $\rho_1/(S_x+S_m)$ . By increasing the exchange rate in the first period and reducing it subsequently, relative to the values it would have in their absence, speculators increase the similarity between the exchange rate and a random walk.

The effects of speculators on exchange rate dynamics are best captured by simple diagrams. Figure 1 superimposes one impulse-response diagrams

on another to show how an individual trade shock generated by an i.i.d. process would affect exchange rates with and without speculators. Figure 2 shows the effect of a single, nonstationary trade disturbance ( $\varpi > 0$ ) on future exchange rates with and without speculators.

The following results present these ideas more rigorously. Each result considers the path of future expected exchange rates after a shock in period 0--that is, they consider

$$\{E_0(e_j | \epsilon_0 > 0; \epsilon_{-i} = 0 \quad \forall i \geq 1)\}_{j \geq 1} .$$

The results cover the cases of (i) stationary shocks and (ii) nonstationary shocks. Proofs for the case of nonstationary shocks are presented in Appendix A. Proofs for the case of stationary shocks are quite similar and are available from the author.

### Effect of Speculative Activity When Shocks Are Nonstationary

(Stationary):

*After an isolated, positive shock to fundamentals,*

(1) *the series of expected exchange rates will converge*

*monotonically to a finite limit, but never reach that limit;*

- (2) *the average difference between the current exchange rate and any future expected exchange rate will be smaller in the presence of speculators than in their absence; and*
- (3) *if speculators' bets are linearly related to expected exchange rate changes, the path of future expected exchange rates will begin between the expected exchange rate path of the no-speculator equilibrium and the (un)conditional expected value, and it will cross the no-speculator path once and only once.*

Only a few important assumptions are embedded in the proofs. The shocks to fundamentals are assumed to be distributed symmetrically around a conditional expected value that converges monotonically to a finite limit. In the case of stationary shocks, the limit is zero; in the case of nonstationary shocks, the limit is nonzero. Since the effects of a shock on the fundamentals themselves are assumed to stabilize ultimately, expected future speculative bets are also assumed to converge to zero.

The strong resemblance between floating exchange rate behavior and a random-walk process fast became a stylized fact once it was noted in the early 1980s (Meese and Singleton 1982). Though some researchers reject the hypothesis (Liu and He 1991; Engle and Hamilton 1990; Clarida and Taylor

1992), exchange rate behavior is sufficiently close to this benchmark that most tests do not reject it. As noted in the introduction, most existing exchange rate models are inconsistent with such behavior unless the exchange rate has near-random-walk fundamentals. Results (2) and (3), which capture the smoothing effect of speculative activity, show that in this model short-term speculators can cause the exchange rate to approximate a random walk for shocks from a variety of generating processes.

Results (2) and (3) have strong implications for the relative performance of alternative econometric models of exchange rates. With sufficient speculation, a "structural" econometric model of the exchange rate, in which its current value is regressed on the first few lagged shocks, would not capture as much of its movement as a simple "random-walk model." Consequently, exchange rates might be forecasted better by a random-walk model than by a structural model, even if the structural forces have been correctly identified. This point, which holds for both stationary and nonstationary shocks, will be addressed at greater length in Section III.

Result (1) has strong implications for the effects of sterilized foreign exchange intervention. Since such intervention is exactly analogous to a one time shock to currency demand, result (1) states that it should have effects

outlasting the effort itself, and that these effects should die out monotonically. Evidence confirming this hypothesis already exists: Dominguez (1990) and Baillie and Osterberg (1991) find that the effects of sterilized intervention last beyond the day of the intervention itself, and Dominguez (1990) finds that the effects of such intervention diminish monotonically over time.

The observed intraday behavior of the yen/dollar exchange rate is also consistent with the implications of this model. Ito and Roley (1986) find that large jumps in this rate are partially reversed within the day, and similar evidence regarding a number of currencies is presented in Goodhart and Figliuoli (1991) and Goodhart et al. (1991). The fact that this negative autocorrelation is much more easily discernible for large exchange rate jumps than for small changes is also consistent with the model.

The analysis so far suggests that it is the presence of speculators that causes the behavior of the exchange rate to differ from that of the fundamentals, but the model as it stands is too general to tell more specifically which aspects of speculators or their environment might be most important in generating exchange rates that resemble a random walk. For this reason, we turn next to a more precise interpretation of speculator

behavior. To highlight the effects of factors other than the distribution of fundamental shocks, disturbances will be assumed to be i.i.d. for the rest of the paper.<sup>5</sup>

## II. A Specific Model of Speculator Behavior

Suppose that all speculators have a standard, constant absolute risk aversion utility function,

$$U = -e^{-\gamma\Pi},$$

here  $\gamma$  is the measure of absolute risk aversion and  $\Pi$  represents speculators' profits:<sup>6</sup>

$$\pi_t = B_t(\cdot) [e_{t+1} - e_t].$$

According to this specification, speculator behavior does not depend on current wealth, but rather on the profits generated from trading activity. This is consistent with the type of speculators being modeled here, since interbank traders and fund managers generally have few, if any, personal funds at risk, and their pay ultimately depends on the profits they make for their clients. The fact that speculators are risk-averse in the model is also consistent with

actual behavior, since speculators do not take infinite positions, and most speculators, observation suggests, usually stay within their position limits. It is also worth noting that these speculators have no investment alternatives other than holding positive or negative positions in currency. This, too, is representative of the options available to interbank traders. The reasons why banks segregate currency position-taking from other position-taking, which may concern the market-specific information and skill required to trade successfully in any highly liquid financial market, are not explicitly modeled here.

Speculators choose their bets with the goal of maximizing expected utility. Under the assumption that the fundamental shocks are normally distributed, an assumption maintained throughout the rest of the paper, speculators' profits will normally be distributed conditionally and consequently the speculators will behave as if they are maximizing the objective function:

$$W_t = E_t\{\pi_{t+1}\} - \frac{\gamma}{2} \text{Var}\{\pi_{t+1}\}.$$

Speculators maximize this objective function with respect to their bet size, taking prices as given. For some readers, this assumption may seem



inconsistent with the fact that the number of speculators is finite. Researchers have demonstrated, however, that in markets with properties of the foreign exchange market, competitive equilibrium is reached with only a few participants. More specifically, the foreign exchange market has many of the critical features of an oral double auction, since about a third of the trades involve brokers who post prices publicly (Federal Reserve Bank of New York 1992). In such market experiments, "the overwhelming result is that these markets converge to the competitive equilibrium even with very few traders" (Plott 1982). The foreign exchange market also has features of a telephone market, in which prices have also been found to converge to the competitive equilibrium when the number of participants is small compared with the number of active foreign exchange traders (see the studies surveyed in Plott 1982). Not only is the assumption of competitive prices realistic in the sense that it is supported by experimental studies, it is also helpful insofar as it provides an explicit, tractable analytic solution to the model while making no difference whatsoever to the qualitative results presented here.<sup>7</sup>

The speculator's optimal bet,  $B_r$ , is proportional to the expected change in the exchange rate:

$$B_t = Q_t[E_t\{e_{t+1}\} - e_t].$$

$Q_t$ , the speculator's optimal bet coefficient, must satisfy

$$Q_t = \frac{1}{\gamma \text{Var}\{e_{t+1}\}},$$

where  $\text{Var}\{e_{t+1}\}$  is the conditional variance of the next period's exchange rate.

