ERROR CORRECTION MECHANISMS AND
SHORT-RUN EXPECTATIONS

Angelos A. Antzoulatos

Error Correction Mechanisms and Short-run Expectations

Angelos A. Antzoulatos
International Macroeconomics Function
Research and Market Analysis Group
Federal Reserve Bank of New York

ABSTRACT

Reflecting the nature of economic decisions, the error-correction mechanism (ECM) in the error-correction representation of a system of co-integrated variables may arise from forward-looking behavior. In such a case, the estimated ECM coefficients may misleadingly appear to be insignificant or to have the opposite-than-expected sign if the variables in the error-correction representation do not adequately capture short-run expectations. This paper explores the nature of this problem with a theoretical model for consumption and demonstrates how severe the problem can be with U.S. data. Because the conditions for similar erroneous inferences are likely to apply to many other settings, the paper also recommends a reexamination of the evidence in cases where the ECM appears to be insignificant or to display the "wrong" sign.

I. Introduction

Since the publication of Engle and Granger's (1987) seminal paper, "Co-integration and Error Correction: Representation, Estimation, and Testing," it has been widely known that a system of co-integrated variables has an error-correction representation in which the vector autoregression (VAR) in differenced variables contains an error-correction mechanism (ECM). In Engle and Granger's paper, the ECM emerges as a statistical property of the data, which supports theoretical explanations that stress an ECM's resemblance to a feedback control rule driven by some partial adjustment mechanism (see, for example, Davidson et al. 1978 and Hendry et al. 1990). Nevertheless, because the nature of economic decisions suggests that the ECM may arise from forward-looking behavior, it may also reflect expectations about future events (see, among others, Campbell 1987 and Alogoskoufis and Smith 1991). Thus, as this paper shows, estimated ECM coefficients can misleadingly appear to be insignificant or to have the opposite-than-expected sign if the other explanatory variables in the error-correction representation generate poor conditional forecasts for the system's endogenous variables. In turn, the erroneous inferences about the ECM coefficients can lead to misspecified econometric models in which the ECM's promise of better short-run forecasts will not materialize.

This paper explores the problem of erroneous inferences regarding the estimated ECM coefficients by using the system of consumption and income. It then demonstrates the potential magnitude of these errors with

U.S. data. Specifically, the second section combines the forward-looking nature of consumption with a partial adjustment mechanism to derive a theoretical model for consumption. In this model, consumption growth is increasing in contemporaneous and (expected) future income growth and decreasing in the ECM, while the ECM is increasing in future income growth. Conversely, in the typical error correction representation, consumption growth is a function of the ECM and of lagged values of consumption and income growth. Provided that these lagged values generate good income-growth forecasts, the estimated ECM coefficient in the error-correction representation will be negative. If not, as in the case of U.S. data, the coefficient will be positively "biased." This bias reflects the positive correlation of future income growth with both the ECM and consumption growth. The bias can be so severe that the ECM coefficient can misleadingly appear to be insignificant or, even worse, positive. How can this problem be addressed? The bias can be reduced by including in the error-correction representation other stationary variables that can help predict income growth.

The empirical evidence, in Section 3, illustrates the problem of erroneous inferences regarding estimated ECM coefficients. The ECM (the lagged residuals of the co-integrating regression of log consumption on log income and a constant) is positively correlated with future income growth. However, its coefficient in the error correction representation is positive and insignificant. We test whether this result is due to the poor income-growth forecasts generated by lagged consumption and income growth terms by including contemporaneous income growth in the regression (the working hypothesis is that expected values differ from realized ones by an

unpredictable stochastic term (Cumby et al. 1983). In this equation, the ECM coefficient becomes negative but remains insignificant. However, the addition of future income growth makes the coefficient significantly negative at the 5 percent level. The addition of another variable that can help predict future income growth (the first lag of the growth rate of the Composite Index of Eleven Leading Indicators) increases the coefficient's significance to the 1 percent level. Overall, in accordance with the theory outlined in Section 2, each additional variable correlated with future income growth helps increase both the significance level and the absolute value of the estimated ECM coefficient.

The forward-looking nature of economic decisions and the difficulty of modeling expectations suggest that the conditions for erroneous inferences about the estimated ECM coefficients are likely to apply to many other settings. For this reason, Section 4 recommends a reevaluation of the evidence in cases where the ECM coefficient appears to be insignificant or with the wrong sign.

II. The Case of Consumption and Income

A. A Theoretical Model for the Error Correction Mechanism

Equations 1 through 3 summarize the problem faced by a forward-looking individual who maximizes the expected value of a time-separable utility function subject to a borrowing constraint:

$$(1) \qquad Max \; E_t \left\{ \sum_{k=0}^{T} \left(\frac{1}{1+\delta} \right)^k u(C_{t+k}) \right\} \quad (\delta > 0)$$

subject to

$$(2) \qquad\qquad C_t \leq X_t$$

$$(3) \qquad X_t = RS_{t-1} + I_t = R(X_{t-1}-C_{t-1}) + I_t \qquad (R \geq 1).$$

The timing and the nature of his decisions are as follows: at the beginning of period t, he decides how much to consume, C_t, in order to maximize his expected utility (equation 1) over the remaining planning horizon. As a result of the borrowing constraint (equation 2) consumption cannot exceed the individual's total assets, X_t. X_t is equal to the savings carried over from $t-1$, $RS_{t-1}=R(X_{t-1}-C_{t-1})$, plus the realized income, I_t, as described by equation 3. For simplicity, and without loss of generality, the real interest rate (R is equal to one plus the real interest rate) and the length of the planning horizon, T, are taken as exogenous. Because the analysis is based on the stochastic implications of the permanent income hypothesis (PIH), no particular assumptions are needed for the income process (Hall 1978). Capital letters denote levels, small letters denote logs and Δ is the difference operator. Thus, $\Delta c_t = c_t - c_{t-1}$ denotes consumption growth at t.

