Federal Reserve Bank of New York
Staff Reports

Anxiety, Overconfidence, and Excessive Risk Taking

Thomas M. Eisenbach
Martin C. Schmalz

Staff Report No. 711
February 2015

This paper presents preliminary findings and is being distributed to economists and other interested readers solely to stimulate discussion and elicit comments. The views expressed in this paper are those of the authors and do not necessarily reflect the position of the Federal Reserve Bank of New York or the Federal Reserve System. Any errors or omissions are the responsibility of the authors.
Abstract

We provide a preference-based rationale for endogenous overconfidence. Horizon-dependent risk aversion, combined with a possibility to forget, can generate overconfidence and excessive risk taking in equilibrium. An “anxiety prone” agent, who is more risk-averse to imminent than to distant risks, has an incentive to distort her future self’s beliefs toward underestimating risk. Such self-deception can be achieved even if the future self is aware of the attempted distortion. We relate our results to the literature on empirically observed overconfidence and excessive risk taking in several domains of financial and other types of decision making.

Key words: overconfidence, dynamic consistency, biases, deception, risk taking
1 Introduction

According to Bondt and Thaler (1994), “[p]erhaps the most robust finding in the psychology of judgment is that people are overconfident.” Systematic underestimation of risk is not only an empirically robust reality (Ben-David et al., 2007, 2013)\(^1\) but also a powerful ingredient of models in financial economics (Harrison and Kreps, 1978; Scheinkman and Xiong, 2003), and possibly the key driver behind entrepreneurial ventures (Cooper et al., 1988; Koellinger et al., 2007). Furthermore, Reinhart and Rogoff (2009) and Akerlof and Shiller (2010) suggest that time-changing confidence needs to be part of realistic models of market dynamics and the business cycle. Yet, the economics literature lacks a theoretical foundation that explains why any such belief distortions can exists in equilibrium. This paper provides such an explanation within the standard paradigm of expected utility maximization.

We show that dynamically inconsistent preferences with respect to risk – which we call “anxiety-prone” – when combined with a possibility to forget can generate overconfidence. We assume that the agent exhibits higher risk aversion for imminent than for distant risks, i.e. horizon-dependent risk aversion (Eisenbach and Schmalz, 2014). An anxiety-prone decision maker prefers risky gambles with sufficiently high returns to safer alternatives as long as they are temporally distant, but reconsiders such risk-taking decisions as the risks approach and the agent gets “chickens out”.\(^2\)

A sophisticated, self-aware decision maker may search for commitment devices that constrain the future self’s action space to make the chosen actions more compatible with current preferences. In the absence of a formal commitment device that limits the choice set, distorting the future self’s beliefs can have a similar effect. If the present self can conceal information about risks from the future self, the latter will underestimate such risks and therefore take higher risks than it otherwise would, thus alleviating the inconsistency between the present self’s preferences and the future self’s actions. This is true even if the future self understands the structure of this game perfectly and tries to “undo” the belief distortion in a fully Bayesian way.

\(^1\)Moore and Healy (2008) disambiguate various alternative interpretations of “overconfidence” and clarify that over-precision of beliefs is the empirically robust finding in the literature.

\(^2\)Anxiety, a statement about preferences, is orthogonal to the belief-based concept of “cold feet” developed in Epstein and Kopylov (2007). It is more closely related to Epstein (2008), who, in contrast to our work and the experimental evidence we discuss below, assumes that risk aversion is higher for distant risks than for imminent risks.
The equilibrium level of overconfidence trades off the costs of insufficient risk-taking due to “anxiety” in some states of the world against those of excessive risk-taking due to overconfidence in others. As a result, in some states of the world, our decision marker’s risk-taking appears “excessive” to an observer who is unaware of the tradeoffs. In other words, if there was no overconfidence leading to the empirically observed excessive risk taking, e.g. by financial decision makers, there would be too little risk taking in equilibrium.

The model follows ideas laid out by Bénabou and Tirole (2002), who show that over-estimating the expected return to effort can be the optimal choice of an impatient agent with quasi-hyperbolic discounting. Such an agent has dynamically inconsistent preferences with respect to intertemporal trade-offs, in Bénabou and Tirole (2002) trading off current costs with future benefits of effort. By contrast, our model generates underestimates of variances, based on dynamically inconsistent preferences with respect to intra-temporal risk trade-offs. Finally, our model is within the standard expected utility framework. This contrasts it with models that allow the prize space to include mental states (e.g. Caplin and Leahy, 2001), information entering the utility function directly (e.g. Pagel, 2014) or preferences over information due to disappointment effects (e.g. Andries and Haddad, 2014; Gul, 1991).

