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# Optimal Interest Rate Rules and Inflation Stabilization versus Price-Level Stabilization

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## Abstract

This paper compares the properties of interest rate rules such as simple Taylor rules and rules that respond to price-level fluctuations—called Wicksellian rules—in a basic forward-looking model. By introducing appropriate history dependence in policy, Wicksellian rules perform better than optimal Taylor rules in terms of welfare and robustness to alternative shock processes, and they are less prone to equilibrium indeterminacy. A simple Wicksellian rule augmented with a high degree of interest rate inertia resembles a robustly optimal rule—that is, a monetary policy rule that implements the optimal plan and is also completely robust to the specification of exogenous shock processes.

Key words: optimal monetary policy, Taylor rule, robust policy

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# 1 Introduction

Is the central bank's objective best achieved by a policy that responds to fluctuations in inflation or the price level? This remains an open question that has regained attention recently among central banks, such as the Bank of Canada which just renewed its "inflation-control target" or even in the US Federal Reserve System (e.g., Evans, 2010). This paper argues that when monetary policy is constrained to following simple rules that respond to inflation or the price level, price-level stabilization delivers superior outcomes on at least three dimensions, in the context of a simple forward-looking model. First, price-level stabilization delivers outcomes that are closest to optimal. Next, such a policy has a robustness feature that delivers desirable outcomes even in the face of key types of model misspecification. Finally, price-level stabilization is more likely to result in a unique bounded equilibrium.

Early studies found it preferable to respond to inflation than to price-level fluctuations in order to minimize the short-run variability of inflation and output (e.g., Lebow *et al.,* 1992; Haldane and Salmon, 1995). The intuition for this result is simple: in the face of an unexpected temporary rise in inflation, price-level stabilization requires the policymaker to bring inflation below the target in subsequent periods. With nominal rigidities, this results in turn in output fluctuations. In contrast, with inflation targeting, the drift in the price level is accepted: bygones are bygones. Price-level stabilization is a "bad idea" according to this conventional view because it would "add unnecessary short term fluctuations to the economy" (Fischer, 1994, p. 282), while it would only provide a small gain in long-term price predictability in the US (McCallum, 1999). However, when agents are forward-looking, committing to a history-dependent policy allows the central bank to affect the private sector's expectations appropriately, hence to improve the performance of monetary policy (Woodford, 2003a,b). This suggests that past deviations of the inflation rate should not be treated as bygones. In fact, in the sticky-price models of Goodfriend and King (2001), Kahn, King and Wolman (2003), and Woodford (2003a), optimal policy involves strong price-level stabilization, though it requires some drift of the price level in the face of some shocks.

In this paper, we consider a basic forward-looking New Keynesian model in which the

social welfare loss function depends on the variability of inflation, the output gap and the interest rate.[1] We seek to determine whether it is best for policy to respond to fluctuations in inflation or in the price level in this model, assuming that the central bank can commit to a policy rule. Simple monetary policy rules are often prescribed as useful guides for the conduct of monetary policy. Most prominently, a commitment to a Taylor rule (after Taylor, 1993) — according to which the short-term policy rate responds to fluctuations in inflation and some measure of the output gap — is known to yield a good welfare performance in a large class of models (see, e.g., papers collected in Taylor, 1999a; Taylor and Williams, 2010). We thus compare the performance of such Taylor rules to that of so-called Wicksellian rules according to which the short-term policy rate depends on deviations of the *price level* from a trend and the output gap.[2] We compare the properties of optimal policy and simple interest-rate rules, focusing our evaluation on two key aspects of policy rules: (i) their welfare implications, and (ii) their robustness to some kind of misspecification that is likely prevalent, i.e., misspecification about the assumed shock processes.

We find that Wicksellian rules perform very well in terms of welfare, indeed better than optimal Taylor rules, by introducing a desirable amount of history dependence in policy. Under price-level stabilization, forward-looking agents understand that current above-target in inflation will be followed by below-target inflation in subsequent periods. This in turn dampens the firms' willingness to change their prices, lowers the variability of inflation and welfare losses.[3]

In addition, while simple Taylor rules are often argued to be robust to various types of model misspecifications (Levin, Williams and Wieland, 1999; Levin and Williams, 2003), we show that their welfare performance can however be very sensitive to the particular assumptions made about the shock processes. Instead, Wicksellian rules are more robust to alternative shock processes. Specifically, we show that (i) optimized coefficients of simple Taylor rules depend critically on the assumed degree of persistence of exogenous disturbances;

[1] This function can be viewed as a quadratic approximation to the representative agent's expected utility.

[2] Wicksellian rules are named after Wicksell (1907) who argued that "price stability" could be obtained by letting the interest rate respond positively to fluctuations in the price level.

[3] Note that the inflation rate used in much of John Taylor's work (e.g., in Taylor, 1993) is a moving average of past quarterly inflation rates, so that his proposed rule incorporates in fact some degree of history dependence. To understand the role of history dependence introduced by the price level, we consider here "Taylor rules" that involve only the contemporaneous inflation rate.