There is no closed-form solution for optimal consumption under uncertainty and borrowing constraints. However, by using the first-order condition of the individual's optimization problem, it is clear that optimal consumption is increasing in contemporaneous assets and expected future income (for a formal analysis, see, among others, Schechtman and Escudero 1977, Deaton 1991 and Antzoulatos 1994a). In mathematical terms, C_{t-1} in equation 4 is increasing in X_{t-1} and $E_{t-1}I_{t+k}$ ($k \geq 0$):

$$(4) \qquad C_{t-1} = C_{t-1}(X_{t-1}; E_{t-1}I_t; E_{t-1}I_{t+1}; E_{t-1}I_{t+2}; \dots).$$

Similarly, $E_{t-1}C_t$ is increasing in $E_{t-1}I_{t+k}$ ($k \geq 0$).

When the individual is at an interior solution at t-1 (nonbinding constraint), expected consumption change, $E_{t-1}\Delta C_t$, will be unrelated to ECM_{t-1} and any other economic variable observed at t-1 or earlier, as the permanent income hypothesis postulates (Hall 1978). This reflects the fact that both C_{t-1} and $E_{t-1}C_t$ are increasing in expected income. The error-correction term, however,

(5) $$ECM_{t-1} = C_{t-1} - I_{t-1}$$

will be increasing in $E_{t-1}I_{t+k}$ $(k \geq 0)$ (equation 4).

However, when the individual faces a binding constraint at t-1 (corner solution), expected consumption change will be increasing in $E_{t-1}I_{t+k}$ $(k \geq 0)$ and decreasing in ECM_{t-1}. At a corner solution, it is $C_{t-1}=X_{t-1}=RS_{t-1}+I_{t-1}$. Thus,
$$ECM_{t-1} = (RS_{t-2}+I_{t-1}) - I_{t-1} = RS_{t-2}.$$

Also, $S_{t-1}=0$, $X_t=I_t$ and

(6) $\quad E_{t-1}\Delta C_t = E_{t-1}C_t - C_{t-1} = E_{t-1}C_t(E_{t-1}X_t;E_{t-1}I_{t+1};E_{t-1}I_{t+2};...) - C_{t-1}$
$\qquad = E_{t-1}C_t(E_{t-1}I_t;E_{t-1}I_{t+1};E_{t-1}I_{t+2};...) - (RS_{t-1}+I_{t-1}).$

Because $E_{t-1}C_t$ is increasing in $E_{t-1}I_{t+k}$ $(k \geq 0)$, expected consumption change will increase in $E_{t-1}I_{t+k}$ $(k \geq 0)$ as well. (For a more formal discussion in the Euler equation framework, see Antzoulatos 1994b.) One caveat arises: if the individual is expected to be at a corner solution at t, $E_{t-1}C_t=E_{t-1}X_t=E_{t-1}I_t$, consumption change will increase in contemporaneous income only. Further,

$E_{t-1}\Delta C_t$ will be decreasing in RS_{t-2} and consequently in ECM_{t-1}.

In terms of the ECM, when the individual is at a corner solution, ECM_{t-1} will tend to be positively correlated with $E_{t-1}I_{t+k}$ $(k \geq 0)$. The reason is that the individual will spend all his savings plus contemporaneous income at t-1—rising C_{t-1} and ECM_{t-1}—when he expects higher income in the future. However, when he gets to a corner solution (that is, when C_{t-1} becomes equal to X_{t-1}) C_{t-1}—and ECM_{t-1}—cannot increase further in anticipation of high future income. Consequently, at the limit, ECM_{t-1} may be uncorrelated with $E_{t-1}I_{t+k}$ $(k \geq 0)$.

The above analysis postulates the following function for expected aggregate consumption change, the sum of individual $E_{t-1}\Delta C_t s$:

(7) $E_{t-1}\Delta C_t = f(ECM_{t-1}, E_{t-1}I_t, E_{t-1}I_{t+1}, E_{t-1}I_{t+2}, ...).$

Because of the individuals who are at a corner solution at t-1, $E_{t-1}\Delta C_t$ will tend to be decreasing in ECM_{t-1} and increasing in $E_{t-1}I_{t+k}$ $(k \geq 0)$. Thus, in the log-linear version of equation 7, given by equation 8 below,[1] β_0 is expected to be negative, while γ_k $(k \geq 0)$ are expected to be positive (the ECM's log version is given by equation 9). Also, under the assumption that realized consumption change differs from that expected by an unpredict-

---

[1] A first-order Taylor expansion of equation 7 around the point $(0, I_{t-1}, E_{t-1}I_t, E_{t-1}I_{t+1}, ...)$
gives $E_{t-1}\Delta C_t = f(0, I_{t-1}, E_{t-1}I_t, E_{t-1}I_{t+1}, ...) + f_1*ECM_{t-1} + f_2*(E_{t-1}I_t - I_{t-1}) +$
$f_3*(E_{t-1}I_{t+1} - E_{t-1}I_t) + f_4*(E_{t-1}I_{t+2} - E_{t-1}I_{t+1}) + ... + u_t$
f_i denotes the partial derivative of $f(.)$ with respect to the i argument.