We review existing experimental and field evidence that supports our assumption that temporal distance affects risk-taking behavior in Section 2. The model we propose in Section 3 generates overconfidence in the sense of underestimating the risk of a random variable as the optimal choice of an “anxious” agent, i.e. an agent with horizon-dependent risk aversion. Section 4 suggests several interpretations of the model and discusses applications. We conclude in Section 5. All proofs are in the appendix.

2 Horizon-Dependent Risk Aversion in Experiments

This section reviews evidence supporting the key assumption of our model, which is that temporal distance affects risk-taking behavior. In particular, subjects tend to be more risk averse when a risk is temporally close than when it is distant, both in across-subject and within-subject studies.

Jones and Johnson (1973) have subjects participate in a simulated medical trial for a new drug; each subject has to decide on a dose of the drug to be administered. The
subjects are told that the probability of experiencing unpleasant side-effects increases with the dose – but so does monetary compensation. More risk averse subjects should then choose lower doses than less risk averse subjects. The study finds that subjects choose higher doses when they are to be administered the next day than when they are to be administered immediately. Interestingly, the difference disappears if the decision can be revisited the next day (no commitment), suggesting that subjects may anticipate their preference reversals. The study also measures higher stress levels for subjects deciding among immediate doses than for subjects deciding among delayed doses.

Welch (1999) documents preference reversals caused by stage fright. He finds that 67% of subjects who agree to tell a joke in front of a class the following week in exchange for $1 “chicken out” when the moment of truth arrives. In contrast, none of those who decline initially change their mind.

A widely used method in experimental economics to elicit risk aversion is the protocol of Holt and Laury (2002). Subjects are presented with a list of choices between two binary lotteries. The first lottery always has two intermediate prizes, e.g. ($10.00, $8.00), while the second lottery always has a high and a low prize, e.g. ($19.25, $0.50). Going down the list, only the respective probabilities of the two prizes change, varying from (0.1, 0.9) to (0.9, 0.1). As probability mass shifts from the second prize to the first prize of both lotteries, the second lottery becomes increasingly attractive compared to the first lottery. Subjects are asked to pick one of two lotteries for each of the probability distributions. The probability distribution at which a subject switches from the “safe” lottery to the “risky” lottery is a proxy for the subject’s risk aversion. Noussair and Wu (2006) use this protocol for a within-subject design with real payoffs, having each subject make choices for resolution and payout to occur immediately and also for risks and payouts that occur three months later. The study finds that more than one-third of subjects are more risk averse for the present than for the future. Coble and Lusk (2010) use the protocol for an across-subject design and find the same pattern with average risk aversion increasing with the temporal proximity of the risk.

In a different type of experiment, Baucells and Heukamp (2010) let subjects choose between two binary lotteries, a “safer” and a “riskier” one. Different treatments vary the delay until the lotteries are resolved and paid out. The study finds that more subjects choose the riskier lottery as the delay increases. Sagristano, Trope, and Liberman (2002) also have subjects choose between two lotteries and find the same effect of temporal
proximity.

Finally, some studies elicit risk aversion by asking subjects for their certainty equivalents for different lotteries; a lower certainty equivalent corresponds to higher risk aversion. In Onculer (2000), subjects state their certainty equivalent for a lottery to be resolved and paid immediately, as well as for the same lottery to be resolved and paid in the future. The study finds that subjects state significantly lower certainty equivalents for the immediate lottery than for the future lottery. Abdellaoui, Diecidue, and Onculer (2011) conduct a similar study with real payoffs and find equivalent results.

3 A Preference-Based Model of Overconfidence

3.1 Preferences

Consider a typical risk-reward tradeoff given by two lotteries \( \tilde{x} \) and \( \tilde{y} \) where \( \tilde{x} = \tilde{y} + \tilde{\varepsilon} + \mu \) with \( \tilde{\varepsilon} \) a mean-zero lottery independent of \( \tilde{y} \) and \( \mu \) a constant so that \( \tilde{x} \) has “higher risk” but also “higher reward” than \( \tilde{y} \). To capture the experimental evidence of agents who prefer the risky lottery \( \tilde{x} \) over \( \tilde{y} \) if both are delayed but prefers the safe lottery \( \tilde{y} \) over \( \tilde{x} \) if both are immediate, we assume the following utility specification:

\[
U_0 = \mathbb{E}[v(x_0) + \delta u(x_1)]
\]

and

\[
U_1 = \mathbb{E}[v(x_1)]
\]

where \( \mathbb{E} \) is the expectations operator, \( \delta \leq 1 \) is a discount factor and. More importantly, \( v \) and \( u \) are von Neuman-Morgenstern utility indexes that depend on whether a risk is imminent or distant. To generate the same choice behavior as experimentally docu-
mented (Figure 1 gives a stylized example), the utility specification has to satisfy the following two conditions:

