On Spatial Dynamics*

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Abstract

It has long been recognized that the forces that lead to the agglomeration of economic activity and to aggregate growth are similar. Unfortunately, few formal frameworks have been advanced to explore this link. We critically discuss the literature and present a simple framework that can circumvent some of the main obstacles we identify. We discuss the main characteristics of an equilibrium allocation in this dynamic spatial framework, present a numerical example to illustrate the forces at work, and provide some supporting empirical evidence.

1. INTRODUCTION

Economists have long discussed the relationship between agglomeration and growth. As Lucas (1988) points out, not only are both phenomena related to increasing (or constant) returns to scale, but in many contexts agglomeration forces are the source of the increasing returns that lead to growth. Krugman (1997), after providing a detailed overview of the different economic forces that can explain both phenomena, identifies probably the most important challenge of this literature: the difficulty of developing a common framework that incorporates both the spatial and the temporal dimensions. In other words, what is needed is a dynamic spatial theory. In this brief paper, we review the recent literature that has emerged to deal with some of the main links between growth and regional economics, discuss the problems that this literature faces, sketch a framework that we believe can be used to further explore the links between the spatial and temporal dimensions, and provide some empirical evidence consistent with the forces present in this framework.

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The dynamics of the distribution of economic activity in space have been studied using three distinct approaches. A first family of models consists of dynamic extensions of *New Economic Geography* models. These models tend to have a small number of locations, typically two. Agglomeration is driven by standard Krugman (1991) pecuniary externalities operating through real wages. The models are usually made dynamic by adding innovation in product quality as in Grossman and Helpman (1991 a,b). There is a wide variety of particular specifications, some of which include capital accumulation or other forms of innovation. Baldwin and Martin (2004) provide a nice survey of this literature. They highlight the possibility of "catastrophic" agglomeration, implying that only one region accumulates factors. More generally, agglomeration and innovation reinforce each other, creating growth poles and sinks. The emergence of regional imbalances is accompanied by faster aggregate growth and higher welfare in all regions.[1]

The contribution of this first strand of the literature is important, as it enhances our understanding of the common forces underlying growth and agglomeration. However, the spatial predictions are rather limited. The focus on a small number of locations does not allow this literature to capture the richness of the observed distribution of economic activity across space, thus restricting the way these models are able to connect with the data. It advances statements about how unequal two regions are, but there is no sense in which one can have a hierarchy of agglomerated areas. One could of course try to generalize these models to more than a few regions. The problem is that the analytical tractability breaks down when one deals with more than two or three regions. Some progress could be made numerically, using dynamic extensions of continuous space *New Economic Geography* frameworks, like the one in Fujita et al. (2001, Chapter 17), but little has been done so far. Therefore, these models remain mostly useful as analytical tools, rather than as guides to doing empirical work.

A second family of models aims to explain the distribution of city sizes. In general, this literature only models, if at all, space *within* cities, but not the location of cities *across* space. Early contributions include Black and Henderson (1999) and Eaton and Eckstein (1997). Black and Henderson (1999) propose a model of a dynamic economy with cities. Increasing returns in the form of externalities create cities and imply, apart from knife-edge parameter conditions, increasing returns at the aggregate level. Hence, as in the papers above, agglomeration leads to explosive growth. In contrast to the first strand of the literature, these theories have the advantage of explicitly modeling

[1] Readers interested in this strand of the literature should consult Baldwin and Martin (2004), Fujita and Thisse (2002, Chapter 11), and some of the particular papers, like Baldwin et al. (2001) and Martin and Ottaviano (1999 and 2001).

the cities in each location and allowing for heterogeneity in city characteristics. This comes at the cost of a black box agglomeration effect in the form of a production externality.

Within this second strand of the literature, the contribution of Gabaix (1999a) is key in establishing the link between the dynamic growth process of cities and the observed distribution of city sizes. He shows that Zipf's Law for cities — the size distribution is approximated by a Pareto distribution with coefficient one — can be explained by models that imply cities exhibiting scale-independent growth. For our purposes, the interesting part of this contribution is not so much the particular size distribution this growth process leads to, but rather the link it establishes between the dynamic growth process of particular production sites and the invariant distribution of economic activity in space. It is the growth process that leads to agglomeration (in the form of a size distribution with a fat left-tail with many large cities), and not the other way around. Following Gabaix (1999a), many papers have built on this basic insight, which had already been used in other applications in macroeconomics. Eeckhout (2004), for example, proposes a simple model in which cities grow by receiving scale-independent shocks, and uses the Central Limit Theorem to show that the resulting size distribution is log normal.

Gabaix (1999a, b) and Eeckhout (2004) postulate the growth rate of cities; they do not propose an economic theory of this growth process. The last generation of models in this second strand of the literature addresses this shortcoming by successfully establishing a link between economic characteristics that determine the growth process and economic agglomeration in cities. Duranton (2007) does so by proposing a growth process through the mobility of industries across cities as a result of innovations in particular sectors. Rossi-Hansberg and Wright (2007) also produce a particular city growth process as a result of adjustment in optimal city sizes and city entry. Córdoba (2008) discusses general properties that these models need to satisfy in order to yield a growth process consistent with particular characteristics of invariant distributions, like Zipf's Law. Some of these papers also establish a reverse link between the growth process and agglomeration. In Rossi-Hansberg and Wright (2007), for example, it is the organization of economic activity in cities that leads to the aggregate constant returns to scale necessary to generate balanced growth. In this sense, agglomeration of economic activity in a particular number and size of cities generates aggregate balanced growth.