With the exception of f_1, all f_i are expected to be positive. Next, since consumption and income appear to be $I(1)$ and co-integrated in logs, the log version of this equation is used, as in Campbell and Mankiw (1990 and 1991).

able stochastic term, u_t should be unrelated to any variables observed at t-1 or before.

$$(8) \Delta ct = constant + \beta_0 ECM_{t-1} + \gamma_0 E_{t-1} \Delta i_t + \gamma_1 E_{t-1} \Delta i_{t+1} + \gamma_2 E_{t-1} \Delta i_{t+2} + ... + u_t.$$

$$(9) \qquad\qquad ECM_{t-1} = c_{t-1} - i_{t-1}.$$

Further, because of the individuals who are at an interior solution at t-1, ECM_{t-1} will tend to be increasing in $E_{t-1} I_{t+k}$ $(k \geq 0)$ even when the ECM_{t-1} is uncorrelated with future income for individuals at a corner solution. As a result, the omission of ECM_{t-1} in equation 8 will induce a negative bias in the regression estimates of γ_k $(k \geq 0)$. This bias reflects the positive relationship between future income and ECM_{t-1} in conjunction with β_0's negative sign, as in the typical case of omitted variables in a regression equation analyzed in Johnston (1984, p. 260). Similarly, the omission of $E_{t-1} I_{t+k}$ $(k \geq 0)$ will induce a positive bias in β_0's estimate which, as a result, can be insignificant or even positive.[2]

In this theoretical model, the partial adjustment of consumption to

---

[2] In the limiting case, when the ECM is uncorrelated with future income for the individuals who are at a corner solution, the bias in aggregate data would arise from the aggregation across all individuals. It would also arise in pooled micro data when the individual income processes are correlated; that is, when they contain a common "aggregate" component in addition to idiosyncratic ones. Of course, if constrained and unconstrained individuals could be identified in micro data, estimating equation 8 for the latter would produce no bias as all coefficients would be zero. But there would be some bias for the constrained individuals only when the ECM is correlated with future income.

expected income, driven by the binding borrowing constraint, induces the negative relationship between Δc_t and ECM_{t-1} and the positive relationship between Δc_t and $E_{t-1}I_{t+k}$ $(k \geq 0)$. Also, the forward-looking nature of consumption induces the positive relationship between ECM_{t-1} and $E_{t-1}I_{t+k}$ $(k \geq 0)$. Nevertheless, the same conclusions, culminating in the same bias for the estimated ECM coefficients, can be derived from more general models that combine forward-looking behavior with some partial adjustment mechanism. Section IIB presents such a model (adapted from Davidson and MacKinnon 1993, p. 680) whose very general structure clearly demonstrates that the problems of erroneous inferences about ECM coefficients may be pervasive.

B. An Alternative Model

Let optimal consumption c_t^* be a function of contemporaneous and expected future income (equation 10). The coefficients μ_k $(k \geq 0)$ are positive, while ε_t is a stochastic disturbance. Because of some sort of adjustment costs, people cannot set actual consumption, c_t, equal to c_t^*. Instead, c_t adjusts towards c_t^* as described by equation 11. (Equation 11 can also be justified by partial adjustment to income shocks resulting from small utility costs of deviation from the optimal consumption path, as in Caballero 1995.) The term $(1-\xi)$ measures the speed of adjustment, while e_t is a stochastic term.

(10) $\qquad c_t^* = \mu + \mu_0 i_t + \mu_1 E_t i_{t+1} + \mu_2 E_t i_{t+2} + \ldots + \mu_p E_t i_{t+p} + \varepsilon_t \, .$

$$(11) \qquad c_t\text{-}c_{t\text{-}1} = (1\text{-}\xi)(c_t^*\text{-}c_{t\text{-}1})+e_t \qquad (0<1\text{-}\xi<1).$$

Subtracting $c_{t\text{-}1}$ from both sides of equation 10, rearranging terms and multiplying both sides of the resulting equation with $(1\text{-}\xi)$ result in the following expression for equation 11:

$$(12) \qquad \Delta c_t = c_t\text{-}c_{t\text{-}1} = (1\text{-}\xi)\mu\text{-}(1\text{-}\xi)(c_{t\text{-}1}\text{-}\lambda i_{t\text{-}1}) + \psi_0(i_t\text{-}i_{t\text{-}1})$$
$$+\psi_1(E_t i_{t+1}\text{-}i_t) + \psi_2(E_t i_{t+2}\text{-}E_t i_{t+1})+ \ldots +\psi_p(E_t i_{t+p}\text{-}E_t i_{t+p\text{-}1}) + [(1\text{-}\xi)\varepsilon_t+e_t]$$

$$= (1\text{-}\xi)\mu - (1\text{-}\xi)(c_{t\text{-}1}\text{-}\lambda i_{t\text{-}1}) + \sum_{k=0}^{p} \psi_k E_t \Delta i_{t+k}+u_t$$

where $\psi_p = (1-\xi)\mu_p, \psi_k = (1-\xi)\sum_{j=0}^{p-k}\mu_{p-j}(k=0,1,2,\ldots,p\text{-}1)$ and

$\lambda(1\text{-}\xi)=\psi_0$. As in equation 8, all the coefficients of the income-growth terms, ψ_k $(0 \le k \le p)$, are positive, while the coefficient of the ECM, $-(1\text{-}\xi)$, is negative. Also, because both equations contain a constant, the income-growth terms can be be expressed as deviations from their means and, thus, reflect short-run income expectations.