For distant lotteries:  \[ \mathbb{E}[\delta u(\tilde{x})] > \mathbb{E}[\delta u(\tilde{y})] \]

For imminent lotteries:  \[ \mathbb{E}[v(\tilde{x})] < \mathbb{E}[v(\tilde{y})] \]

Given the definitions of \( \tilde{x} \) and \( \tilde{y} \), these conditions can be satisfied only with \( v \) more risk averse than \( u \):

\[ -\frac{v''(x)}{v'(x)} \geq -\frac{u''(x)}{u'(x)} \quad \text{for all } x. \]

Note that the discount factors \( \delta \) plays no role in the two conditions above. This illustrates that intra-temporal risk tradeoffs and inter-temporal consumption tradeoffs are conceptually very different; the experimental evidence can therefore not be addressed by relaxing the standard assumption of geometric discounting.

As an example, let \( v(x) = \sqrt{x} \) and \( u(x) = x \) and set \( \delta = 1 \). Then the agent is risk averse with respect to current uncertainty and risk neutral with respect to future uncertainty. Now consider the following two lotteries:

\[ \tilde{x} = \begin{cases} 4 & \text{with prob. } \alpha \\ 0 & \text{with prob. } 1 - \alpha \end{cases} \quad \text{and} \quad \tilde{y} = 1 \]

Then \( v \) prefers the risky \( \tilde{x} \) to the safe \( \tilde{y} \) if \( \alpha > \frac{1}{2} \) while \( u \) prefers \( \tilde{x} \) to \( \tilde{y} \) if \( \alpha > \frac{1}{4} \) and there is disagreement between \( v \) and \( u \) for all \( \alpha \in (\frac{1}{4}, \frac{1}{2}) \) as illustrated in Figure 1. In particular, suppose that \( \alpha = \frac{1}{3} \) and that the lotteries are resolved and paid out in period 1. Then a sophisticated agent will choose the safe option \( \tilde{y} \) in period 1 but would prefer to commit to the risky option \( \tilde{x} \) in the initial period 0. In fact, the agent is willing to pay up to \( \frac{1}{3} \) to commit to the risky option in period 0 but then is willing to pay up to \( \frac{5}{9} \) to avoid the risky option once period 1 arrives.

### 3.2 Environment

In this paper we are interested in the case where outright commitment devices are not available to the agent and the current self may try to distort the future self’s beliefs to manipulate its actions. In particular, the current self would like to convince the future
self that risks are lower than they actually are. This would lead the future self to take riskier decisions which are more in line with the current self’s preferences. However, if the future self has access to additional information, the distorted beliefs may lead to decisions that are excessively risky, even from the current self’s point of view. To analyze such intra-personal manipulation, we place our agent in a setting similar to the model of Bénabou and Tirole (2002).3

There are two periods \( t = 0, 1 \). In period 1 the agent has to choose between a risky or a safe alternative. The risky alternative is given by a lottery with random payoff \( x \). The lottery is characterized by its distribution function \( G_\theta \) where \( \theta \in \{H, L\} \) denotes a state of the world that determines how risky the lottery is. We assume that \( G_H \) is a mean-preserving spread of \( G_L \) so the risky alternative is unambiguously riskier in state \( H \) than in state \( L \). The prior probability of the high-risk state \( H \) is given by \( \pi \). The safe alternative, on the other hand, is given by a constant payoff \( a \).

When facing the decision in period 1, the agent evaluates the risk using utility \( v \) and therefore wants to take the risky alternative whenever

\[
\mathbb{E}_\theta[v(x)] > v(a),
\]

where \( \mathbb{E}_\theta \) denotes the expectation with respect to \( G_\theta \). Denoting the certainty equivalent of \( G_\theta \) given the utility function \( v \) by \( c^\theta_v \), this condition can be rewritten as

\[
c^\theta_v > a.
\]

The agent wants to take the risky alternative, whenever its certainty equivalent \( c^\theta_v \) is greater than the safe alternative \( a \).

When thinking about the decision ahead of time in period 0, the agent evaluates the risk using utility \( u \) and therefore wants the future self to take the risky alternative whenever

\[
\mathbb{E}_\theta[u(x)] > u(a)
\]

\[\Leftrightarrow\]

\[c^\theta_u > a.\]

\[3\text{For earlier work studying belief manipulation in a setting with } \beta-\delta \text{ time inconsistency see Carrillo and Mariotti (2000).}\]
As in the simple numerical example above, we have potential disagreement between self 0 and self 1.