The main limitation of the dynamic frameworks in this second strand of the literature is the lack of geography. Production happens in particular sites, but these sites are not ordered in space and the trade links between them are either frictionless or uniform. Cities are the units in which production is organized. The internal structure of cities is sometimes modeled as an area with land as a factor of production and agents facing transport and/or commuting costs. However, geography is only modeled *within* cities, not *across* them. In this sense, these models do not present dynamic spatial theories that can be contrasted to the observed distribution of economic activity in space.

The third strand of the dynamic spatial literature incorporates fully forward-looking agents and factor accumulation into models with a continuum of geographically ordered locations.[2] It also allows for either capital mobility or some form of spillovers or diffusion between regions (see, e.g., Boucekkine et al., 2009, Brock and Xepapadeas, 2008a, b, Brito, 2004, and Quah, 2002). Apart from these interactions, points in space are still completely isolated from each other. We review the particular structure of these problems in Section 3 below. For now it suffices to say that progress here has been mostly restricted to formulating the necessary and sufficient conditions for efficient allocations and, in some cases, the corresponding conditions characterizing rational expectation equilibria. Few substantive results have been advanced.

The remainder of this paper is organized as follows. In Section 2 we go further into the importance of developing spatial frameworks that can be compared with the data, some of the difficulties of doing this, and the comparison with trade frameworks, like that in Eaton and Kortum (1999). Section 3 discusses some of the setups with continuous space that have been analyzed for the case of forward-looking agents. Section 4 then proposes a simple endogenous growth spatial framework in which innovation decisions are optimally not forward-looking, and it uses a numerical example to shed light on the different forces present in this framework. Section 5 presents some basic evidence from the US on the forces highlighted in Section 4, and Section 6 concludes.

2. THE IMPORTANCE OF SPACE

Incorporating geographically ordered space (or land) is important for two main reasons. Land at a particular location is a rival and non-replicable input of production, and land is geographically ordered in a way that matters for economic activity. The latter claim has been documented extensively: patents cite geographically close-by patents (Jaffe et al., 1993), firms co-locate (Ellison and Glaeser, 1997, and Duranton and Overman, 2005 and 2008), and in general there is ample evidence

[2] We discuss in more detail the importance of using a continuum of locations in the next section, but the evidence seems to suggest that the observed patterns are very different when land, and not only cities, is incorporated into the analysis. In particular, Holmes and Lee (2008) show that the distribution of employment across equal sized squares in space has a significantly lower tail than the one for cities. They also show that for space, and in contrast with cities, growth rates are not independent of scale (Gibrat's Law).

of substantial trade costs, mobility costs, commuting costs and other costs that increase with distance. The use of land as a non-replicable input of production requires, perhaps, some additional explanation. Economic activity at a particular location is, of course, endogenous, so the factors operating at a given location can be replicated. Nevertheless, since land is an input of production, increasing factors at a given location leads to decreasing returns to scale and therefore dispersion.

It is obviously difficult to incorporate space into dynamic frameworks because it increases the dimensionality of the problem. Another difficulty of incorporating a continuum of locations in geographic space is that, in the presence of mobility frictions like transport or commuting costs, clearing factor and goods markets is not trivial. The reason is that how many goods or factors are lost in transit depends on mobility and trade patterns, which in turn depend on factor prices that are the result of market clearing. Hence, to impose market clearing it is necessary to know the number of goods lost in transit. That is, factor prices at each location depend on the equilibrium pattern of trade and mobility at all locations. This yields a problem that in many cases is intractable.

The trade literature has circumvented this difficulty by analyzing the case of a finite (though potentially large) number of locations in the presence of random realizations of productivity for a continuum of goods (see, e.g, Eaton and Kortum, 2002). In such a framework, the only relevant equilibrium variable is the share of exported and imported goods, which is well determined by the properties of the distributions of the maximum of the productivity realizations. This has proven to be an effective way to deal with this difficulty. However, it does not allow us to talk about trade in particular sectors, since only aggregate trade flows are determined in equilibrium. This is an important drawback if we want to study geography models that focus on spatial growth across industries. Since the empirical evidence shows that different sectors exhibit very different spatial growth patterns, this is a relevant issue (see, e.g., Desmet and Fafchamps, 2006, and Desmet and Rossi-Hansberg, 2009a).

Another way of solving this problem is to clear markets sequentially. Suppose space is linear and compact. Then we can start at one end of the space interval and accumulate production minus consumption in a given market (properly discounted by transport or commuting costs) until we reach the end of the interval. At the boundary, 'excess supply' has to be equal to zero in order for markets to clear. This method, proposed in Rossi-Hansberg (2005), is fairly easy to apply, but it can only be used in one-dimensional (or two-dimensional and symmetric) compact setups. In Section 4 we sketch a model that uses this form of market clearing. Extending this formulation to non-symmetric two-dimensional spatial setups (like reality!) is a theoretical challenge.

3. SPATIAL MODELS WITH FORWARD-LOOKING AGENTS

The few papers that have studied a fully dynamic setup with a continuum of locations normally focus on the problem of a planner who allocates resources. We present two examples below. Spatial interactions are introduced in two different ways: a first one by allowing for capital mobility, and a second one by assuming a spatial capital externality. Neither of them introduces land as an input of production, although given that technology is not necessarily assumed to be constant returns to scale, it could be easily incorporated through absentee landlords.