The derivation of equation 12 illustrates the analytical foundations of the error-correction representation. Although equation 12 departs from the strict stochastic implications of the permanent income hypothesis, it is consistent with the forward-looking nature of consumption and the empirical regularities found in the U.S. consumption data. That is, contrary to the PIH postulate that consumption growth should—at a first approximation—be unpredictable, consumption growth is correlated with contemporaneous and future income growth (Antzoulatos 1994b). Also, log consumption and

income are integrated of order one and appear to be co-integrated (this means that the term $(c_{t-1} - \lambda i_{t-1})$ is stationary).

C. Short-Run Income Expectations and the Error Correction Representation

If c_t and i_t are $I(1)$ and co-integrated, as they appear to be, the typical single-equation error-representation for consumption growth would be:

$$(13) \qquad \Delta c_t = \alpha + \phi_1 \Delta i_{t-1} + \ldots + \phi_p \Delta i_{t-p+1} + \theta_1 \Delta c_{t-1} + \ldots$$
$$+ \theta_p \Delta c_{t-p+1} + \beta ECM_{t-1} + u_t$$

The coefficient β, which corresponds to β_0 in equation 8 and to $-(1-\xi)$ in equation 12, should be negative. (If not, under the assumption that the ECM_{t-1} is positively correlated with future income, which is confirmed by the data, the system of consumption and income would be unstable.) Also, the comparison of equations 8 and 12 with 13 indicates that the terms Δi_{t-m} and Δc_{t-m} $(m=1...p)$ can be thought of as proxies of $E_{t-1} \Delta i_{t+k}$ $(k \geq 0)$. If these proxies do not generate good forecasts for Δi_{t+k} $(k \geq 0)$, β will be a positively biased estimate of β_0, a reflection of the ECM_{t-1}'s positive correlation with $E_{t-1} \Delta i_{t+k}$ $(k \geq 0)$ in conjunction with the latter's positive correlation with Δc_t. This is similar to the typical case of omitted variables in a regression equation (Johnston 1984, p. 260).

Consequently, when Δi_{t-m} and Δc_{t-m} $(m=1...p)$ do not generate good forecasts for $E_{t-1} \Delta i_{t+k}$ $(k \geq 0)$, β may appear to be insignificant or even positive. However, the inclusion of other stationary variables that are correlated with expected income growth will help reduce β's "bias." Such variables, labeled here as exogenous, allow a richer specification of the

system's short-run dynamics and more efficient estimation without undermining the analytical foundations of the error-correction representation.

D. An Example

A more concrete example helps illustrate the nature of the bias, the importance of good income proxies, and the efficiency gains afforded by exogenous variables which can help improve income growth forecasts. For simplicity, but without loss of generality, let Δc_t in equation 8 be a function of ECM_{t-1}, $E_{t-1}\Delta i_t$ and $E_{t-1}\Delta i_{t+1}$ only. Let also Δc, ECM, $E\Delta i_t$ and $E\Delta i_{t+1}$ denote the vectors of realizations of these variables; X the matrix with columns ECM, $E\Delta i_t$ and $E\Delta i_{t+1}$; and γ the coefficient vector $\gamma' = (\beta_0, \gamma_0, \gamma_1)$.

The theoretical model for consumption growth—the equivalent of 8—is given by:

$$(14) \qquad\qquad \Delta c = X\gamma + u.$$

Next, let $z_{0,t-1}$ and $z_{1,t-1}$ be two proxies of expected future income, as in equation 13. These proxies can be lagged values of income or consumption growth or exogenous stationary variables. Let also the matrix Z of the regressors in equation 13 have as columns ECM, z_0 and z_1. The OLS coefficients $\zeta' = (\beta, \zeta_0, \zeta_1)$ are:

$$(15) \qquad\qquad \zeta = (Z'Z)^{-1}Z'\Delta c.$$

Substituting equation 14 into equation 15 gives (under the assumption $Z'u=0$):

$$(16) \quad \zeta = (Z'Z)^{-1}Z'\Delta c = (Z'Z)^{-1}Z'(X\gamma + u) = (Z'Z)^{-1}Z'X\gamma$$
$$+ (Z'Z)^{-1}Z'u = (Z'Z)^{-1}Z'X\gamma = \Theta\gamma$$

The $(3,3)$ matrix $\Theta = (Z'Z)^{-1}Z'X$ has as columns the regression coefficients of ECM, $E\Delta i_t$, and $E\Delta i_{t+1}$ (matrix X) on ECM, z_0 and z_1. Let Θ be

$$\Theta = \begin{array}{ccc} 1 & \pi & \omega \\ 0 & \pi_0 & \omega_0 \\ 0 & \pi_1 & \omega_1, \end{array}$$

which implies the following regression equations:

$$(17) \qquad ECM_{t-1} = 1ECM_{t-1} + 0z_{0,t-1} + 0z_{1,t-1};$$

$$(18) \qquad E_t\Delta i_t = \pi ECM_{t-1} + \pi_0 z_{0,t-1} + \pi_1 z_{1,t-1}; \text{ and}$$

$$(19) \qquad E_t\Delta i_{t+1} = \omega ECM_{t-1} + \omega_0 z_{0,t-1} + \omega_1 z_{1,t-1} .$$

So, the coefficient β in equation 16, the equivalent of the ECM coefficient in the error-correction representation (equation 13), will be:

$$(20) \qquad \beta = \beta_0 + \pi\gamma_0 + \omega\gamma_1 .$$

Because ECM_{t-1} and Δi_{t+k} $(k=0,1)$ are positively correlated, one can reasonably assume that the coefficients π and ω in equations 18 and 19 are

positive.[3] Taking into account that γ_0 and γ_1 are expected to be positive, β's bias and the possibility that β is insignificant or positive are increasing in the value of π and ω. Thus, the bias will be highest when $z_{i,t-1}$ $(i=0,1)$ are unrelated to $E_{t-1}\Delta i_{t+k}$ $(k=0,1)$ in which case π_i and ω_i $(i=0,1)$ are equal to zero while π and ω attain their highest values. In general, as the explanatory value of $z_{i,t-1}$ $(i=0,1)$ for $E_{t-1}\Delta i_{t+k}$ $(k=0,1)$ increases, the value of π and ω (and the bias) will decrease. The bias will be totally eliminated only when $z_0=E_{t-1}\Delta i_t$ and $z_1=E_{t-1}\Delta i_{t+1}$; in this case, $\pi_0=\omega_1=1$ and $\pi=\omega=\pi_1=\omega_0=0$.