**Lemma 1.** Since \( v \) is more risk averse than \( u \), we have \( c^0_u > c^0_v \) for both \( \theta \in \{H, L\} \) so the agent in period 0 (self 0) and the agent in period 1 (self 1) will disagree about the right course of action whenever \( a \in [c^0_v, c^0_u] \).

To make this problem interesting, we assume that the payoff of the safe alternative \( a \) is not known to the agent until period 1. Self 0 only knows the prior distribution \( F \) on \( [a, \bar{a}] \) but self 1 observes the realized value of \( a \). The state of the world \( \theta \), on the other hand, is revealed to the agent at the beginning of period 0 in form of a perfectly informative “red flag” warning signal \( s \) if the state is high-risk:

\[
s = \begin{cases} 
R & \text{if } \theta = H \\
\emptyset & \text{if } \theta = L
\end{cases}
\]

If self 0 receives a red flag, it can choose to forget the signal with probability \( \phi \in [0, 1] \),

\[
\phi = \Pr[\hat{s} = \emptyset | s = R],
\]

where \( \hat{s} \) is self 1’s recollection of the signal. We assume that self 1 is fully aware of self 0’s incentive to forget warning signals, so self 1 expects a forgetting probability \( \phi^e \). Self 1 is fully Bayesian so conditional on not remembering a red flag signal, self 1 assigns a posterior probability to the state of the world being high-risk given by:

\[
\hat{\pi}(\phi^e) = \frac{\pi \phi^e}{\pi \phi^e + 1 - \pi}
\]

### 3.3 Intra-Personal Game

Given our setup, self 0 and self 1 are playing a form of Stackelberg game. First self 0 chooses the forgetting probability \( \phi \) taking into account self 1’s behavior and then self 1 decides between the risky and the safe alternative taking into account self 0’s behavior. Figure 2 illustrates the timeline of the intra-personal game; Figure 3 in the appendix gives an extensive form representation. We are interested in the perfect Bayesian equilibria of this intra-personal game.
First, we derive self 1’s best response in period 1, taking as given an expected forgetting probability $\phi^e$. If self 1 remembers seeing a red flag, $\hat{s} = R$, she knows that the state of the world is high-risk and chooses the risky alternative if $c_H^v > a$. If self 1 doesn’t remember seeing a red flag, $\hat{s} = \emptyset$, she uses the Bayesian posterior $\hat{\pi}(\phi^e)$ and chooses the risky alternative if $c_v(\phi^e) > a$ where $c_v(\phi^e)$ is the certainty equivalent of the risky alternative given $\phi^e$, implicitly defined by:

$$\mathbb{E}[v(x) | \hat{\pi}(\phi^e)] = v(c_v(\phi^e))$$

Second, we derive self 0’s best response in $t = 0$, taking as given self 1’s behavior for an expected $\phi^e$. If self 0 receives a warning signal and chooses a forgetting probability $\phi$, her expected utility is:

$$(1 - \phi) \left[ \int_{a}^{c_H^v} \mathbb{E}_H[u(x)] dF(a) + \int_{c_H^v}^{\pi} u(a) dF(a) \right] + \phi \left[ \int_{a}^{c_v(\phi^e)} \mathbb{E}_H[u(x)] dF(a) + \int_{c_v(\phi^e)}^{\pi} u(a) dF(a) \right]$$

With probability $1 - \phi$ the agent remembers the warning signal in period 1 and uses the certainty equivalent $c_H^v$ as the threshold; then she chooses the risky alternative for payoffs of the safe alternative below the threshold, $a \in [a, c_H^v)$, and chooses the safe alternative for payoffs above the threshold, $a \in [c_H^v, \bar{a}]$. With probability $\phi$ the agent forgets the warning signal and uses the certainty equivalent $c_v(\phi^e)$ as the threshold,
choosing the risky alternative for \( a \in [\underline{a}, c_v(\phi^e)] \) and the safe alternative for \( a \in [c_v(\phi^e), \bar{a}] \).

We denote by \( D(\phi^e|v) \) the derivative of self 0’s expected utility with respect to her choice variable \( \phi \) conditional on the value \( \phi^e \) expected by self 1. This marginal benefit of forgetting is given by:

\[
D(\phi^e|v) := \int_{c_v(\phi^e)}^{c_H} \left( E_H[u(x)] - u(a) \right) dF(a)
\]

This expression has a very natural interpretation. The warning signal affects self 1’s decision only for realizations of the safe alternative \( a \in [c_H, c_v(\phi^e)] \). In this interval, self 1 chooses the risky alternative whenever she remembers seeing a red flag and the safe alternative otherwise. The effect on self 0’s expected utility of forgetting the warning signal more often is exactly the difference in utility from the risky action compared to the safe action for the values of \( a \) where the decision is affected.