The spatial setup is the real line and time is continuous. Let $c(\ell, t)$ denote consumption, $L(\ell, t)$ population, and $k(\ell, t)$ capital at location ℓ and time t. A central planner then maximizes the sum of utilities of all agents, all of whom discount time at rate β. Production requires only capital, $k(\ell, t)$, which depreciates at rate δ. Total factor productivity is given by $Z(\ell, t)$. The change in capital at a particular location is therefore equal to production minus depreciation minus consumption plus the capital received from other locations. Boucekkine et al. (2009) show how this last term can be expressed as the second partial derivative of capital across locations: essentially, it is just the difference between the flow of capital from the regions to the left minus the flow of capital flowing to the regions to the right. This law of motion of capital, a parabolic differential equation, and in particular the spatial component entering through the second order term, introduces space into the problem. In addition, capital at all locations at time 0 is assumed to be known, and since the real line is infinite, a transversality condition on capital is also required. Hence, the problem solved by Boucekkine et al. (2009) becomes:

$$\max_{c} \int_0^\infty \int_{\mathbb{R}} U(c(\ell, t)) L(\ell, t) e^{-\beta t} d\ell dt$$

subject to

$$\frac{\partial k(\ell, t)}{\partial t} - \frac{\partial^2 k(\ell, t)}{\partial \ell^2} = Z(\ell, t) f(k(\ell, t)) - \delta k(\ell, t) - c(\ell, t)$$

$$k(\ell, 0) = k_0(\ell) > 0$$

$$\lim_{\ell \to \pm\infty} \frac{\partial k(\ell, t)}{\partial \ell} = 0.$$

Brock and Xepapadeas (2008b) and Brito (2004) solve similar problems, but with different preferences. In fact, Boucekkine et al. (2009) show that for general preferences this is an 'ill-posed' problem in the sense that the initial value of the co-state does not determine its whole dynamic path. This is a general problem in spatial setups. One can address this issue either by considering particular solutions (like the type of cyclical perturbation analysis found in many studies) or by putting strong restrictions on preferences. Boucekkine et al. (2009) show that some progress can be made by focusing on the linear case.

Brock and Xepapadeas (2008b) study a similar problem in a compact interval R, given by

$$\max_c \int_0^\infty \int_R U\left(k\left(\ell,t\right),c\left(\ell,t\right),X\left(\ell,t\right)\right)L\left(\ell,t\right)e^{-\beta t}d\ell dt$$

subject to

$$\frac{\partial k\left(\ell,t\right)}{\partial t} = f\left(k\left(\ell,t\right),c\left(\ell,t\right),X\left(\ell,t\right)\right)$$

$$X\left(\ell,t\right) = \int_{\ell\in R}\omega\left(\ell-\ell'\right)k\left(\ell',t\right)d\ell'$$

$$k\left(\ell,t\right) = k_0\left(\ell\right) > 0$$

where $X\left(\ell,t\right)$ is an externality that affects production and utility, and f now refers to production minus consumption plus an additional term reflecting the direct effect of the externality on the law of motion of capital. In contrast to the problem of Boucekkine et al. (2009), there is no capital mobility, which eliminates a huge difficulty. Instead, the spatial component is introduced through the externality, which is just a kernel of capital at all locations. This is an interesting problem, since it incorporates diffusion, although not mobility. As in the previous case, the authors can derive the Pontryagin necessary conditions for an optimum and, under more restrictive assumptions, sufficient conditions. Solving for stable steady states remains, nevertheless, an exercise of finding whether or not uniform steady states are stable. This is progress, although it does not amount to a complete analysis of the problem.

The lack of a complete solution to the problems above is hardly the fault of the authors working on them. These problems are complicated and, absent more structure, it is hard to extract general insights. The main problem seems to be that agents are forward-looking and thus need to understand the whole future path to make current decisions. Modeling space implies understanding the whole distribution of economic activity over space and time for each feasible action. One way around this difficulty is to impose enough structure — either on the diffusion of technology or on the mobility of agents and land ownership — so that agents do not care to take the future allocation paths into account, given that they are out of their control and do not affect the returns from current decisions. In the next section we present an example of such a framework.

4. AN ALTERNATIVE MODEL WITH FACTOR MOBILITY AND DIFFUSION

In Desmet and Rossi-Hansberg (2009b) we introduce a model in which locations accumulate technology by investing in innovation in one of two industries and by receiving spillovers from other locations. The key to making such a rich structure computable is that diffusion, together with labor mobility and diversified land ownership, implies that agents and firms need not be forward-looking when they decide where to locate and how much to invest in innovation every period. The result is a model in which locations are changing occupations and employment density continuously, but in the aggregate the economy converges on average to a balanced growth path.

Desmet and Rossi-Hansberg (2009b) study an economy with two sectors and analyze the sectoral interaction in generating innovation. They use the model to explain the observed evolution in the spatial distribution of economic activity in the US. To give a sense of the forces at work in that model, we present here a simpler version of the setup with only one good (and therefore no specialization decision or cross-industry innovation effects). In this version of the model, factor mobility is frictionless, and trade is just the result of agents holding a diversified portfolio of land across locations.

Land is given by the unit interval $[0,1]$, time is discrete, and total population is $\bar{L}$. We divide space into 'counties' (connected intervals in $[0,1]$), each of which has a local government. Agents solve

$$\max_{\{c(\ell,t)\}_0^\infty} E\sum_{t=0}^{\infty}\beta U(c(\ell,t))$$

subject to

$$w(\ell,t)+\frac{\bar{R}(t)}{\bar{L}}=p(\ell,t)\,c(\ell,t)\text{ for all }t\text{ and }\ell.$$

where $p(\ell,t)$ is the price of the consumption good and $w(\ell,t)$ denotes the wage at location ℓ and time t. Total land rents per unit of land at time t are denoted by $\bar{R}(t)$, so that $\bar{R}(t)/\bar{L}$ is the dividend from land ownership received by agents, assuming that agents hold a diversified portfolio of land in all locations. Free mobility implies that utilities equalize across regions each period.