---

[3] By analogy to Johnston's (1984, pp. 81-86) analysis, the value of π and ω is increasing in the correlation coefficient between ECM_{t-1} and $E_{t-1}\Delta i_{t+k}$ $(k=0,1)$, and decreasing in the correlation coefficients between ECM_{t-1} and $z_{i,t-1}$ $(i=0,1)$ and between $E_{t-1}\Delta i_{t+k}$ $(k=0,1)$ and $z_{i,t-1}$ $(i=0,1)$.

As a reminder, Johnston analyzes the case with three interrelated variables Y, X_2 and X_3 in the model:

$$Y = \beta_1 + \beta_2 X_2 + \beta_3 X_3 .$$

Let r_{12}, r_{13} and r_{23} denote their simple correlation coefficients and s_i $(i=1,2,3)$ denote their standard errors, with the subscript 1 referring to the Y variable.

Johnston shows that the estimates of β_2 and β_3, denoted as b_2 and b_3, satisfy the relationships:

$$b_2 = \frac{r_{12}-r_{13}r_{23}}{1-r_{23}^2}\frac{s_1}{s_2} ; \qquad b_3 = \frac{r_{13}-r_{12}r_{23}}{1-r_{23}^2}\frac{s_1}{s_3} .$$

Focusing on the first relationship, the value of b_2 is increasing in the correlation coefficient between Y and X_1 (r_{12}), and decreasing in the correlation coefficient between Y and X_3 and between X_2 and X_3 $(r_{13}$ and $r_{23})$.

Johnston's analysis further implies that the value of π and ω will decrease as the explanatory value of z_0 and z_1 for $E_{t-1}\Delta i_{t+k}$ $(k=0,1)$ increases.

III. Results

All series are constructed from CITIBASE data. Real per-capita consumption, nondurables and services, and personal disposable income are measured in 1982 dollars. The sample is restricted to 1953:1 through 1988:4 to hedge against the distortive impact on the income series of the Korean War period and the revisions of the original consumption and income series (Campbell and Mankiw 1990). Nevertheless, the results are qualitatively the same for the whole sample period, 1947 through 1990. Also, the three-month Treasury Bill rate, secondary market, and the growth rate of the Composite Index of Eleven Leading Indicators and the University of Michigan's Consumer Confidence Index are used as instruments. These instruments help predict the growth rate of personal disposable income despite the fact that income growth exhibits very little autocorrelation. The critical values for the ADF tests are taken from Charemza and Deadman (1992, Tables 2 and 3). Throughout this section, the numbers in parentheses below the estimated coefficients correspond to t-statistics, while one, two, and three asterisks denote significance at the 10, 5, and 1 percent levels, respectively.

Table 1 summarizes the correlation coefficients between income growth and some variables of interest. The first row indicates that Δi_t exhibits very little autocorrelation: with the exception of the third lead, all the autocorrelation coefficients are well below 0.10. The second row confirms the expectation that consumption growth should be positively correlated with contemporaneous and future income growth. Taken together, the first two rows indicate that lagged values of Δi_t and Δc_t will generate poor forecasts of future income growth, Δi_{t+k} $(k \geq 1)$. The third row confirms the expectation that ECM_t should be positively correlated with Δi_{t+k} $(k>1)$.

Finally, the last row shows that the growth rate of the Composite Index of Eleven Leading Indicators, denoted as $DL3_t$, is positively correlated with Δi_{t+k} $(k>1)$. As such, $DL3_t$ is expected to help reduce the bias of the ECM_t coefficient in the error-correction representation for consumption growth.

Further, the correlation coefficients between ECM_t and Δi_{t-k}, Δc_{t-k}, $DL3_{t-k}$ $(k=0,1,2,3,...)$ are below 0.2 and frequently below 0.1. These low coefficients imply that π and ω in equations 18 and 19 will likely be positive; consequently, β will be a positively biased estimate of β_0 (see the discussion in the previous footnote).

Table 1:

Correlation Coefficients

	Δi_t	Δi_{t+1}	Δi_{t+2}	Δi_{t+3}	Δi_{t+4}
Δi_t	1.000	0.055	0.012	0.158	-0.040
Δc_t	0.529	0.299	0.092	0.238	0.197
ECM_t		0.314	0.217	0.188	0.197
$DL3_t$		0.288	0.134	0.245	0.157

Notes:
a) Source: CITIBASE.
b) Sample period: 1953:1 through 1988:4.
c) Δi, Δc: growth rates, per capita personal disposable income and consumption (nondurables and services), 1982$.
d) ECM_t: residuals of co-integrating regression of c_t on i_t and a constant.
e) $DL3_t$: growth rate, third month of tth quarter, Composite Index of Eleven Leading Indicators.