There are three possibilities for perfect Bayesian equilibria in this intra-personal game, characterized by the equilibrium forgetting probability \( \phi^* \):

**Honesty:** If \( D(0|v) \leq 0 \), there is an equilibrium with \( \phi^* = 0 \). In this equilibrium the agent never ignores red flags and doesn’t influence her future self’s beliefs.

**Overconfidence:** If \( D(1|v) \geq 0 \), there is an equilibrium with \( \phi^* = 1 \). In this equilibrium the agent always ignores red flags and makes her future self maximally overconfident.

**Mixed:** If \( D(\bar{\phi}|v) = 0 \) for some \( \phi \in (0, 1) \), there is an equilibrium with \( \phi^* = \bar{\phi} \).

In this equilibrium the agent plays a mixed strategy, ignoring the red flag with probability \( \bar{\phi} \), and makes her future self partially overconfident.

We have the following result on existence of equilibria.

**Proposition 1.** One of the extreme equilibria always exists, either the honesty equilibrium or the overconfidence equilibrium or both. If both extreme equilibria exist, a mixed equilibrium also exists.
3.4 Apparent Over- and Underconfidence

In any equilibrium $\phi^*$ with overconfidence, an outside observer will find the agent using one of two cutoffs. If the state is high-risk and the warning signal was remembered the agent is observed using the cutoff $c^H_v$ which makes her seem perfectly calibrated. However, if the warning signal was forgotten or the state of the world is low-risk the agent is observed using the cutoff $c_v(\phi^*)$.

The two possibilities for observing cutoff $c_v(\phi^*)$ and the fact that $c^H_v < c_v(\phi^*) < c^L_v$ generate different interpretations for the outside observer. If the state of the world is high-risk but the warning signal was forgotten, the agent using $c_v(\phi^*)$ appears overconfident to the outside world since – based on her preference $v$ – she is expected to use $c^H_v < c_v(\phi^*)$. If the state of the world is low-risk, the agent using $c_v(\phi^*)$ appears underconfident since she is expected to use $c^L_v > c_v(\phi^*)$. This yields the following corollary.

**Corollary 1.** In any equilibrium with overconfidence, agents can appear to be over- or underconfident ex post, depending on the true state of the world:

- Agents can only appear overconfident if the environment truly is high-risk.
- Agents can only appear underconfident if the environment truly is low-risk.

3.5 Excessive Risk-Taking

Equilibria with partial or maximal overconfidence can display excessive risk taking. In these equilibria it can be the case that the future self ends up taking risks which even the less risk averse current self would have avoided. To an observer who is unaware of the agent’s intra-personal conflict and resulting equilibrium level of overconfidence, the agent seems to be taking risks that are greater than can be explained even based on the less risk averse preference $u$.

This seemingly paradoxical situation of an anxious agent taking excessive risks can arise if the true state of the world is high-risk, $\theta = H$, and the agent forgets the warning signal, $\hat{s} = \emptyset$. In this case, self 0 would like the cutoff $c^H_u$ to be used but self 1 actually uses the cutoff $c_v(\phi^e)$. Whenever the payoff of the safe alternative is between the two cutoffs, $a \in (c^H_u, c_v(\phi^e))$, the agent takes risks in period 1 that even self 0 considers
excessive. Of course, the paradox is due to the fact that self 0 knows the state of the world to be high-risk while self 1 has to rely on her Bayesian posterior.

**Corollary 2.** In an equilibrium with overconfidence and $c_u^H < c_v(\phi^*)$, the agent will be observed to take excessive risks, i.e. she will appear less risk averse than both $v$ and $u$.

Excessive risk-taking can arise since the condition for an equilibrium with overconfidence, $D(\phi^*|v) \geq 0$, does not necessarily imply $\mathbb{E}_H[v(x)] > u(a)$ for all $a < c_v(\phi^*)$, i.e. that self 0 wants the risky alternative where self 1 chooses it. Therefore, excessive risk taking arises in all equilibria $\phi^*$ with $c_u^H < c_v(\phi^*)$, i.e. the equilibrium cutoff used by self 1 is greater than the cutoff self 0 would use. To an outside observer who knows that the state is $H$, the anxious agent using the cutoff $c_v(\phi^*)$ appears as if she were less risk averse than the non-anxious preference $u$. This is not true, however – rather, the anxious agent using the cutoff $c_v(\phi^*)$ is systematically overconfident.