#### A. *Market Equilibrium*

Three more conditions define equilibrium in this market. First, expected utility from foreign exchange speculation,  $E\{W\}$ , can be no greater than expected utility from other activities,  $W^*$ . If for some reason  $E\{W\}$  were to exceed  $W^*$ , then more people would choose to become foreign exchange speculators. This motivation seems to be exactly what is driving the mutual fund "gunslingers" to enter the foreign exchange market: "For the five years that ended in December, 1991, the compounded annual return of the Ferrell foreign exchange index--a benchmark designed to measure the performance of foreign exchange managers--was 31.6%, compared with a compounded annual return of 15.38%, including dividends, for the Standard

and Poor's 500 index" (Roman 1992). Eventually, the entry of new speculators will drive  $E\{W\}$  down to the same level as  $W^*$ , at which point there will be no other new entrants. By contrast, if  $E\{W\} < W^*$ , foreign exchange speculators will leave the market, but it is possible that, even after all speculators have left foreign exchange,  $E\{W\}$  will never reach  $W^*$ . In sum,

$$E\{W\} \leq W^* .$$

This condition effectively endogenizes the number of speculators. Because it takes time to hire and educate new staff, the endogenous response of speculators to perceived differences in expected utility across markets does not happen instantaneously, as illustrated in the previous quote. Thus, the full endogeneity of the number of speculators can be viewed as a condition of long-run equilibrium, with short-run equilibrium characterized by an approximately fixed number of speculators. This paper will generally focus on the properties of long-run equilibrium.

The second equilibrium condition is the familiar requirement of market clearing:

$$C_x - S_x e_t + \epsilon_t + NQ_t[E_t\{e_{t+1}\} - e_t] - NQ_{t-1}[E_{t-1}\{e_t\} - e_{t-1}] = C_m + S_m e_t .$$

The third equilibrium condition is that exchange rate behavior be consistent with speculators' expectations.

The model can be solved analytically using the method of undetermined coefficients. The solution shows that there exists an equilibrium in which speculators' optimal bet coefficient  $Q$  is constant over time at  $1/\gamma\sigma^2\xi^2$ , where  $\sigma^2\xi^2$  represents the conditional volatility of  $e_{t+1} - e_t$ . Only this equilibrium will be considered in further analysis.

The equilibrium exchange rate in either the short or the long run can be represented as a moving average of past shocks:

$$e_t = \alpha + \beta(L)\epsilon_t,$$

where  $L$  is the lag operator. The unconditional mean of the exchange rate,  $\alpha = (C_x - C_m)/(S_x + S_m)$ , is unchanged by the presence of speculators. This is because short-term speculators in the model, as in actual practice, ultimately liquidate any position, which means that any positive influence speculators exert today by purchasing currency will eventually be offset when that currency is sold, and vice versa. (This result also depends on the linearity of the model.)

The moving average coefficients,  $\beta_i$ ,  $i \geq 0$ , must satisfy the second-order difference equation:

$$\beta_{i+1} - \frac{(S_x + S_m + 2NQ)}{NQ} \beta_i + \beta_{i-1} = 0.$$

Any solution must have the form  $\beta_i = \xi_1 \lambda_1^i + \xi_2 \lambda_2^i$ , where the  $\lambda_i$  ( $i = 1, 2$ ) are the roots to the polynomial  $\lambda^2 - \lambda(S_x + S_m + 2NQ)/NQ + 1 = 0$ . There is in fact a continuum of solutions jointly indexed by  $(\xi_1, \xi_2)$ . It is algebraically easy to show that one of the roots, call it  $\lambda_2$ , is necessarily greater than unity, implying that unless  $\xi_2 = 0$ , the entire system will be explosive. Assuming a stable system, we can then establish a unique value for  $\xi_1$  by noting that the moving average coefficients must also satisfy the initial condition  $\beta_0(S_x + S_m + 2NQ) = 1 + NQ\beta$ . Dropping the subscript "1" on  $\lambda_1$  and  $\xi_1$ , since such subscripts are now redundant, the solution to the system can be expressed as follows:

$$\beta_i = \frac{(1-\lambda)}{S_x + S_m} \lambda^i,$$

In the short run,

$$(3a) \quad 0 \leq \lambda = \frac{N(S_x + S_m)}{\gamma \sigma^2}.$$

In the long run,

$$(3b) \quad 0 \leq \lambda = \frac{1-2\gamma W^*}{1+2\gamma W^*} < 1 .$$

This model shows that, even though the shocks are i.i.d., the exchange rate will be a function of all past shocks, the influence of which declines geometrically at the rate  $\lambda = N(S_x + S_m) / \gamma \sigma^2$ . Such predictable mean reversion is usually assumed to be inconsistent with efficient markets: For example, Ito and Roley (1988) remark that "it is difficult to rationalize such behavior." Likewise, Lehman (1990) has observed that "systematic changes in fundamental valuation [of equities] over intervals like a week should not occur in efficient markets." The results presented here imply, however, that predictable, noninstantaneous mean reversion (or mean aversion) could characterize an efficient foreign exchange market. That is, even if speculators are rational utility maximizers, prices may move in a predictable fashion.

### ***B. Origins of Near-Random-Walk Exchange Rate Behavior***

As explained in the introduction, many standard general equilibrium exchange rate models, such as Frenkel and Mussa's monetary model (1980)

with rational expectations, imply that exchange rates can only behave like a random walk if the exchange rates' fundamentals also follow a random walk. In the present model, the exchange rate's autocorrelation will depend on many additional factors, including speculators' risk aversion,  $\gamma$ , expected welfare in other markets,  $W^*$ , the variance of fundamentals,  $\sigma^2$ , and the exchange rate sensitivity of imports and exports,  $S_x + S_m$ . This section examines these factors to see which will tend to increase the resemblance between the exchange rate and a random walk.<sup>8</sup>

To explore this issue, let us reconsider the example presented earlier, which begins with the assumption that there are no speculators. In the example, a positive disturbance of one generates an exchange rate rise of ten and is expected to lead subsequently to a decline in the exchange rate, so any potential speculator sells short. If there is only one speculator, the exchange rate's initial response to the unit shock might only be nine, in which case the exchange rate would be increased by one when these positions are unwound.

Influence of  $N$ : Consider the consequences of introducing one more speculator to the initial example. The total amount sold by speculators in the first period is larger than before, and consequently the exchange rate's initial rise (relative to its conditional mean) is smaller compared with the one-

speculator equilibrium (only seven, perhaps) and the exchange rate's second-period rise (relative to its conditional mean) is larger. If the number of speculators is very high, the impact of a given shock is almost constant in all future periods, and the exchange rate more closely resembles a random walk. Of course, the number of speculators is not exogenous in the long run, which the rest of this section will focus on, but this analysis will be important as we turn to the other parameters.

Influence of  $\gamma$ : A decline in speculators' risk aversion has an effect similar to that of raising their number, since it increases total speculative currency demand for a given expected exchange rate change.<sup>9</sup> This is true for the short and the long run.