A series of ADF tests established that c_t and i_t are $I(1)$. The lower critical values at the 1 percent level for 100 and 150 observations are -2.70 and -2.68 (Table 2, m=0) without intercept, and -2.90 and -2.79 (Table 3, m=0) with intercept. The number of available observations is approximately 140. Regressing consumption growth, Δc_t, on c_{t-1} and Δc_{t-k} $(k=1,2)$ gave a t-statistic for c_{t-1} of 5.13 which, obviously, is not significantly negative. The inclusion of an intercept in the regression gave a t-statistic of -0.07 which is not significantly negative either. Next, regressing the second difference, $\Delta\Delta c_t$, on Δc_{t-1} and $\Delta\Delta c_{t-k}$ $(k=1,2)$ resulted in a t-statistic for Δc_t of -3.07 without and -5.20 with an intercept. Both are below the lower critical values at the 1 percent level. Similarly, in the regression of Δi_t on i_{t-1} and Δi_{t-k}, $k=1,2$, the t-statistics of i_{t-1} were 4.79 without and -0.29 with intercept. In the $\Delta\Delta i_t$ regression, the appropriate t-statistics were -3.99 and -5.64, respectively, which are below the lower critical values at the 1 percent level. The stationarity tests were also conducted with three and four lags of Δc_t and Δi_t. Because the conclusions regarding the order of integration for c_t and i_t are the same, only the results for $k=1,2$ are reported to save space.

The results of the co-integrating regression, the equivalent of equation 9 above, at the first step of Engle and Granger's (1987) two-step estimator, are summarized below:

$$c_t = -0.041 + 0.912i_t + v_t \ ; \qquad R^2 = 0.997, \ D.W. = 0.378 \ .$$
$$ (-4.48) \ \ (209.6)$$

In general, the OLS estimate of i_t's coefficient is biased. However, as

Davidson and McKinnon (1993, p. 724) remark, the bias seems to be least severe when the R^2 is close to 1, as is the case here.

Regressing the change in the residuals, Δv_t, on v_{t-1} and Δv_{t-1} established that the null of non-co-integration between c_t and i_t can be rejected at the 5 percent level. In more detail, the t-statistic of v_{t-1} was -3.30, while the lower critical value for 100 and 150 observations at the 5 percent level with one estimated parameter (Table 2, m=1) is -2.87.

To establish that the joint distribution of c_t and i_t is an error-correction system, in the second step of Engle and Granger's estimator, consumption growth is regressed on v_{t-1} and five lags of Δc_t and Δi_t. An F test indicated that, with the exception of Δc_{t-1}, all the lagged terms were jointly insignificant. More specifically, the F test for the joint hypothesis $\Delta c_{t-k}=\Delta i_{t-m}=0$ *(k=2,3,4,5 and m=1,2,3,4,5)* was F(9,139)=1.29, far below the critical values at all conventional significance levels. Proceeding with a "general to specific" modeling approach, the more parsimonious equation is estimated below:

$$\Delta c_t = 0.005 + 0.023v_{t-1} + 0.236\Delta c_{t-1} \ ; \qquad R^2 = 0.06, \ D.W. = 2.02 \ .$$
$$\quad (6.24)^{***} \ \ (0.63) \quad (2.84)^{***}$$

In this equation, the estimated ECM coefficient, $\beta=0.023$, is not only insignificant, but its sign is opposite of that expected. Moreover, there is strong evidence of high-order serial correlation in the residuals. The positive coefficient of Δc_{t-1} probably reflects Δc_{t-1}'s positive correlation with Δi_{t+k} *(k ≥ 0)*.

To test whether the positive sign and the statistical insignificance of β are due to the fact that lagged values of Δc_t and Δi_t are poor proxies for

future income growth, Δi_t is included in the set of the regressors. (In the spirit of equation 20, this manipulation will render $\pi = 0$ and reduce β's positive bias.) The working hypothesis is that, under rational expectations, expected income growth differs from that realized by an unpredictable stochastic term (Cumby et al. 1983). Because this term is likely to be correlated with the error in the error-correction representation, the new equation should be estimated with instrumental variables (IV). In addition, since the time aggregation of consumption may induce an $MA(1)$ term in the error, the instruments should be lagged at least two periods (see Campbell and Mankiw 1990 and 1991 for details). The instruments are a constant, c_{t-2}, i_{t-2}, r_{t-k} $(k=2,4)$, $\Delta c_{T,t-k}$ $(k=2,4)$ and Δi_{t-k} $(k=2,5)$. r_t denotes the quarterly average of the three-month Treasury Bill rate (secondary market), while $\Delta c_{T,t}$ denotes the growth rate of total consumption. Also, c_{t-1} and i_{t-1} are used instead of the residuals of the co-integrating regression.

As the equation below indicates, the coefficients of c_{t-1} and i_{t-1} have the expected sign, that is, they imply a negative β, but are insignificant, while the coefficient of Δi_t is positive and significant at the 1 percent level. On the positive side, this model—as well as the next one—passes diagnostic tests for serial correlation and autoregressive conditional heteroskedasticity (ARCH) in the residuals. For parsimony, the equation does not include Δc_{t-1}, which is insignificant.

$$\Delta c_t = -0.001 \quad -0.059 c_{t-1} \quad +0.054 i_{t-1} \quad +0.480 \Delta i_t \; ; \quad R^2 = 0.196 \quad \text{D.W.} = 2.16 \,.$$
$$ (-0.17) \quad (-1.27) \quad\quad (-1.27) \quad\quad (4.59)^{***}$$