Why such excessive risk taking is an equilibrium outcome can be illustrated as follows. From Lemma 1 we know that the certainty equivalents always satisfy $c_v^H > c_u^H$. In an equilibrium with excessive risk taking we also have $c_u^H < c_v(\phi^*)$. Given these two inequalities we can decompose the marginal effect of forgetting more often on self 0’s utility as follows:

$$D(\phi^*|v) = \int_{c_v(\phi^*)}^{c_v^H} (\mathbb{E}_H[u(x)] - u(a)) dF(a)$$

$$= \int_{c_v(\phi^*)}^{c_v^H} (\mathbb{E}_H[u(x)] - u(a)) dF(a) - \int_{c_v(\phi^*)}^{c_u^H} (\mathbb{E}_H[u(x)] - u(a)) dF(a)$$

In an equilibrium $\phi^*$ with excessive risk taking we have $D(\phi^*|v) \geq 0$. Given the decomposition above, this implies the following inequality:

$$\int_{c_v^H}^{c_u^H} (\mathbb{E}_H[u(x)] - u(a)) dF(a) \geq \int_{c_v(\phi^*)}^{c_v^H} (\mathbb{E}_H[u(x)] - u(a)) dF(a)$$

For values of the safe alternative $a \in (c_v^H, c_u^H)$ self 1 only takes risk if manipulated and this is desirable, captured by the utility benefit on the left-hand side. For values $a \in (c_u^H, c_v(\phi^*))$ self 1 takes excessive risks that self 0 doesn’t want to take and this is undesirable, captured by the utility cost on the right-hand side. For the excessive
risk taking to occur in equilibrium, the benefit of more risk taking when desired has to outweigh the cost of too much risk taking when not desired.

3.6 Comparative Statics

The existence of each kind of equilibrium depends on the all the main primitives of the model, which yields the following comparative statics.

**Proposition 2.** Overconfidence is more likely – in the sense that the honesty equilibrium is less likely to exist, the overconfidence equilibrium is more likely to exist, and a mixed equilibrium has more overconfidence – in any of the following situations:

1. If the agent is more prone to anxiety – in the sense that \( u \) remains unchanged but \( v \) is even more risk averse.

2. If the high-risk state is more likely ex ante – in the sense that \( \pi \) is higher.

3. If the high-risk state is more risky – in the sense that \( G_L \) remains unchanged but a mean-preserving spread is added to \( G_H \).

Somewhat counterintuitively, we find that agents who are more prone to anxiety when facing immediate risk who are the ones that are more likely to exhibit overconfidence. In terms of the environment agents are in, we find that a riskier environment – both ex ante and ex post – is more conducive to overconfidence.

We can interpreting the results of Proposition 2 in the cross-section of environments faced by different agents. The fact that agents in riskier environments are more likely to exhibit overconfidence (part 2 of Proposition 2) is reminiscent of work on cognitive dissonance such as Akerlof and Dickens (1982) which involves the assumption of psychic utility such as the fear of accidents. Our framework also applies to environment where the agent’s job involves risk-taking without risk to bodily harm. For example, the finance profession should be particularly likely to feature overconfident agents, as documented by Ben-David et al. (2013), see also Lo and Repin (2002); Lo et al. (2005).

A benefit of the fully rational framework and the perfect Bayesian equilibrium analysis is that we can interpret the comparative statics of Proposition 2 not only in the cross-section of agents or environments but also in the time series. As long as the realization of the state of the world \( s \) is i.i.d. so there is no role for learning, we
can imagine the simple two-period setup being repeated in sequence with parameters characterizing the environment changing over time.

Empirically, Reinhart and Rogoff (2009) as well as Akerlof and Shiller (2010) argue that confidence is too high at the peak of booms when actual risks are high while confidence is too low in the trough of crises when actual risk is low. This prediction is consistent with our result that agents appear overconfident when risks are high and underconfident when risks are low. Further, as overconfident traders have a greater demand for risk than unbiased types do, overconfidence sustains and reinforces excessive risk levels. Conversely, under-confidence helps sustain price levels below fundamentals in a crisis.

4 Interpretation

The above model describes overconfidence as resulting from a choice to forget risk signals, according to a well-specified maximization problem. As is true in most economic modeling, this model is not necessarily meant to describe the actual mental process of the decision maker. Rather, the outcome of the maximization problem represents the observed behavior and beliefs as if the decision maker consciously made the trade-offs described. We now discuss how real-world decision makers might – consciously or not – implicitly implement the self-deception game presented in reduced form above.