The inputs of production are land and labor. Production per unit of land is given by

$$x(L(\ell,t))=Z(\ell,t)\,L(\ell,t)^{\mu},$$

where $\mu<1$, $Z(\ell,t)$ denotes TFP, and $L(\ell,t)$ is the amount of labor per unit of land used at location ℓ and time t. The problem of a firm at location ℓ is thus given by

$$\max_{L(\ell,t)} \left(1-\tau\left(\ell,t\right)\right)\left(p\left(\ell,t\right)Z\left(\ell,t\right)L\left(\ell,t\right)^{\mu}-w\left(\ell,t\right)L\left(\ell,t\right)\right),$$

where $\tau\left(\ell,t\right)$ denotes taxes on profits charged by the county government.

The government of a county can decide to buy an opportunity to innovate by taxing local firms $\tau\left(\ell,t\right)$. In particular, a county can buy a probability $\phi \leq 1$ of innovating at a cost $\psi\left(\phi\right)$ per unit of land. This cost $\psi(\phi)$ is increasing and convex in ϕ, and proportional to wages. If a county innovates, all firms in the county have access to the new technology. A county that obtains the chance to innovate draws a technology multiplier $z(\ell)$ from a Pareto distribution with lower bound 1, leading to an improved level of TFP, $z_\ell Z_i\left(\ell,t\right)$, where

$$\Pr\left[z<z_\ell\right]=\left(\frac{1}{z}\right)^{a}.$$

The risk-neutral government of county G, with land measure I, will then maximize

$$\max_{\phi(\ell,t)} \int_G \frac{\phi\left(\ell,t\right)}{a-1} p\left(\ell,t\right)Z\left(\ell,t\right)L\left(\ell,t\right)^{\mu} d\ell - I\psi\left(\phi\right) \tag{1}$$

The benefits of the extra production last only one period. Since a county is by assumption small and innovation diffuses geographically, a county's innovation decision today does not affect its expected level of technology tomorrow. This implies that governments need not be forward-looking when choosing the optimal level of investment in innovation. Note the scale effect in the previous equation: high employment density locations will optimally innovate more (and so will high-price and high-productivity locations). This is consistent with the evidence presented by Carlino et al. (2007). They show that a doubling of employment density leads to a 20% increase in patents per capita.

The timing of the problem is key. Innovation diffuses spatially between time periods. So, before the innovation decision, location ℓ has access to

$$Z_i\left(\ell,t+1\right)=\max_{r\in[0,1]} e^{-\delta|\ell-r|}Z\left(r,t\right)$$

which of course includes its own technology. Agents then costlessly relocate, ensuring that utility is the same across all locations. After labor moves, counties invest in innovation. Assuming wages are set before the innovation decision, the fact that agents hold a diversified portfolio of land in all locations implies that they need not be forward-looking when deciding where to locate. Note also that by holding a diversified portfolio of land, rents are redistributed from high-productivity to low-productivity locations. As a result, high-productivity locations run trade surpluses, and low-productivity locations run trade deficits.

In addition to the geographic diffusion of innovations, transport costs are another source of agglomeration. For simplicity we assume iceberg transport costs, so if one unit of the good is transported from ℓ to r, only $e^{-\kappa|\ell-r|}$ units of the good arrive in r. Hence, if goods are produced in ℓ and consumed in r, $p(r,t) = e^{\kappa|\ell-r|}p(\ell,t)$. As described in Section 2, goods markets clear sequentially. Define $H_i(\ell,t)$ by $H_i(0,t) = 0$ and by the differential equation

$$\frac{\partial H(\ell,t)}{\partial \ell} = \theta(\ell,t)\,x(\ell,t) - c(\ell,t)\left(\sum_i \theta(\ell,t)\,L(\ell,t)\right) - \kappa\,|H(\ell,t)|\,.$$

Then, the goods market clears if $H(1,t) = 0$. The labor market clearing condition is given by

$$\int_0^1 L(\ell,t)\,d\ell = \overline{L}, \text{ all } t.$$

Computing an equilibrium of this economy is clearly feasible. Given initial productivity functions, we can solve for production in all locations, for the wages that equalize utility and clear the national labor market, for the prices that clear the goods market, and for the resulting average land rents, which are added to agents' income. This determines the location of agents and the investments in innovation. After productivity is realized, we compute actual production, actual distributed land rents, and trade. Overnight there is diffusion, which determines the new productivity function. Since decisions are based on current outcomes only, computing an equilibrium involves solving a functional fixed point each period, but it does not involve calculating rational expectations.

What can we learn from this model? Although the model is extremely simple, it has two forces that are interesting when thinking about spatial dynamics. On the one hand, although technology is constant returns in land and labor, it exhibits local decreasing returns to labor, because locally land cannot be replicated. This is a form of local congestion that spreads employment across space given identical technology levels. On the other hand, agglomeration is the result of the diffusion of technology. Areas with high levels of employment innovate more, since the incentives to innovate are larger there. Since diffusion decreases with distance, areas close to high-employment clusters become high-productivity areas. This attracts employment and leads to more innovation. As usual, the balance between the congestion and agglomeration forces determines the spatial landscape.

The same forces that lead to particular spatial employment patterns also explain aggregate growth. Dispersion implies more uniform, but smaller, incentives to innovate. In contrast, concentration implies that less locations innovate, but each of them innovates more. More diffusion implies that the second (extensive) effect is less important and that aggregate growth is generally greater.