Influence of  $W^*$ : A decline in  $W^*$ , expected welfare in other markets, has by definition no effect on exchange rate behavior in the short run. In the long run, such a change causes speculators elsewhere to shift into foreign exchange, which, as was just shown, increases the tendency of the exchange rate to resemble a random walk.

Influence of  $\sigma_\epsilon^2$ : A change in the variance of fundamentals affects the exchange rate through speculators' willingness to bet. Suppose, for the moment, that the number of speculators is fixed. A decline in the variance of



fundamentals reduces the variance of profits and induces speculators to bet more for a given expected exchange rate change. In this way, a decline in  $\sigma^2_\epsilon$  increases the persistence of the effects of shocks in the short run when the number of speculators has yet to respond to changed circumstances.

This increased persistence reduces expected speculator utility by reducing expected profits. If utility in other markets is not changed by the decline in  $\sigma^2_\epsilon$ , speculators will eventually leave the foreign exchange market, causing  $\lambda$  to return to its original value. The full model therefore implies that  $\lambda$  is invariant with respect to  $\sigma^2_\epsilon$  in the long run.

Influence of  $S_x+S_m$ : This primitive affects the exchange rate's ability to mimic a random walk through its role as scale parameter for the model as a whole. This role is best understood by returning to the exchange rate's behavior in the absence of speculators (see equation 1). In this scenario, the exchange rate's response to a disturbance  $\epsilon_t$  is  $\epsilon_t/(S_x+S_m)$ . Since speculative activity merely redistributes over time the effect that would occur in the absence of speculators,  $1/(S_x+S_m)$  is also the sum over time of the exchange rate's responses to  $\epsilon_t$  when speculators are present.

With this in mind, consider the effects of a rise in  $S_x+S_m$ . In the short run, when the number of speculators is fixed, (i) all the moving average

coefficients decline, an outcome that (ii) decreases the variance of speculators' profits. This decrease, in turn, (iii) induces speculators to bet more aggressively, so that (iv)  $\lambda$  goes up. In the long run, this (v) reduces expected utility to foreign exchange speculation, and (vi) causes an exodus of speculators from the foreign exchange market to other markets, so that (vii) ultimately there is no effect at all on  $\lambda$ .

In this context, it is interesting to note that  $S_x + S_m$  also affects the size of the exchange rate's initial responses to shocks. In the present model, the initial response of the exchange rate is proportionately lower than the initial shock whenever  $\beta_0 = \xi = (1-\lambda)/(S_x + S_m) < 1$ , and vice versa. Proportionate exchange rate responses have often seemed to exceed proportionate changes in fundamentals, an observation that has spawned important insights, such as the Dornbusch Overshooting Model (1976). This outcome requires that  $S_x + S_m$  be "large," or

$$(4) \quad S_x + S_m > \gamma\sigma^2 / (N - \gamma\sigma^2).$$

Other economists have found that exchange rates' responses to shocks are surprisingly small (Goodhart 1988, Frenkel 1981), at least in the London

market. This finding is consistent with the present model when equation 4 is reversed.

In sum, exchange rates in the model will more closely resemble a random walk when:

- the number of speculators is in some sense high
- speculators are relatively tolerant of risk or
- goods traders are relatively sensitive to exchange rate levels.

Are these conditions consistent with reality? It is commonly known that short-term price elasticities of international trade are fairly low, an observation that is encouraging. The fact that speculative activity in the foreign exchange market is extensive relative to any other market seems to be encouraging at first blush, but in fact the standard of comparison is not appropriate to the model. The following section takes another approach to examining the empirical plausibility of these results.

### **III. Empirical Requirements of Observed Exchange Rate Behavior**

This section uses simulations of the model to show that a structural model of exchange rates with i.i.d. fundamentals can generate exchange rate

behavior that (a) is econometrically indistinguishable from a random walk and (b) forecasts more accurately through a random-walk model than through a structural model. The simulations continue to assume that shocks are distributed i.i.d.; since most of the macroeconomic variables likely to be important to exchange rates are strongly autocorrelated, this assumption is a very conservative one. The composite parameter  $\lambda = N(S_X + S_M) / \gamma \sigma^2$  determines the persistence with which any given shock affects the exchange rate, so by varying  $\lambda$  we can examine the circumstances under which model-generated exchange rates might resemble observed exchange rates.

**A. *Failure to Reject the Random-Walk Hypothesis for Exchange Rates***

As  $\lambda$  approaches unity, the effects of individual shocks become more persistent and the exchange rate is therefore more likely to resemble a random walk. Table 1 documents the actual frequency with which exchange rates from the model are econometrically indistinguishable from a random walk, for various values of  $\lambda$ . It is well known that unit root tests are not very powerful, and in fact the random-walk hypothesis is generally not rejected when  $\lambda$  exceeds 0.85 or 0.90. The tests' ability to discriminate increases quickly as  $\lambda$  falls below this range. Table 1 shows that exchange

rates generated by i.i.d. fundamentals can be mistaken econometrically for random-walk exchange rates, if there is sufficient speculative activity.

Construction of this Table 1 began with the creation of 100 series of i.i.d. normal disturbances, 351 observations each. These were combined with the first 100 moving average coefficients for a given set of exogenous parameters to create 100 series of exchange rates, 251 observations long.<sup>10</sup> Without loss of generality, the unconditional mean of the exchange rate was taken to be zero. This procedure was then repeated for a number of parameter combinations, using the same underlying set of i.i.d. disturbances each time.

The  $F$ - and  $d$ -tests are included following the example of Hakkio (1986).  $DF$ ,  $ADF(3)$ , and  $ADF(8)$  refer, respectively, to Dickey-Fuller and Augmented Dickey-Fuller tests with three and eight lagged first differences. The  $\Phi$  tests are documented in Dickey and Fuller (1981) and their nonparametric extensions (labeled with a "Z") are documented in Perron (1988). (Further details regarding these tests are presented in Appendix B.)

### ***B. Forecasting Performance of Random-Walk and Structural Models***

The parameter  $\lambda$  also determines the relative success of structural and random-walk models in forecasting exchange rates. A structural model in this context regresses the current exchange rates on a series of recent shocks. To understand the contribution of  $\lambda$  to the relative success of these two models, consider two exchange rates,  $A$  and  $B$ , for which  $\lambda = 0.5$  and  $\lambda = 0.9999$ . The important influences on  $A$  include the first four or five lags of  $\epsilon_t$ , so a structural model ought to be fairly successful with this exchange rate. The important influences on  $B$  include the first few hundred lags, so a similar structural model of  $B$  would have low explanatory power.

Although a structural model is likely to be more successful with  $A$  than with  $B$ , the reverse would be true for a random-walk model. The most recent exchange rate will effectively summarize the current influence of past shocks for  $B$ , giving recent exchange rates great explanatory power. By contrast, recent exchange rates would be much less closely related to current exchange rates for  $A$ , and the random-walk model would likely perform poorly.

Tables 2a and 2b compare the performance of the random-walk and structural models. The structural models include the first five lagged shocks as regressors. Table 2a considers the models' ability to "explain" past

exchange rates, while Table 2b considers their forecasting ability. Both tables indicate that the random-walk model starts to outperform the structural model when  $\lambda$  is between 0.85 and 0.90.