4.1 Choice of Information Environment

One interpretation for the model’s belief-manipulation framework is as a reduced-form metaphor for the choice of the agent’s informational environment. Specifically, given a preference for a biased posterior, an anxiety-prone agent will attempt to implement information and communication systems that render her misinformed about risks.

In an organizational context, management scholars and practitioners have remarked about the scarcity of openly expressed critical upward feedback. Indeed, the lack of informal and open upward feedback is the reason for the establishment of formal, anonymous upward feedback mechanisms investigated by the personnel psychology literature.

\footnote{It may be difficult to imagine a conscious decision to forget. Yet, remembering something often takes a conscious effort. Not making such an effort, which likely results in forgetting, can therefore be viewed as a conscious choice.}
(Atwater et al., 1995; Smither et al., 1995; Walker and Smither, 1999; Atwater et al., 2000). Lack of upward feedback is often said to be implicitly or explicitly mandated by the head of the organization (“killing the messenger”).

In the context of our model, an anxiety-prone leader will indeed design incentives for subordinates to systematically hide risk signals from her, especially in particularly risky states of nature. As a result, the more severe the dynamic inconsistency in the leader’s preferences and the higher the actual risk level, the less upward feedback subordinates will provide.

While these examples resonate with informal accounts of the informational environments in Wall Street firms before the recent crises, there is also direct evidence on the biased choice of information from financial decision making. Karlsson et al. (2009) find that investors look up their portfolio performance less often after receiving a signal about increased risks. This behavior is known as the “Ostrich Effect”. Bhattacharya et al. (2012) find that retail investors have little demand for unbiased advice – especially those who need it the most.

4.2 Self-Manipulation with Alcohol and Drugs

A second interpretation of how the belief manipulation of our model may be implemented in practice is through the use of alcohol and other drugs. This section gives a brief review of psychological evidence on the effect of alcohol and other drugs on (i) risky behavior, (ii) forgetting and confidence, and (iii) performance changes. In addition, we discuss evidence on the strategic use of alcohol and other drugs by anxiety-prone individuals to induce effects (i)–(iii).

The finding that alcohol is associated with more risky behavior is robust across domains. In the field, alcohol consumption has been shown to lead to risky sexual behavior (Halpern-Felsher et al., 1996; Cooper, 2002), accident-related injuries (Cherpitel et al., 1995), and dangerous driving patterns (Donovan et al., 1983). Pathological gambling is more common among people with alcohol use disorders, and vice versa (Grant

---

5A historical leader with apparently small propensity to anxiety is Queen Elizabeth I., who is said to have rebuked a jester “for being insufficiently severe with her” (Otto, 2001).

6The original finding is that investors tend to not look up their portfolio’s performance after market-wide declines, about which they are likely to become informed via generic news reports. Note that (i) price drops may be caused by increases in risk levels, but also (ii) falling prices increase volatility estimates. Thus, in any case, falling prices are a signal for increased risk.
et al., 2002; Petry et al., 2005). In the lab, Lane et al. (2004) establish causality from alcohol consumption to risky behavior.

Riskier behavior can be driven either by reductions in risk aversion, or by a decreased perception of risk. Cohen et al. (1958) show that the more risky driving behavior caused by alcohol consumption is associated with a higher degree of overconfidence. Supporting the channel suggested by our model, alcohol has also been shown to lead to forgetting, especially of negative signals (Nelson et al., 1986; Maylor and Rabbitt, 1987).

In the context of our model, the performance of anxiety-prone individuals should then improve with moderate levels of drug-induced overconfidence. James et al. (1977) as well as Brantigan et al. (1982) show that the use of beta-blockers improves the performance of musicians with stage fright.

Lastly, there is evidence that drugs are used strategically to induce performance changes, and particularly so for individuals with greater degrees of horizon-dependent risk aversion. Steptoe and Fidler (1987) find that 17 percent of professional musicians with high performance anxiety reported taking sedatives as a method of coping, compared with 4 percent with medium and none of the respondents with low performance anxiety. Anecdotal evidence on the “widespread use of [...] cocaine by professional traders” (Bossaerts, 2009) is consistent both with strategic self-manipulation and with our observations about cross-sectional overconfidence across environments.7

5 Conclusion

Using standard tools in economics, this paper shows that horizon-dependent risk aversion preferences (“anxiety”) supply a rationale for overconfident beliefs, wherein selective information processing is used as a tool to accomplish self-delusion. The model predicts salient features of organizational design, individuals’ choice of information systems, and drug use, as well as observed equilibrium levels of overconfidence. An application to equilibrium asset pricing models is left for future research.