More important, the inclusion of future income growth (which is

equivalent to setting $\omega = 0$ in 20) makes the coefficients of c_{t-1} and i_{t-1} significant at the 5 percent level. In addition, all coefficients have the theoretically predicted sign:[4]

$$\Delta c_t = -0.003 \ -0.093c_{t-1} \ +0.085i_{t-1} \ +0.430\Delta i_t \ +0.231\Delta i_{t+1} \ ; \quad R^2 = 0.319, \ D.W. = 1.88 \ .$$
$$\quad (-0.80) \quad (-2.04)^{**} \quad (2.04)^{**} \quad (4.34)^{***} \quad (2.27)^{**}$$

The results were similar for the whole sample, 1947 through 1990. The ECM coefficient was marginally significant at the 10 percent level (t-statistic -1.68) in the unrestricted equation, which included lagged values of Δc_t and Δi_t. The inclusion of $DL3_{t-1}$ in the set of the regressors raised ECM's significance to the 5 percent level (t-statistic -2.1). The addition of Δc_t further raised the significance level to 1 percent (t-statistic -3.25). Also, in

---

[4] To conform with the majority of the literature, the last two equations are also estimated with OLS. As with IV estimation, β turns negative in the first equation but is still insignificant. In the second, β becomes significant at the 5 percent level.

$$\Delta c_t = 0.003 \quad -0.039v_{t-1} \ +0.308\Delta i_t \qquad R^2 = 0.287, \ D.W. = 1.90 \ ;$$
$$\quad (7.38)^{***} \ (-1.18) \quad (7.41)^{***}$$

$$\Delta c_t = 0.002 \quad -0.069v_{t-1} \ +0.311\Delta i_t \ +0.169\Delta i_{t+1} \qquad R^2 = 0.375, \ D.W. = 1.92 \ .$$
$$\quad (5.07)^{***} \ (-2.18)^{**} \quad (7.96)^{***} \quad (4.44)^{***}$$

Further, the inclusion of $DL3_{t-1}$ increases both the t-statistic of β and its absolute value. This provides further evidence about the hypothesis that "exogenous" variables that can help predict income growth may help reduce the bias of the ECM coefficient:

$$\Delta c_t = -0.002 \ -0.077v_{t-1} \ +0.283\Delta i_t \ +0.157\Delta i_{t+1} \ +0.043DL3_{t-1} \ ; \quad R^2 = 0.410, \ D.W. = 2.03 \ .$$
$$\quad (4.82)^{***} \ (-2.48)^{***} \quad (7.21)^{***} \quad (4.21)^{***} \quad (2.86)^{***}$$

accordance with the theory, the ECM coefficient kept increasing in absolute value as more regressors correlated with future income were added.

Finally, an error-correction model was estimated for income. The original model included six lags of consumption and income growth, as well as the error-correction mechanism and the first lag of the growth rate of the Composite Index of Eleven Leading Indicators. A series of F tests indicated that only the first three lags of consumption growth and the error-correction mechanism had significant predictive power for income growth. As one might expect from the theoretical analysis and the correlation coefficients in Table 1, the coefficient of ECM_{t-1} was positive and significant at the 1 percent level. The ECM_{t-1}'s coefficient positive sign provides further support for the argument that a positive β in equation 13 would indicate an unstable system.

IV. Conclusion

This paper uses a consumption example to argue that biased error-correction coefficients may be pervasive in error-correction models that are meant to capture behavior that is influenced in an essential way by expectations. The paper uses a simple theoretical model to show that the ECM coefficient in the error-correction representation of aggregate consumption will be biased upwards if some consumers are borrowing-constrained and some are not. If some consumers are borrowing-constrained, expected consumption growth is negatively correlated with the ECM and positively correlated with contemporaneous expectations of future income growth. Similarly, the ECM is positively correlated with contemporaneous expectations of future income if some consumers are not borrowing-constrained. Hence, the esti-

mated ECM coefficient will be biased upwards if the other right-hand-side variables in the error-correction representation for consumption are poor proxies of future income expectations. The crucial aspects of the model are the forward-looking nature of consumption and the partial adjustment induced by binding borrowing constraints. Naturally, the model's conclusions can be justified by other theoretical paradigms that combine forward-looking behavior with some partial adjustment mechanism.

The paper also demonstrates that the theoretical predictions are borne out by U.S. data on consumption (nondurables and services) and disposable income. Lagged growth rates of these two series are poor proxies of future income growth and, presumably, of consumers' income expectations. In the typical error-correction representation, which includes only these (poor) proxies in the right-hand side, the ECM coefficient is positive but insignificant. However, when better predictors of future income growth are added to the right-hand-side, the estimated ECM coefficient eventually turns negative and significant, as predicted by the theory.

More generally, the paper demonstrates the risks inherent in treating the ECM as a mere statistical property of the data, without regard to the economic forces behind it. It also holds the promise of helping to alleviate the problems of inefficient estimation, misspecified econometric models, and poor short-run forecasts that arise from erroneous inferences about the estimated ECM coefficients. Given the forward-looking nature of economic decisions and the difficulty of modeling expectations, such problems are expected to be pervasive. Therefore, a reexamination of the evidence seems warranted in cases where the ECM appears to be insignificant or to have the opposite-than-expected sign.

References

Alogoskoufis, George, and Ron Smith. 1991. "On Error Correction Models: Specification, Interpretation, Estimation." *Journal of Economic Surveys* 5, no 1: 97-128.

Antzoulatos, Angelos A. 1994a. "Credit Rationing and Rational Behavior." *Journal of Money, Credit, and Banking* 26, no 2: 182-202.

Antzoulatos, Angelos A. 1994b. "Borrowing Constraints, Income Expectations and the Euler Equation: Theory and Evidence." *Economics Letters* 45: 323-27.