These tables were constructed using the same exchange rate series underlying Table 1. First, a structural model was estimated for each exchange rate. The structural model's explanatory power on the first 221 observations and its out-of-sample forecasting ability on the final thirty observations were then compared with corresponding properties of the random-walk model. The out-of-sample forecasts employed a static forecasting procedure, following the literature.

Together, Tables 1 and 2 show that exchange rates in this model are most likely to resemble those actually observed during the floating rate period when the exchange rate's first-order autocorrelation coefficient,  $\lambda$ , is above 0.90. As indicated in the previous section, this finding could be associated with highly autocorrelated fundamentals or with a combination of primitives such that exchange rates were highly autocorrelated.

#### IV. Extensions of the Model

This section amends the model to incorporate costs of speculation and multiple shocks, important aspects of the trading environment.

##### A. *Costs of Speculation*

Speculation costs come in a number of types and sizes: There are costs for space, personnel, and communications, and there are net borrowing costs in the form of bid/ask spreads. Regardless of their form, however, positive speculation costs are likely to reduce the exchange rate's ability to resemble a random walk.

Imposing fixed costs, such as one time expenses for physical facilities, will raise the level of expected profits required for speculators to achieve the same level of expected utility as their counterparts in other markets. In equilibrium there will be fewer speculators and a decrease in the persistence of a shock's effect on the exchange rate.

To consider the implications of variable costs, suppose that these costs were to rise with the square of each bet: Today's cost would be  $CB^2$ , where  $C$  is a positive constant. In addition to ensuring that marginal costs are



positive regardless of whether the speculator is long or short, this functional form can act as a proxy for the presence of position limits on individual interbank traders.

In the presence of such variable costs, the optimal bet coefficient,  $Q$ , has a new formula:

$$Q = \frac{1}{2C + \gamma \text{Var}\{e_{t+1}\}} .$$

The costs discourage betting, so for any given set of exogenous parameters the equilibrium will have a somewhat lower value of  $\lambda$  and the resemblance between the exchange rate and a random walk will be less than in the absence of these costs.

### ***B. Permanent and Transitory Elements of Exchange Rates***

Recent research has indicated that exchange rates have multiple "components", or factors. Campbell and Clarida (1987), for example, find that dollar exchange rates can be described as comprising a permanent and a transitory component. Similar results could be generated in this model simply by introducing multiple, mutually uncorrelated disturbances to importers and exporters. Such factors would find their way into our analysis

quite naturally if the real side of this market were modeled in greater detail, since factors such as oil prices, the weather, and gulf wars would then appear individually.

Suppose, for concreteness, that importers and exporters are affected by one true random-walk disturbance and one i.i.d. disturbance. The random-walk factor would create a "permanent" component of the exchange rate. The contribution of this factor would be identical whether or not speculators are present, since the factor does not affect the expected change in the exchange rate.<sup>11</sup> The i.i.d. factor would contribute a "transitory" or stationary moving average component. The durability of the transitory component (or, more accurately, its associated value of  $\lambda$ ) would be smaller than it would otherwise be because the random-walk disturbance increases the risk to speculation without affecting expected profitability. This induces speculators to reduce their bet coefficient,  $Q$ , a move that in turn lowers  $\lambda$ .

### *C. Applications to Other Financial Markets*

The applicability of the model to other financial markets is immediately suggested by the fact that it is nothing but a noise trader model. The model may be especially useful in understanding markets for

commodities, such as precious metals and agricultural products, where near-random-walk behavior has also been observed (Kolb 1990). These markets share two salient characteristics of the model developed here: They have no known final price, in contrast to the bond market, and they have a source of demand that is not exclusively dependent on expected price changes.

There are a number of empirical observations about equity markets that this model may explain. Kim, Nelson, and Startz (1991), among others, have found that mean reversion in equity prices declined following World War II. It is frequently hypothesized that this might be associated with the "resolution of major uncertainties about the survival of the U.S. economy" (Kim et al.), a conjecture that is consistent with our conclusion that a broad-based decrease in fundamentals' volatility is likely to be associated with greater financial price autocorrelation.

Lo and MacKinlay (1988), who find mean aversion for weekly equity returns, note that this mean aversion is greater for small stocks than for large stocks, and weaker since 1974 than between 1962 and 1974. To apply the model to these observations, we would note first that mean aversion would imply shocks, which contain a unit-root process. In this context, we could explain the greater mean aversion for small stocks by appealing to the greater

inherent variability of the underlying performance of small companies, and we could explain the decline in mean aversion since the mid-1970s by appealing to the surge in trading after the elimination of fixed commissions in 1976.

## V. Concluding Comments

This paper develops a model in which short-term speculators take center stage. The analysis suggests that this type of speculation may be shaping many of the familiar properties of short-term exchange rate behavior. Short-term speculation may help the exchange rate resemble a random walk, it may reduce the forecasting ability of structural models relative to a random walk, and it may cause an extended response of exchange rates to sterilized intervention.

The model here departs from conventional exchange rate models by imposing the requirement of flow, rather than stock, equilibrium. That is, exchange rates are determined by the condition that the amount of currency supplied to the market for trading in any given period must equal the amount that is desired in trade, rather than the requirement that agents be willing to

hold a predetermined supply of currency or other assets. (Imposing flow equilibrium in this model of very short run exchange rate dynamics is not equivalent to denying the importance of stock equilibrium, which could well be a defining condition of longer term dynamics.)

Flow models as a group have been distrusted by international economists during the past two decades or so, and for good reason: Earlier flow models included many ad hoc assumptions, especially with regard to asset market behavior. The inadequacies of these assumptions were highlighted by the more rigorous asset market models that succeeded them. However, neither monetary models, which stress the fixed currency supply, nor asset market models, which stress fixed supplies of bonds, as well as money, have been particularly successful in explaining observed short run exchange rate dynamics. For this reason, it seemed natural to seek an alternative approach. The flow approach, in particular, was attractive because it is widely accepted in the general asset market literature. Further, some empirical evidence now exists showing that prices in the equity market are indeed affected by flow demand and supply (Bagwell, Breen, and Korajczyk 1993).

Analyzing the behavior of short-term speculators in a flow-equilibrium setting makes it possible to show that the activities of such speculators can eliminate the isomorphism between the behavior of fundamentals and that of the exchange rate. Regardless of the fundamentals' generating processes, speculative activity will bring the exchange rate to a closer resemblance with a random walk. The factors affecting speculator behavior, such as the number of speculators, their risk aversion, the attractiveness of other professional opportunities, and the costliness of speculating, are introduced as potential influences on the extent to which the exchange rate will appear to follow a random walk.

Though this model is not intended to provide a complete story of exchange rate determination, exchange rates generated by it have been shown to be consistent with many observed short run properties of floating rates. Briefly, these properties are as follows: (i) large changes in the yen/dollar exchange rate are typically reversed in part within the next hour (Ito and Roley 1986); (ii) the effects of sterilized foreign exchange intervention last for more than just a day (Dominguez 1990, Baillie and Osterberg 1991); (iii) exchange rates seem to be mean-reverting (Engel and Hamilton 1990, Cutler, Poterba, and Summers 1991); (iv) exchange rates comprise multiple

independent factors (Campbell and Clarida 1987); and (v) exchange rates may "undershoot" (Goodhart 1988) or "overshoot" (Dornbusch 1976) in response to fundamental shocks."