References


Appendix

Proof of Proposition 1. The belief \( \hat{\pi}(\phi^e) \) is continuous and increasing in \( \phi^e \). Therefore the certainty equivalent \( c_v(\phi^e) \) is continuous and decreasing in \( \phi^e \). Finally, this implies that \( D(\phi^e|v) \) is continuous and increasing in \( \phi^e \). We then have either \( D(1|v) \geq 0 \) or \( D(0|v) \leq 0 \) or both so one of the extreme equilibria \( \phi^* \in \{0, 1\} \) always exists. For the case where \( D(1|v) \geq 0 \) and \( D(0|v) \leq 0 \), there exists a \( \phi \in (0, 1) \) such that \( D(\phi|v) = 0 \) so the mixed equilibrium \( \phi^* = \phi \) also exists. \( \square \)

Lemma 2. Consider two von Neumann-Morgenstern utility functions \( v_1 \) and \( v_2 \). If \( v_2 \) is more risk averse than \( v_1 \), then \( D(\phi^e|v_2) > D(\phi^e|v_1) \) for all \( \phi^e \).

Proof of Lemma 2. If \( v_2 \) is more risk averse than \( v_1 \), then \( c_{v_2}^H < c_{v_1}^H \) and \( c_{v_2}(\phi^e) < c_{v_1}(\phi^e) \) for all \( \phi^e \). This implies that for all \( \phi^e \)

\[
D(\phi^e|v_2) = \int_{c_{v_2}^H}^{c_{v_2}(\phi^e)} (E_H[u(x)] - u(a)) dF(a) \\
> \int_{c_{v_1}^H}^{c_{v_2}(\phi^e)} (E_H[u(x)] - u(a)) dF(a) \\
= D(\phi^e|v_1),
\]
Lemma 3. Consider two von Neumann-Morgenstern utility functions \( v_1 \) and \( v_2 \). If \( v_2 \) is more risk averse than \( v_1 \) and if there are \( \bar{\phi}_1 \) and \( \bar{\phi}_2 \) such that \( D(\bar{\phi}_1|v_1) = 0 \) and \( D(\bar{\phi}_2|v_2) = 0 \), then \( \bar{\phi}_1 > \bar{\phi}_2 \).

Proof of Lemma 3. If \( v_2 \) is more risk averse than \( v_1 \), then \( c_{v_2}^H < c_{v_1}^H \) by Lemma 1 so the integral in \( D(\bar{\phi}_2|v_2) \) has a smaller lower bound. Since \( (\mathbb{E}_H[u(x)] - u(a)) \) is a strictly decreasing function of \( a \), for \( D(\bar{\phi}_1|v_1) = D(\bar{\phi}_2|v_2) = 0 \) it is necessary that \( c_{v_2}(\bar{\phi}_2) > c_{v_1}(\bar{\phi}_1) \), i.e., that the integral in \( D(\bar{\phi}_2|v_2) \) must have a greater upper bound. Since \( c_{v_2}(\phi) < c_{v_1}(\phi) \) for a given \( \phi \), and \( c_{v}(\phi) \) is decreasing in \( \phi \) for \( v_1 \) and \( v_2 \), this implies \( \bar{\phi}_2 < \bar{\phi}_1 \). □

Proof of Proposition 2. For part 1, from Lemma 2 we know that \( D(1|v_2) > D(1|v_1) \) for \( v_2 \) more risk averse than \( v_1 \). Therefore an overconfidence equilibrium exists for \( v_2 \) if it exists for \( v_1 \). Again using Lemma 2 we know that \( D(0|v_2) > D(0|v_1) \) for \( v_2 \) more risk averse than \( v_1 \). Therefore an honesty equilibrium exists for \( v_1 \) if it exists for \( v_2 \). Finally, if a mixed equilibrium exists for \( v_1 \) and \( v_2 \), characterized by \( \bar{\phi}_1 \) and \( \bar{\phi}_2 \) respectively, then by Lemma 3 we have \( \bar{\phi}_1 > \bar{\phi}_2 \).

For part 2, note that \( \hat{\pi}(\phi^e) \) is increasing in \( \pi \) similarly as in \( \phi^e \) so analogously to the proof of Proposition 1, we know that \( D(\phi^e|v) \) is increasing in \( \pi \). This implies that for higher \( \pi \) the condition \( D(0|v) \leq 0 \) for an honesty equilibrium is harder to satisfy, the condition \( D(1|v) \geq 0 \) for an overconfidence equilibrium is easier to satisfy and any solution to \( D(\bar{\phi}|v) = 0 \) will be for a higher \( \bar{\phi} \).

For part 3, note that adding a mean-preserving spread to the distribution \( G_H \) has same effect on certainty equivalents as more disagreement between \( v \) and \( u \) so the arguments for part 1 apply analogously. □