Perhaps surprisingly, higher trade costs imply more concentrated production, which in turn may lead to more growth. Although higher trade costs imply static efficiency losses, they also lead to dynamic gains through increased concentration and innovation, an effect reminiscent of the one in Fujita and Thisse (2003). A clear empirical implication emerges from the theory: more concentration of employment in surrounding areas leads to higher innovation and growth. This effect is the result of two forces. First, more concentration as a result of, say, transport costs, leads to more innovation. Second, more innovation in certain areas leads, through diffusion, to productivity growth in neighboring areas (see, e.g., Ciccone (2002) for evidence on this mechanism).[3]

The model presented above has only one industry, so by construction it is not suited to study cross-industry effects. In Desmet and Rossi-Hansberg (2009b) we present a version of the model with two industries. In that case, another spatial link between the distribution of economic activity and growth emerges. Locations near clusters of firms in one sector, say, manufacturing, experience high prices of the other good, say, services, since their proximity to manufacturing locations allows them to sell services paying small trade costs. This channel works through trade: neighboring areas that are specialized in manufacturing will import services, thus pushing up the relative price of services. As a result, locations close to manufacturing clusters tend to have high employment and high prices in services and therefore will tend to innovate in services. Hence, being near clusters in the other industry is also a source of growth and innovation. However, note that this force operates through imports, whereas the diffusion force operates through employment. In the next section we present some evidence supporting these predictions.

Figure 1 presents a numerical simulation from the framework with two sectors, manufacturing and services. The model used to compute the figure is identical to the one presented in Desmet and Rossi-Hansberg (2009b), and we use the basic calibration in that paper with a diffusion parameter $\delta = 50$. The figure shows a contour map of productivity in time and space. Space is the unit interval, and we run the model for 100 periods. We use initial conditions that imply that locations close to the upper bound are good in manufacturing, whereas all locations have an initial productivity in services equal to 1. These initial conditions imply that manufacturing starts innovating first and only in the upper regions. As we argued, diffusion implies that regions that innovate are clustered. As a result, productivity growth happens in concentrated areas. This is an expression of the first effect discussed above.

In period 63 some scattered service areas, which are close to manufacturing clusters, start in-

[3]Duranton and Overman (2005, 2008) present detailed and strong evidence of co-location in the UK. This is some of the best evidence of regional agglomeration mechanisms within and between industries. Unfortunately, it does not directly address the link between growth and regional agglomeration.

novating. This innovation happens in clusters too and, more important, next to manufacturing areas. Relative prices of services are high next to clusters of manufacturing production as a result of transport costs and trade. This leads to endogenously higher employment and more innovation in services. This is an expression of the second effect discussed above.

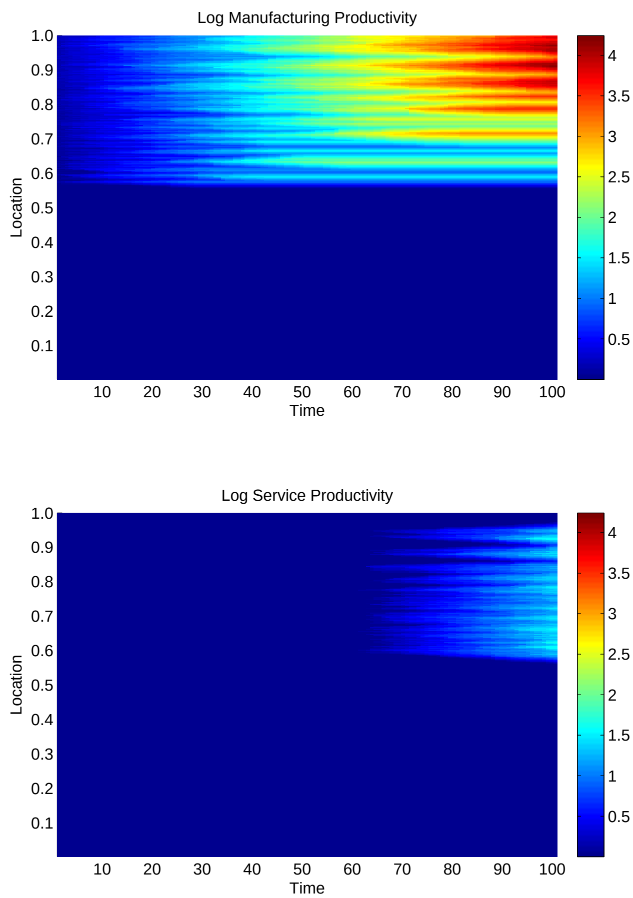

Figure 1: An Example

It is important to understand how productivity growth in the service sector gets jump-started. Assuming an elasticity of substitution less than one, the sector with the higher relative productivity growth loses employment share. Initially, when only manufacturing is innovating, the share of employment in services is gradually increasing. Since gains from innovation in a given sector depend on employment in that sector, at some point the service sector becomes large enough, allowing for innovation to take off. This mechanism provides an endogenous stabilization mechanism that tends to increase the productivity of one of the sectors when the economy experiences fast productivity growth in the other sector. The result is that by period 100 both sectors are growing at a roughly constant rate of around 3%.

5. SOME EMPIRICAL EVIDENCE

The model in Section 4 illustrates two main forces that mediate spatial dynamics. The first one is a 'spillover' effect by which locations close to other locations in the same sector grow faster because they benefit from innovation investments close by. The second is a 'trade' effect by which locations close to areas that import a particular good experience high prices for that good, thus providing incentives to innovate in that sector. If these effects are the cornerstone of spatial dynamics, as the model above postulates, we should be able to find them in the data.