The results presented here are more general than the model used for illustration. The crucial aspect of the trading sequence in the model presented above is a series of trades that closely replicate each other. There are two common aspects of the activity of marketmaking that share this crucial aspect. First, marketmakers frequently "lay off" their trades by quickly engaging in a reversing trade with another marketmaker. This tendency is thought by at least some market participants to lead to long sequences of very similar trades among marketmakers. Such a sequence of trades is easy to imagine: a British importer buys \$10 million in exchange for pounds from a marketmaker at Bank A, which then neutralizes the effect of that trade on its position by buying \$10 million from a marketmaker at Bank B, which then buys \$10 million from Bank C, and so forth. Eventually, counterparties are found who are willing to keep the extra pounds, and the sequence ends. In the meantime, the initial upward influence on the value of the dollar would have been maintained far beyond the instant of the initial transaction.

The second mechanism that would tend to propagate shocks through time is the use of the currency swap. The daily volume of these transactions in New York was roughly \$60 billion during 1992, almost four times the volume of outright forwards and over half the volume of spot transactions, and the bulk of these transactions have a maturity of under one week (Federal Reserve Bank of New York 1992). Since a swap is by definition a transaction that is reversed in full, any impulse can be propagated through these contracts if prices are determined in flow equilibrium.

The model can be reinterpreted to incorporate diversity among speculators by construing the disturbances as the foreign exchange activity of irrational speculators, following Kyle (1985), rather than as import and export shocks. This addition to the model is important since in reality much foreign exchange trade takes place among speculators, rather than between speculators and goods traders, as the basic form of the model requires. Furthermore, many authors have found that survey data on exchange rate forecasts are not consistent with rationality (Frankel and Froot 1987 and 1990; Ito 1990; Takagi 1991), so assuming that all speculators are completely rational is not likely to be realistic. With shocks interpreted as the activity of irrational traders, the model indicates that irrational behavior can alter



financial price dynamics, even in the presence of fully rational agents, consistent with the rest of the noise trader literature.

This interpretation of the model may also help evaluate the foreign exchange market's concern that mutual funds' trading "could temporarily create sharp price swings" (Roman 1992). Managers of mutual funds are reported to rely heavily on "technical analysis" to determine their betting (Allen and Taylor 1990), a strategy regarded by many economists as equivalent to noise trading. If one accepts the equivalence of technical analysis and noise trading (which may be inappropriate in the foreign exchange market: see Sweeney 1986, and Levich and Thomas 1991), then, by showing that noise traders actually generate exchange rate movements, the noise trader interpretation of the model supports this concern.

The approach to analyzing the effects of short-term speculators suggested here could be extended in many potentially interesting directions. Interest rate disturbances could be included; importers' and exporters' behavior could incorporate the long lags that exist between exchange rate changes and the response of imports and exports; finally, the signals available to speculators could incorporate noise.

## Endnotes

1. An alternative version of this model, in which interest rates are important, shows that the presence of interest rates does not affect the conclusions of this paper regarding the random-walk behavior of exchange rates.
2. Why is there no loss of generality in assuming  $S_x + S_m > 0$ ? Consider the consequences of  $S_x + S_m < 0$ . In this case one can redefine the fundamental shocks as  $\epsilon'_t = -\epsilon_t$ , and also redefine the semi-elasticities as  $S'_x = -S_x$  and  $S'_m = -S_m$ . The new model, in which  $S'_x + S'_m > 0$  is mathematically identical to the old model, except for a slight change in the representation of the exchange rate's unconditional mean. The old version of the mean would have been  $\alpha = (C_x - C_m)/(S_x + S_m)$ , while the new version is  $\alpha' = - (C_x - C_m)/(S'_x + S'_m)$ .
3. Importers could also be subject to shocks, but this would increase notational complexity with no corresponding gain in generality.
4. The distinction between "goods-and-services traders" and "speculators" is not as clear in reality as it is presented here. In particular, goods-and-services traders are likely to incorporate their views on future exchange rate changes in their decisions on the timing of hedging purchases. This does not undermine the conclusions of this paper. The mixed motives can be incorporated in this model by reinterpreting the parameter  $N$ , which represents the number of speculators, as the extent of speculative concerns, aggregating over all market participants.
5. The solution to the model with autocorrelated disturbances is available from the author. Its behavior is almost identical to the behavior described here.
6. This utility function is common in speculative dynamics models. See, for example, Stein (1987) and De Long, Schleifer, Summers, and Waldman (1990).
7. If traders operate as if participating in a Cournot-type market, incorporating in their decisions their effects on exchange rates while assuming their competitors' bets are fixed, then the optimal bet coefficient differs only slightly from that associated with the competitive model presented in the text. In particular, the optimal bet coefficient is slightly smaller in the Cournot model, at  $Q = 1/[\gamma \text{Var}(e_{t+1}) + 2/(S_x + S_m)]$ , than the bet coefficient described below in the text. An analysis of the Cournot model's comparative statics shows that the qualitative results of the competitive model carry through unchanged in the Cournot model.

8. The results are simply the comparative statics of the model, many of which follow immediately from the model's solution, equation 3. Other results are available from the author.

9. This is true whether or not it is assumed that the risk aversion of speculators in other markets changes in parallel with the risk aversion of foreign exchange speculators.

10. Two hundred fifty-one observations provide 240 to 250 degrees of freedom, depending on the particular econometric exercise. This range corresponds roughly to the degrees of freedom available from monthly exchange rate series covering the floating rate period, or daily exchange rates for a little more than a year.

11. This can be seen clearly by examining equation 2, which shows that speculators' net foreign exchange demand in a given period is a function only of current and previous expected exchange rate changes, which in turn can only be a function of the predictable components of the exchange rate itself. Thus, the random-walk disturbances will not affect speculators' bets, and will enter the market equilibrium condition only through their influence on the importers' and exporters' foreign exchange demands.

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## Appendix A

*Proof of general properties of the exchange rates under mean-averting disturbances.*

For notational convenience the initial time  $t$  will be taken to be 0.

### I. Underlying Assumptions and Their Implications

(i) Fundamental Shocks: We examine the effects over time of a positive shock at  $t=0$ :

$$\epsilon_{-i} = 0 \quad \forall i \geq 1. \quad \epsilon_0 > 0. \quad E_0\{\epsilon_j\}_{j \geq 1} \equiv \rho_j(\epsilon_0).$$

Disturbances are mean averting but not explosive:

$$\epsilon_0 < \rho_j(\epsilon_0) < \rho_{j+1}(\epsilon_0). \quad 0 < \lim_{j \rightarrow \infty} \rho_j(\epsilon_0) \equiv \rho(\epsilon_0) < A < \infty.$$

Future disturbances are distributed symmetrically around their conditional mean,  $\rho_j(\epsilon_0)$ .

(ii) Bets: The speculators' bet function,  $B(x)$ , where  $x \equiv E_t\{e_{t+1}\} - e_t$ , is an odd function,  $B(x) \in C^\infty(\mathfrak{R})$  and  $B'(x) \geq 0$ . This further implies  $B(0) = 0$ . Finally, it will be assumed that  $\lim_{j \rightarrow \infty} E_0\{B_j\} = 0$ .