Caballero, Ricardo J. 1995. "Near-Rationality, Heterogeneity and Aggregate Consumption." *Journal of Money, Credit, and Banking* 27, no 1: 29-48.

Campbell, John Y. 1987. "Does Saving Anticipate Declining Labor Income? An Alternative Test of the Permanent Income Hypothesis." *Econometrica* 55, no 6: 1249-73.

Campbell, John Y., and N. Gregory Mankiw. 1990. "Permanent Income, Current Income, and Consumption." *Journal of Business and Economic Statistics* 8, no 3: 265-79.

Campbell, John Y., and N. Gregory Mankiw. 1991. "The Response of Consumption to Income: A Cross-Country Investigation." *European Economic Review* 35, no 4: 723-56.

Charemza, Wolciech W., and Derek F. Deadman. 1992. *New Directions in Econometric Practice*. United Kingdom: Edward Elgar Publishing Limited.

Clarida, Richard H. 1994. "Cointegration, Aggregate Consumption, and the Demand for Imports: A Structural Econometric Investigation." *American Economic Review* 84, no 1: 298-08.

Cumby, Robert E., John Huizinga, and Maurice Obstfeld. 1983. "Two-Step Two-Stage Least Squares Estimation in Models with Rational Expectations." *Journal of Econometrics* 21: 333-55.

Davidson, E.H., and David F. Hendry. 1981. "Interpreting Econometric Evidence: The Behavior of Consumers' Expenditure in the U.K." *European Economic Review* 16: 177-94.

Davidson, Russell, and James G. MacKinnon. 1993. *Estimation and Inference in Econometrics*. New York: Oxford University Press.

Davidson, J.E.H., D.F. Hendry, F. Srba, and S. Yeo. 1978. "Econometric Modelling of the Aggregate Time-Series Relationship between Consumers' Expenditure and Income in the United Kingdom." *Economic Journal* 88: 661-92.

Deaton, Angus. 1991. "Saving and Liquidity Constraints." *Econometrica* 59, no 5: 1221-48.

Engle, Robert F., and C.W.J. Granger. 1987. "Co-Integration and Error Correction: Representation, Estimation, and Testing." *Econometrica* 55, no 2: 251-76.

Engle, Robert F., and Byung Sam Yoo. 1987. "Forecasting and Testing in Co-Integrated Systems." *Journal of Econometrics* 35: 143-59.

Gali, Jordi. 1990. "Finite Horizons, Life-Cycle Savings, and Time-Series Evidence on Consumption." *Journal of Monetary Economics* 26: 433-52.

Granger, C.W.J. 1981. "Some Properties of Time Series Data and their Use in Econometric Model Specification." *Journal of Econometrics* 16: 121-30.

Hall, Robert E. 1978. "Stochastic Implications of the Life Cycle-Permanent Income Hypothesis: Theory and Evidence." *Journal of Political Economy* 86, no 6: 501-17.

Hamilton, James. 1994. *Time Series Analysis*. Princeton, N.J.: Princeton University Press.

Hendry, D.F., J.H. Muellbauer, and A. Murphy. 1990. "The Econometrics of DHSY." In J.D. Hey and O. Winch, eds., *A Century of Economics: 100 Years of the Royal Economic Society and the Economic Journal*. Oxford: Basil Blackwell.

J. Johnston. 1984. *Econometric Methods*. 3rd Edition. McGraw-Hill, Inc.

Nelson, Charles R. 1987. "A Reappraisal of Recent Tests of the Permanent Income Hypothesis." *Journal of Political Economy* 95, no 3: 641-46.

Philips, Peter C.B., and Mico Loretan. 1991. "Estimating Long-Run Equilibria." *Review of Economic Studies* 58: 407-43.

Schechtman, Jack, and Vear L.S. Escudero. 1977. "Some Results on 'An Income Fluctuation Problem.'" *Journal of Economic Theory* 16: 151-66.

Staff Reports

CENTRAL BANK INDEPENDENCE AND DISINFLATIONARY CREDIBILITY: A MISSING LINK?
by Adam Posen. May 1995.

THE COSTS AND BENEFITS OF DUAL TRADING
by Hun Y. Park, Asani Sarkar, and Lifan Wu. June 1995.

SHORT-TERM SPECULATORS AND THE ORIGINS OF NEAR-RANDOM-WALK
EXCHANGE RATE BEHAVIOR
by C.L. Osler. July 1995.

HEAD AND SHOULDERS: NOT JUST A FLAKY PATTERN
by C.L. Osler and P.H. Kevin Chang. August 1995.

USING OPTION PRICES TO ESTIMATE REALIGNMENT PROBABILITIES IN THE
EUROPEAN MONETARY SYSTEM
by Allan M. Malz. September 1995.

AN EFFICIENT, THREE-STEP ALGORITHM FOR ESTIMATING ERROR-CORRECTION MODELS
WITH AN APPLICATION TO THE U.S. MACROECONOMY
by Michael D. Boldin. October 1996.

THE RELATIONSHIP BETWEEN GOVERNMENT FINANCIAL CONDITION AND EXPECTED TAX
RATES REFLECTED IN MUNICIPAL BOND YIELDS
by Sangkyun Park. November 1995.

PRESERVING FIRM VALUE THROUGH EXIT: THE CASE OF VOLUNTARY LIQUIDATIONS
by Michael J. Fleming and John J. Moon. December 1995.

THE EFFECTS OF INFLATION ON WAGE ADJUSTMENTS IN FIRM-LEVEL DATA:
GREASE OR SAND?
by Erica L. Groshen and Mark E. Schweitzer. January 1996.