Using US county data for the period 1980-2000 from the Bureau of Economic Analysis, we first construct two kernels to measure the importance of the 'spillover' and the 'trade' effect. For each county, the first kernel sums employment over all other counties, exponentially discounted by distance. To compute the second kernel, we first measure county imports in a particular sector as the difference between the county's consumption and production in that sector.[4] For each county, the second kernel then sums sectoral imports over all counties, exponentially discounted by distance.[5] This constitutes a measure of the excess demand experienced by a county in a particular sector. With these two kernels in hand, we run the following regression:

$$\log Emp_\ell^i(t+1) - \log Emp_\ell^i(t) = \alpha + \beta_1 \log Emp_\ell^i(t) + \beta_2 \log(EK_\ell^i(t)) + \beta_3 \log(IK_\ell^i(t))$$

where $Emp_\ell^i(t)$ denotes employment, $EK_\ell^i(t)$ the employment kernel, and $IK_\ell^i(t)$ the imports kernel, for sector i, county ℓ and period t.[6,7]

Table 1 presents the results for different discount rates. We fix the discount rate for the employment kernel at 0.1 (implying the effect declines by half every 7 km), and let the decay parameter for

[4] A county's consumption in a given sector is obtained by multiplying the national share of earnings in that sector by the county's total earnings. A county's production in a given sector is simply measured by its earnings in that sector. Note that this calculation does not take into account international trade, most of which is in goods. However, since this changes the level of imports in a similar way in all counties, it should not affect our calculations significantly.

[5] Note that, according to the theory, the discount rate should be related to transport costs.

[6] Since the import kernel measures a discounted sum of imports in a given sector, this measure may be positive or negative. We can therefore not simply take the natural logarithm. In the regression we use the natural logarithm of the kernel when the kernel is positive and minus the natural logarithm of the absolute value of the kernel when it is negative.

[7] Since we include the log of employment in county ℓ as a separate regressor, the employment kernel does not include employment in county ℓ. In contrast, the import kernel does include imports by county ℓ.

the import kernel vary between 0.07 and 0.14 (implying the effect declines by half every 5 to 10 km). We present four sets of regressions, the first two present the results for the service sector for the decades 2000-1990 and 1990-1980, and the last two present the same regressions for the industrial sector (manufacturing plus construction).

Decay Emp. Kernel:				0.1 (half life 7 km)				
Decay Imp. Kernel:	0.07	0.08	0.09	0.10	0.11	0.12	0.13	0.14
Half-Life Imp. Kernel (km):	9.9	8.7	7.7	6.9	6.3	5.8	5.3	5.0
Dependent variable: Log(Service Employment 2000)-Log(Service Employment 1990)								
Log(Serv. Emp. 1990)	0.00346	0.00383	0.00409	0.00426	0.0043	0.00443	0.00451	0.00458
	[1.29]	[1.43]	[1.53]	[1.59]	[1.61]	[1.66]*	[1.69]*	[1.72]*
Log(Serv. Emp. Kernel 1990)	0.00624	0.00603	0.00587	0.00576	0.00572	0.00563	0.00557	0.00552
	[4.55]***	[4.40]***	[4.28]***	[4.20]***	[4.17]***	[4.11]***	[4.07]***	[4.03]***
Log(Serv. Imp. Kernel 1990)	-0.00028	0.00014	0.00044	0.00065	0.00073	0.00089	0.00101	0.00113
	[0.74]	[0.36]	[1.15]	[1.67]*	[1.85]*	[2.26]**	[2.55]**	[2.87]***
Constant	0.18715	0.18406	0.18195	0.1805	0.18018	0.17918	0.17855	0.17789
	[8.08]***	[7.95]***	[7.86]***	[7.80]***	[7.79]***	[7.75]***	[7.73]***	[7.71]***
Observations	2277	2277	2277	2277	2277	2277	2277	2277
R-squared	0.0131	0.013	0.0135	0.0141	0.0144	0.0151	0.0157	0.0165
Dependent variable: Log(Service Employment 1990)-Log(Service Employment 1980)								
Log(Serv. Emp. 1980)	0.04007	0.04012	0.04028	0.04034	0.04026	0.04024	0.04023	0.0402
	[13.89]***	[13.91]***	[13.99]***	[14.02]***	[14.00]***	[14.00]***	[14.00]***	[13.99]***
Log(Serv. Emp. Kernel 1980)	0.01013	0.01003	0.00987	0.00977	0.00977	0.00973	0.00975	0.00978
	[6.89]***	[6.82]***	[6.72]***	[6.67]***	[6.67]***	[6.65]***	[6.66]***	[6.68]***
Log(Serv. Imp. Kernel 1980)	0.00153	0.00176	0.00208	0.00232	0.00236	0.00249	0.00251	0.00245
	[3.72]***	[4.22]***	[4.96]***	[5.49]***	[5.58]***	[5.86]***	[5.90]***	[5.76]***
Constant	-0.19616	-0.19598	-0.19655	-0.19644	-0.19573	-0.19524	-0.19523	-0.19522
	[8.12]***	[8.11]***	[8.15]***	[8.15]***	[8.12]***	[8.11]***	[8.11]***	[8.11]***
Observations	2616	2616	2616	2616	2616	2616	2616	2616
R-squared	0.1191	0.1204	0.1227	0.1245	0.1248	0.1259	0.1261	0.1255
Dependent variable: Log(Industry Employment 2000)-Log(Industry Employment 1990)								
Log(Ind. Emp. 1990)	-0.053	-0.0526	-0.05222	-0.05191	-0.05166	-0.05162	-0.0514	-0.05125
	[12.70]***	[12.59]***	[12.49]***	[12.41]***	[12.34]***	[12.32]***	[12.26]***	[12.21]***
Log(Ind. Emp. Kernel 1990)	0.00624	0.00617	0.00607	0.00602	0.00605	0.00592	0.00584	0.00567
	[2.08]**	[2.05]**	[2.02]**	[2.00]**	[2.01]**	[1.97]**	[1.94]*	[1.88]*
Log(Ind. Imp. Kernel 1990)	0.00626	0.00624	0.00628	0.00638	0.00639	0.00632	0.00633	0.00626
	[10.71]***	[10.54]***	[10.49]***	[10.59]***	[10.53]***	[10.36]***	[10.36]***	[10.22]***
Constant	0.56213	0.55913	0.55678	0.55473	0.55276	0.55297	0.55164	0.55117
	[19.06]***	[18.92]***	[18.82]***	[18.74]***	[18.65]***	[18.64]***	[18.59]***	[18.55]***
Observations	2543	2543	2543	2543	2543	2543	2543	2543
R-squared	0.1221	0.121	0.1206	0.1213	0.1209	0.1197	0.1197	0.1187
Dependent variable: Log(Industry Employment 1990)-Log(Industry Employment 1980)								
Log(Ind. Emp. 1980)	-0.03445	-0.03389	-0.03371	-0.03338	-0.03287	-0.03256	-0.0323	-0.03207
	[7.35]***	[7.22]***	[7.18]***	[7.10]***	[6.99]***	[6.92]***	[6.86]***	[6.81]***
Log(Ind. Emp. Kernel 1980)	0.03753	0.03786	0.03795	0.03806	0.0382	0.0382	0.03824	0.03828
	[10.77]***	[10.89]***	[10.92]***	[10.97]***	[11.03]***	[11.05]***	[11.08]***	[11.10]***
Log(Ind. Imp. Kernel 1980)	0.00176	0.00223	0.0024	0.00265	0.00297	0.00313	0.00332	0.00348
	[2.45]**	[3.06]***	[3.26]***	[3.59]***	[4.00]***	[4.21]***	[4.47]***	[4.68]***
Constant	0.1047	0.09882	0.09711	0.09398	0.08948	0.0871	0.08495	0.08312
	[3.21]***	[3.02]***	[2.97]***	[2.88]***	[2.73]***	[2.66]***	[2.60]***	[2.54]**
Observations	2857	2857	2857	2857	2857	2857	2857	2857
R-squared	0.0417	0.0428	0.0432	0.044	0.045	0.0456	0.0463	0.047