According to the market equilibrium condition, the exchange rate in each period will equal

$$e_t = \frac{C_x - C_m}{S_x + S_m} + \frac{NB_t - NB_{t-1} + \epsilon_t}{S_x + S_m}$$

or

$$e_t = \alpha + \frac{NB_t - NB_{(t-1)} + \epsilon_t}{S_x + S_m}.$$

The assumption that speculators are active in the market implies  $N > 0$ .

At any time such as  $t = 0$ , there will be an expected exchange rate and an expected bet for each future period, and these will be related as follows:

$$(A1) \quad E_0\{e_j\} = \alpha + \frac{E_0\{NB_j\} - E_0\{NB_{j-1}\} + \rho_j(\epsilon_0)}{S_x + S_m}.$$

## II. Proof of Results (1) through (3) in the Text

*Result (1): The series of expected future exchange rates will converge monotonically but will never reach its limit.*

The proof will have three parts, (a), (b), and (c):

(a): *The series of expected future exchange rates converges to the exchange rate's conditional expected value:  $\lim_{j \rightarrow \infty} E_0\{e_j - \alpha - \rho(\epsilon_0)/(S_x + S_m)\} = 0$ .*

Proof: Note first that

$$\lim_{j \rightarrow \infty} E_0\{e_j - \alpha - \rho(\epsilon_0)/(S_x + S_m)\} = (1/(S_x + S_m)) \lim_{j \rightarrow \infty} E_0\{NB_j - NB_{j-1}\} = 0,$$

where the first equality follows from equation A1 and the second follows from assumption (ii). This implies the required result directly:

$$\lim_{j \rightarrow \infty} E_0\{e_j\} = \alpha + \lim_{j \rightarrow \infty} \rho_j(\cdot)/(S_x + S_m) = \alpha + \bar{\rho}(\epsilon_0)/(S_x + S_m).$$

(b): *The series of expected future exchange rates cannot oscillate.*

Proof: Assume the contrary: There is some triplet of expected future exchange rates such that they are expected to rise and then fall:

$$E_0\{e_{j-1}\} < E_0\{e_j\} > E_0\{e_{j+1}\} .$$

Rationality implies certain properties for the expected future bets in period  $j-1$  and  $j$ :

$$E_0\{B_{j-1}\} > 0 , \text{ and } E_0\{B_j\} < 0 .$$

From these properties of expected future bets, we can infer more about the expected future exchange rate of period  $j$  relative to its value in the absence of speculators. In particular, the net bet in period  $j$  must be negative, since it is the difference between the negative absolute bet in period  $j$  and the positive absolute bet in  $j-1$ :

$$E_0\{e_j\} - \alpha - \rho_j(\epsilon_0)/(S_x+S_m) = [N/(S_x+S_m)] E_0\{B_j-B_{j-1}\} < 0 .$$

We now know that if expected future exchange rates oscillate in a rising-then-falling pattern then the exchange rate must be expected to begin below  $\alpha + \rho_j(\epsilon_0)/(S_x+S_m)$ , rise once again while remaining below  $\alpha + \rho_j(\epsilon_0)/(S_x+S_m)$ , and then fall once again. The reverse must be true of any series of expected future exchange rates that oscillate in the mirror image pattern of falling-then-rising around a point  $\hat{f}$ : the exchange rate must be expected to begin the pattern above  $\alpha + \rho_j(\epsilon_0)/(S_x+S_m)$ , fall while still remaining above  $\alpha + \rho_j(\epsilon_0)/(S_x+S_m)$ , and then rise again.

Clearly, the first pattern cannot precede the second, because this would require that  $\rho_j(\epsilon_0) > \rho_j(\epsilon_0)$ , while  $\hat{f} < j$ , contradicting the assumption that the effects of an initial shock rise monotonically, as stated in (i). Thus, if the series of expected exchange rates begins by rising, it can never oscillate.

Suppose the series of expected exchange rates begins by falling between periods 0 and 1. To be consistent with rationality, speculators' initial bets must be nonpositive,  $B_0 \leq 0$ .

If the series of expected exchange rates is to change direction from falling to rising, it must begin above  $\alpha + \epsilon_0/(S_x + S_m)$  (because it must be expected to be above  $\alpha + \rho_f(\epsilon_0)/(S_x + S_m)$  when it begins to rise, as just shown, and  $\epsilon_0 < \rho_f(\epsilon_0)$  by assumption of mean aversion). Since all bets before period 0 must be zero (by assumptions (i) and (ii)), the only way  $e_0$  can be above  $\alpha + \epsilon_0/(S_x + S_m)$  is for the initial bet to be positive,

$$e_0 > \alpha + \epsilon_0/(S_x + S_m) \Rightarrow NB_0/(S_x + S_m) > 0 .$$

But if the initial bet is positive, we have contradicted our earlier conclusion that  $B_0 \leq 0$ , and this implies that the exchange rate cannot oscillate if it begins by falling.

Since the series of expected exchange rates cannot oscillate if it begins by rising or if it begins by falling, it cannot oscillate at all.

*(c): The series of expected exchange rates never reaches the exchange rate's conditional mean:*

$$E_0\{e_j - \alpha - \bar{\rho}(\epsilon_0)/(S_x + S_m)\} > 0, \forall j > 0 .$$

**Proof:** Suppose the contrary, so that there is some point  $K < \infty$  in the future before which the expected exchange rate is less than  $\alpha + \bar{\rho}(\epsilon_0)/(S_x + S_m)$  and for which  $E_0\{e_K\} = \alpha + \bar{\rho}(\epsilon_0)/(S_x + S_m)$ . Since the series of expected exchange rates cannot oscillate, once  $E_0\{e_K\} = \alpha + \bar{\rho}(\epsilon_0)/(S_x + S_m)$ , all future expected exchange rates must also equal  $\alpha + \bar{\rho}(\epsilon_0)/(S_x + S_m)$ .

The information provided so far is sufficient to show  $E_0\{B_{K-1}\} > 0$  while all  $E_0\{B_{K+j}\} = 0 \quad \forall j \geq 0$ .

Knowing the properties of the bet series, we can infer more about the series of expected future exchange rates. In particular, we can infer that our premise,  $E_0\{e_N\} = \alpha + \bar{\rho}(\epsilon_0)/(S_x+S_m)$ , must be wrong, because:

$$E_0\{e_K - \alpha - \bar{\rho}(\epsilon_0)/(S_x+S_m)\} = 0 \text{ implies that}$$

$$\frac{1}{S_x+S_m}[\rho_K(\epsilon_0) - \bar{\rho}(\epsilon_0) + E_0\{NB_K - NB_{K-1}\}] = 0,$$

or

$$\rho_K(\epsilon_0) - \bar{\rho}(\epsilon_0) = E_0\{NB_{K-1} - NB_K\} = E_0\{NB_{K-1}\}.$$

But  $\rho_K(\epsilon_0) - \bar{\rho}(\epsilon_0) < 0$  if  $\epsilon_0 > 0$ , by assumption (i), implying a contradiction:  $0 > \rho_K(\epsilon_0) - \bar{\rho}(\epsilon_0) = E_0\{NB_{K-1}\} > 0$ .