Absolute value of t statistics in brackets
** significant at 10%; ** significant at 5%; *** significant at 1%*

Table 1: The Effect of Employment and Import Kernels on US Employment Growth Rates

To illustrate our results, focus on the case of a decay parameter in the import kernel of 0.1 (identical to the one in the employment kernel). In services, we find that for the 1990s a 1% increase in the initial employment kernel leads to a 0.006% increase in county service employment between 1990 and 2000. The coefficient on the employment kernel does not change much across different decay parameters and across both sectors. We obtain a different result for the 1980-1990 decade, where the coefficients are still positive and significant, but the coefficient in industry is substantially larger.

We also find a positive and robust 'trade' effect. In 1980-1990 the effect seems to be similar in both industries. A 1% increase in the import kernel implies roughly a 0.002% increase in employment growth over the decade. In the 1990s, the effect is larger in industry and smaller in services. In almost all specifications the 'trade' effect is positive and significant. However, note that the model above leaves out another potential effect, namely, the growth effect of easier access to inputs in the same industry. This effect would imply, on its own, negative coefficients on the import kernel. The only case in which we obtain such a negative coefficient is when we use a very low spatial discounting coefficient for the import kernel of services in 1990-2000. Since in that case the negative coefficient is statistically insignificant, we conclude that the trade effect seems to dominate the growth effects from easier access to inputs.[8]

Table 2 presents regressions similar to the ones in Table 1, but we now take sectoral earnings growth as the dependent variable. The results are similar, and, if anything, the coefficients are larger than for employment growth. According to the theory this should be the case, since the productivity and employment effect on innovation are complementary, as are the price and employment effects (see Equation 1). As before, for virtually all decay parameters we find positive and significant 'spillover' and 'trade' effects.

6. CONCLUSION

In this paper we have discussed the theoretical problems involved in the study of spatial dynamics. The literature consists of a set of frameworks that have only been partially understood and analyzed. To deal with some of the main obstacles in this literature, we have presented a simple framework that allowed us to underscore two key links between space and time, for which we have provided empirical support. In particular, we have shown that both the 'spillover' and the 'trade' innovation

[8] Dumais, et al. (2002) provide firm-level evidence of a 'spillover' effect in the manufacturing industry. For a detailed discussion of the effect of current employment on sectoral growth rates see Desmet and Rossi-Hansberg (2009a).

effects seem to be present in US county data.