*Result (2): The average difference between the current exchange rate and any future expected exchange rate is smaller in the presence of speculators than in their absence.*

**Proof:** Recursive substitution using equation A1 shows that the aggregate difference between the expected path of the exchange rate with and without speculators is proportional to the negative of speculators' initial aggregate bet:

$$\sum_{j=1}^{\infty} [E_0\{e_j\} - \alpha - \frac{\rho_j(\epsilon_0)}{S_x+S_m}] = -\frac{NB_0}{S_x+S_m} < 0.$$

The initial bet must be positive to be consistent with rationality, since it has already been shown that the exchange rate must rise monotonically after period 0. Thus, the exchange rate in period 0 must exceed the value it would have in the absence of speculators. Further, the average future difference between the exchange rate's expected path with and without speculators is negative. Given that the exchange rate rises monotonically, we have enough information to infer that

$|e_0 - E_0\{e_j\}|, j > 0$ , is on average smaller in the presence of speculators than in their absence.

*Result (3): If speculators' bets are linearly related to expected exchange rate changes, then the path of future expected exchange rates must begin above the expected exchange rate path of the no-speculator equilibrium and cross the no-speculator path only once.*

Proof: Once again, the proof will be subdivided into three parts.

(a) It has already been shown that the exchange rate must begin above the no-speculator initial value, because it has been shown that  $B_0 > 0$ .

(b) The exchange rate must rise more slowly than it would in the absence of speculators, until the two paths cross. Proof:

If the exchange rate is expected to rise by more than it would in the absence of speculators between two periods  $j-1$  and  $j$ , then

$$E_0\{e_j\} - \alpha - \rho_j(\epsilon_0)/(S_x + S_m) > E_0\{e_{j-1}\} - \alpha - \rho_{j-1}(\epsilon_0)/(S_x + S_m) > 0.$$

This expression implies, by the linearity of speculators' bets, that  $|E\{B_{j-1}\}| < |E\{B_j\}|$ ; therefore, the exchange rate's rate of increase is expected to rise between periods  $j-1$  and  $j$ . In turn, this requires that

$$E_0\{e_{j+1}\} - \alpha - \rho_{j+1}(\epsilon_0)/(S_x + S_m) > E_0\{e_j\} - \alpha - \rho_j(\epsilon_0)/(S_x + S_m) > 0,$$

an expression implying that  $|E\{B_j\}| < |E\{B_{j+1}\}|$ . Thus, the exchange rate's increase is expected to rise further between periods  $j$  and  $j+1$ . Inductive reasoning shows that the exchange rate's rate of increase must be expected to accelerate indefinitely, but such explosive growth is inconsistent with the fact that the expected exchange rate has a finite limit,  $\lim_{j \rightarrow \infty} E_0\{e_j\} = L$ .

(c) Proof that in the presence of speculators the exchange rate cannot rise above the no-speculator path once it has crossed below that path.

It has been shown so far that in the presence of speculators the expected exchange rate must begin by rising more slowly than it would have in their absence, and it must continue to rise more slowly at least until it crosses the no-speculator path. Thereafter, it cannot rise above the no-speculator path, because if it did so the rate of increase of the expected exchange rate would exceed that of the no-speculator equilibrium, a result that is impossible given the analysis under (b) above.

## Appendix B

### *I. Brief Description of Unit Root Tests*

(1) The  $F$ -test asks whether the coefficients in the following regression are all zero:

$$de_t = \alpha + \sum_{i=1}^T \rho_i de_{t-i} + v_t$$

where  $de_{t-i} \equiv e_{t-i} - e_{t-i-1}$ .  $T$  is set at 9.

(2) A " $d$ -test," suggested by Sargan (1983) and based on the von Neumann ratio, compares average squared first differences of the exchange rate with its unconditional variance.

(3 and 4) The simple Dickey-Fuller test,  $DF$ , asks whether  $\rho = 0$  in this equation:

$$(B1) \quad e_t = \alpha + \rho e_{t-1} + \zeta_t$$

(Fuller 1976). Confidence intervals for the standard  $t$ -test on  $\rho$  are modified to account for the infinite variance of the right-hand-side variable under the



null hypothesis. Phillips and Perron (1988) proposed a modification of this test, " $Z(DF)$ ," which imposes "weaker conditions ... on the stochastic innovations driving the system" (Perron 1988).

(5) Dickey and Fuller (1979, 1981) modified their original test to include lags of first differences of the regressors as right-hand-side variables. These "augmented Dickey-Fuller" tests apply the original Dickey-Fuller confidence intervals to the standard  $t$ -statistic for  $\rho$  found when estimating the following regression:

$$e_t = \alpha + \rho e_{t-1} + \sum_i \beta_i de_{t-i} + \eta_t$$

These tests were run with three lagged first-differences, " $ADF(3)$ ," and with eight lagged first differences, " $ADF(8)$ ."

(6 and 7) Dickey and Fuller (1981) propose a joint test of the full null hypothesis  $H_0: (\alpha, \rho) = (0, 1)$  against the diffuse alternative  $(\alpha, \rho) \neq (0, 1)$ , in equation  $B1$ . The statistic " $\Phi_1$ " is compared with Monte-Carlo derived confidence intervals (1981).  $Z(\Phi_1)$  modifies the original statistic  $\Phi_1$  to allow for more general innovation processes (Perron 1986).

The remaining tests are based on equation  $B2$ :

$$(B2) \quad e_t = \alpha + \beta_t + \rho e_{t-1} + \delta_t$$

where  $t$  is a linear trend term.

(8) Phillips and Perron (1988) suggest a non-parametric version of the standard  $t$ -test of the null hypothesis  $\rho = 1$  in (4),  $Z(\rho)$ .

(9 and 10) Dickey and Fuller (1981) suggest the " $\Phi_2$ " test of the null hypothesis  $(\alpha, \beta, \rho) = (0, 0, 1)$  and provide confidence intervals (1981). Perron (1986) suggests the non-parametric relative of  $\Phi_2$ ,  $Z(\Phi_2)$ .

(11 and 12) Dickey and Fuller (1981) also suggest the " $\Phi_3$ " test of the null hypothesis  $(\alpha, \beta, \rho) = (\alpha, 0, 1)$ . Likewise, Perron (1986) suggests  $\Phi_3$ 's non-parametric relative  $Z(\Phi_3)$ .

## ***II. Observations Concerning the Relative Power of Unit Root Tests***

The results of the unit root tests can serve as a measure of the power of these tests for particular moving average processes. The least powerful of these tests are the  $F$  test, the standard Dickey-Fuller tests, and the Augmented Dickey-Fuller test with a high number of lags.

The  $d$ -test is particularly powerful, consistent with the analysis of Dickey and Fuller (1981): "[t]he statistic  $d$  is an appropriate test when the alternative is that  $e_t$  is a stationary first-order autoregressive time series. It displays good power for this alternative (that is, when  $\alpha = 0$  and  $\rho < 1$ )." The augmented Dickey-Fuller test with only three lagged first differences is also quite powerful.

Phillip and Perron's modifications of Dickey and Fuller's " $\Phi$ " tests do not strongly affect these results because the exchange rate in these simulations is actually distributed normally, while the tests' primary contribution is to allow for non-normal distributions.

**Table 1: Frequency with Which a Random Walk Hypothesis  
Would Not Be Rejected**

Test	Significance	Significance					
		0.97 <sup>1</sup>	0.95	0.90	0.85	0.80	0.75
<i>F</i>	1%	98%	96%	90%	76%	69%	16%
	5	95	90	70	46	37	4
<i>d</i>	1	89	72	13	0	0	0
	5	77	35	0	0	0	0
<i>DF</i>	1	100	100	97	92	83	11
	5	100	100	95	68	56	1
<i>Z(DF)</i>	1	100	98	96	77	34	12
	5	99	97	85	44	17	6
<i>ADF(3)</i>	1	98	90	49	9	0	0
	5	92	68	8	0	0	0
<i>ADF(8)</i>	1	99	97	72	53	31	18
	5	96	78	45	16	7	1
$\Phi_1$	2	92	85	33	11	2	0
	10	74	58	12	0	0	0
<i>Z(Φ<sub>1</sub>)</i>	2	88	83	27	9	4	0
	10	82	42	11	0	0	0
<i>Z(ρ)*</i>	1	87	84	15	11	1	0
	5	79	64	3	2	0	0
$\Phi_2^*$	2	91	95	73	33	27	0
	10	81	85	37	15	11	0
<i>Z(Φ<sub>2</sub>)*</i>	2	87	92	74	33	23	0
	10	80	79	36	17	12	0
$\Phi_3^*$	2	90	94	58	26	19	0
	10	78	76	29	10	3	0
<i>Z(Φ<sub>3</sub>)*</i>	2	89	92	53	26	18	0
	10	77	76	27	11	4	0

<sup>1</sup> Including a trend term increases the likelihood of statistical rejection of the random walk hypothesis if the number of observations is not very large. The reason is that the trend term picks up behavior that is actually purely random, thereby polluting the tests. This is especially true when  $\lambda$  is close to unity. Those tests that are based on equations incorporating a trend term are marked with an \*.

**Table 2a: Frequency with Which a Random Walk Model Explains Past Exchange Rates Better Than a Structural Model (% of 100)**

Criterion:	$\lambda$					
	0.98	0.95	0.90	0.875	0.85	0.80
Mean squared error	100%	100%	90%	35%	1%	0%
Mean absolute error	100	100	89	38	1	0
Addendum: Mean $R^2$ of "Structural model"	0.27	0.48	0.73	0.81	0.86	0.93

**Table 2b: Frequency with Which a Random Walk Model Forecasts Exchange Rates Better Than a Structural Model**

Criterion:	$\lambda$					
	0.98	0.95	0.90	0.875	0.85	0.80
Mean squared error	97%	94%	60%	44%	3%	0%
Mean absolute error	97	93	64	34	10	0

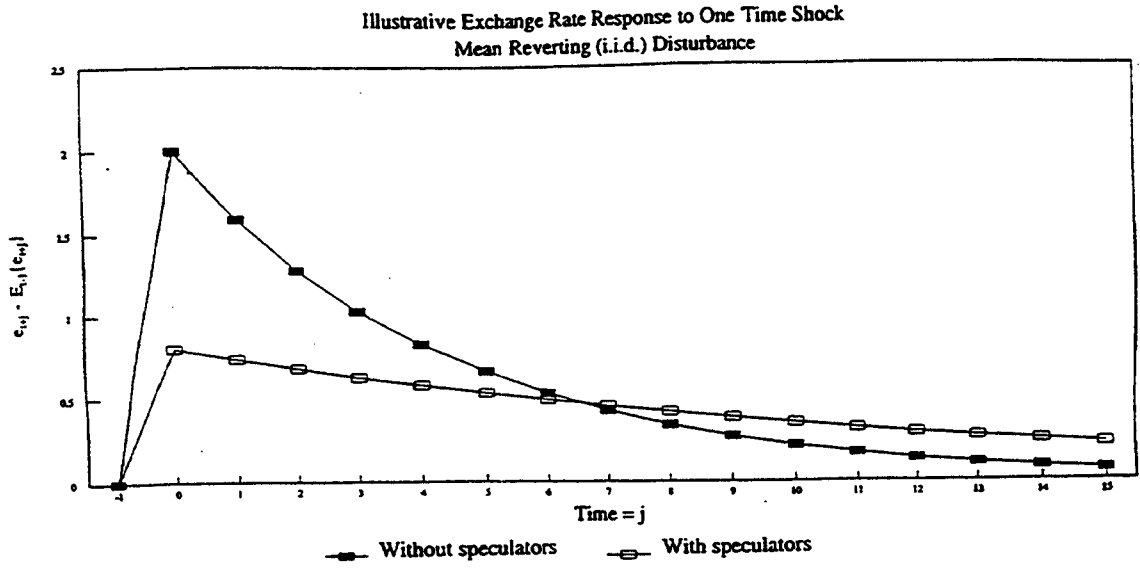


Figure 1

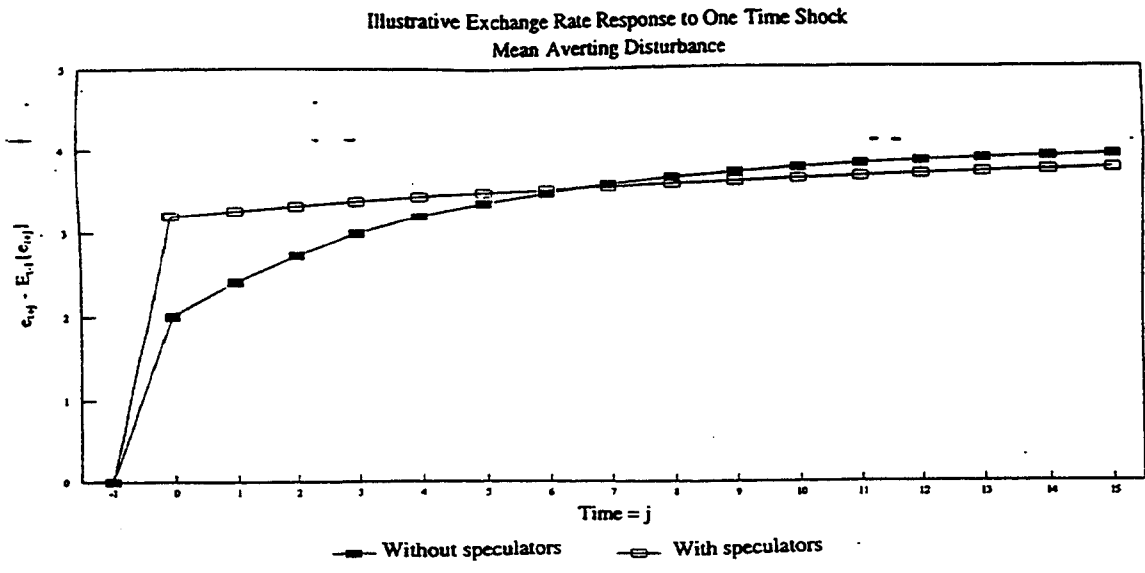


Figure 2