Decay Emp. Kernel:	0.1 (half life 7 km)							
Decay Imp. Kernel:	0.07	0.08	0.09	0.10	0.11	0.12	0.13	0.14
Half-Life Imp. Kernel (km):	9.9	8.7	7.7	6.9	6.3	5.8	5.3	5.0
Dependent variable: Log(Service Earnings 2000)-Log(Service Earnings 1990)								
Log(Serv. Emp. 1990)	0.04436	0.04472	0.04494	0.04517	0.04518	0.04528	0.04532	0.04536
	[12.86]***	[12.98]***	[13.06]***	[13.14]***	[13.15]***	[13.18]***	[13.20]***	[13.22]***
Log(Serv. Emp. Kernel 1990)	0.00964	0.00929	0.00905	0.00882	0.00874	0.00862	0.00854	0.00847
	[5.15]***	[4.97]***	[4.84]***	[4.73]***	[4.68]***	[4.62]***	[4.58]***	[4.54]***
Log(Serv. Imp. Kernel 1990)	0.00062	0.00117	0.00156	0.00193	0.00204	0.00222	0.00237	0.00251
	[1.22]	[2.27]**	[3.01]***	[3.70]***	[3.89]***	[4.21]***	[4.48]***	[4.73]***
Constant	0.08251	0.08013	0.07879	0.07725	0.07742	0.07682	0.07669	0.0765
	[2.84]***	[2.77]***	[2.72]***	[2.67]***	[2.68]***	[2.66]***	[2.66]***	[2.65]***
Observations	2745	2745	2745	2745	2745	2745	2745	2745
R-squared	0.0911	0.0923	0.0936	0.0951	0.0956	0.0965	0.0972	0.098
Dependent variable: Log(Service Earnings 1990)-Log(Service Earnings 1980)								
Log(Serv. Emp. 1980)	0.07938	0.07944	0.07965	0.0797	0.07957	0.07947	0.07941	0.07932
	[20.86]***	[20.91]***	[20.99]***	[21.03]***	[21.02]***	[21.01]***	[21.00]***	[20.97]***
Log(Serv. Emp. Kernel 1980)	0.01383	0.01362	0.0134	0.01325	0.0132	0.01315	0.01316	0.01322
	[7.12]***	[7.02]***	[6.92]***	[6.85]***	[6.83]***	[6.81]***	[6.82]***	[6.84]***
Log(Serv. Imp. Kernel 1980)	0.00241	0.00285	0.00326	0.0036	0.00375	0.0039	0.00394	0.00386
	[4.43]***	[5.19]***	[5.89]***	[6.47]***	[6.73]***	[6.98]***	[7.04]***	[6.88]***
Constant	-0.17965	-0.17902	-0.17955	-0.17907	-0.17744	-0.17611	-0.17554	-0.17503
	[5.65]***	[5.64]***	[5.67]***	[5.66]***	[5.61]***	[5.57]***	[5.56]***	[5.54]***
Observations	2647	2647	2647	2647	2647	2647	2647	2647
R-squared	0.2021	0.2043	0.2066	0.2087	0.2097	0.2107	0.211	0.2103
Dependent variable: Log(Industry Earnings 2000)-Log(Industry Earnings 1990)								
Log(Ind. Emp. 1990)	0.02312	0.02384	0.02453	0.02486	0.02524	0.02546	0.02584	0.02613
	[3.32]***	[3.42]***	[3.52]***	[3.56]***	[3.62]***	[3.65]***	[3.70]***	[3.74]***
Log(Ind. Emp. Kernel 1990)	0.02212	0.02232	0.0225	0.02239	0.02251	0.02247	0.02252	0.02242
	[4.36]***	[4.41]***	[4.45]***	[4.43]***	[4.45]***	[4.44]***	[4.46]***	[4.44]***
Log(Ind. Imp. Kernel 1990)	0.00607	0.0064	0.00673	0.00678	0.00688	0.00691	0.00704	0.00704
	[6.15]***	[6.41]***	[6.67]***	[6.69]***	[6.72]***	[6.72]***	[6.83]***	[6.82]***
Constant	0.08837	0.08216	0.0765	0.07448	0.07109	0.06955	0.06641	0.06462
	[1.85]*	[1.72]*	[1.60]	[1.55]	[1.48]	[1.45]	[1.38]	[1.34]
Observations	2752	2752	2752	2752	2752	2752	2752	2752
R-squared	0.0271	0.0282	0.0295	0.0295	0.0297	0.0297	0.0302	0.0301
Dependent variable: Log(Industry Earnings 1990)-Log(Industry Earnings 1980)								
Log(Ind. Emp. 1980)	-0.02419	-0.02376	-0.02347	-0.02324	-0.02272	-0.02231	-0.02196	-0.02165
	[3.89]***	[3.81]***	[3.76]***	[3.72]***	[3.63]***	[3.55]***	[3.49]***	[3.44]***
Log(Ind. Emp. Kernel 1980)	0.05805	0.05818	0.05834	0.05837	0.05852	0.0586	0.05868	0.05875
	[12.87]***	[12.91]***	[12.96]***	[12.97]***	[13.02]***	[13.06]***	[13.09]***	[13.11]***
Log(Ind. Imp. Kernel 1980)	0.00168	0.00191	0.00213	0.00223	0.00249	0.00269	0.00287	0.00302
	[1.84]*	[2.07]**	[2.28]**	[2.37]**	[2.64]***	[2.83]***	[3.01]***	[3.17]***
Constant	0.32398	0.31993	0.31693	0.31486	0.31006	0.30633	0.30321	0.30033
	[7.36]***	[7.23]***	[7.15]***	[7.09]***	[6.96]***	[6.86]***	[6.78]***	[6.70]***
Observations	2758	2758	2758	2758	2758	2758	2758	2758
R-squared	0.0587	0.059	0.0593	0.0595	0.0599	0.0603	0.0607	0.061

Absolute value of t statistics in brackets
** significant at 10%; ** significant at 5%; *** significant at 1%*

Table 2: The Effect of Employment and Import Kernels on US Earnings Growth Rates

Undoubtedly, much work is still needed. First, we need to understand the basic frameworks better. In particular, we need to extract a set of robust insights from a model rich enough to be compared with the data. This requires a model with many locations and a distribution of economic activity varied enough to calculate standard statistics. Having two or three regions without land markets is not enough. Second, we need better ways of comparing these statistics with the data. What are the main attributes of the evolution of the distribution of economic activity in space that we should compare with the data? What are the main statistics across industries that can inform us on spatial-dynamic linkages? Essentially, we need a tighter connection with the data that goes beyond reduced-form regressions like the ones in Section 5. These are mayor challenges for the next fifty years of regional science!

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