(ii) such optimized Taylor rules result in an indeterminate equilibrium for some parameter configurations; (iii) the welfare performance relative to the first best deteriorates sharply in the event that the economy is hit by shocks with a higher persistence than the typical historical shocks. In contrast, optimized Wicksellian rules (i) are less sensitive to the assumed shock persistence, (ii) generally result in a determinate equilibrium, and (iii) maintain a very good welfare performance in the face of changes in shock processes or in the face of misspecified shocks. This sensitivity of optimized simple Taylor rules is arguably undesirable to the extent that in practice central banks may not want to commit to policy rules that perform well only in the face of a few typical shocks, as they may not be able to conceive at the time of commitment all possible shocks that will affect the economy in the future. Policymakers might thus be more inclined to commit to a rule that is robust to the statistical properties of the exogenous disturbances.

As shown in Giannoni and Woodford (2003a,b, 2010), it is possible under general conditions to derive a *robustly optimal rule* that implements the optimal equilibrium and that is completely independent of the specification of the exogenous shock processes. We show that this rule is a close cousin of the simple Wicksellian rule augmented with a large amount of interest-rate inertia, in the model considered. This latter rule remains extremely simple and introduces about the right amount of history dependence, regardless of the persistence of exogenous disturbances. It should thus be particularly appealing to policymakers who search for simple rules but worry about unforeseeable circumstances (shocks) affecting the economy in the future.

Several other studies have highlighted the potential advantages of price-level stabilization. While most studies have shown numerical results in a variety of models (e.g., Williams, 2003), our analysis of a micro-founded macroeconomic model that incorporates key tradeoffs faced by policymakers and an accurate evaluation of agents' welfare allows us to derive analytical results that provide a clear intuition about the welfare implications of simple Taylor rules and Wicksellian rules, and their sensitivity to various assumptions. This paper also uniquely emphasizes the robustness properties of simple rules focused on price-level stabilization in the face of key model misspecifications. It also shows how inappropriate simple Taylor rules can be in the face of misspecification of the assumed shock processes.

We assume that the central bank is able to credibly commit to a policy rule. The desirability of price-level stabilization stems essentially from the fact that the central bank's credible commitment to eventually undo unexpected changes in the price-level induces firms to maintain relatively stable prices. This is differs very much for results obtained in another branch of the literature which assumes instead that the policymaker acts under full discretion. These studies consider attributing to policymakers alternative objective functions so to mitigate the discretionary bias in policy and hence raise social welfare. They compare the effects of a regime in which the policymaker is assigned a loss function that involves inflation variability, to a regime in which the loss function involves price-level variability. Svensson (1999), Dittmar *et al.* (1999), Dittmar and Gavin (2000), Cecchetti and Kim (2004), Vestin (2006) show that assigning a loss function involving price-level variability may yield lower inflation variability and a more favorable inflation-output gap trade-off. Our analysis differs from this line of research by assuming that the central bank seeks to maximize *social* welfare, and determines which simple systematic policy strategy yields the best welfare results.

Policymakers such as Evans (2010) have recently suggested committing temporarily to stabilizing the price level around its long-run trend, in order to offset the adverse effects from a zero lower bound on nominal interest rates which appears to have constrained monetary policy. Supporting arguments for this view have been proposed in Eggertsson and Woodford (2003), Wolman (2005) and Billi (2008), who have argued that a commitment to price-level stabilization (possibly around a drifting path) may be an effective way of preventing deflations and exiting from deflationary traps. While we don't explicitly account for the zero-lower bound on interest rates here, our contribution suggests that a commitment to price-level stabilization may also be desirable even after the lower-bound constraint on interest rates has ceased to bind.

The rest of the paper is organized as follows. Section 2 reviews the model used in our analysis. Section 2 characterizes the optimal plan. Section 3 determines simple optimal Taylor rules and discusses their properties. Section 4 derives simple optimal Wicksellian rules and compare their implications to optimal Taylor rules and the optimal plan, in terms of their dynamic responses to disturbances, their welfare implications, the sensitivity of the optimal policy coefficients to the degree of persistence in the exogenous disturbances. Section

5 introduces interest-rate inertia. It first presents a simple rule that implements the optimal equilibrium and that is robust to the specification of the process of exogenous disturbances, and then argues that it resembles a Wicksellian rules with a large degree of interest-rate inertia. Section 7 concludes.

# 2 A Simple Structural Model

We consider a variant of the simple New Keynesian model that has been widely used in recent studies of monetary policy, following Goodfriend and King (1997), Rotemberg and Woodford (1997), Clarida, Galí and Gertler (1999), and Woodford (2003a,b).

## 2.1 Structural equations

The behavior of the private sector is summarized by two structural equations, an intertemporal IS equation relating relates spending decisions to the interest rate, and a New Keynesian aggregate supply equation[4]

$$y_t - g_t = \mathrm{E}_t \left( y_{t+1} - g_{t+1} \right) - \sigma^{-1} \left( i_t - \mathrm{E}_t \pi_{t+1} \right), \tag{1}$$

$$\pi_t = \kappa \left( y_t - y_t^n \right) + \beta \mathrm{E}_t \pi_{t+1}, \tag{2}$$

where $y_t$ denotes the log of (detrended) real output, $\pi_t$ is the quarterly inflation rate, $i_t$ is the nominal interest rate (all three variables expressed in deviations from their values in a steady-state with zero inflation and constant output growth), $g_t$ is an exogenous variable representing autonomous variation in spending such as government spending, $\beta \in (0,1)$ denotes the discount factor of the representative household, and $\sigma, \kappa > 0$. While (1) can be obtained as an approximation to the representative household's Euler equation for optimal timing of expenditures, (2) results from a log-linear approximation to the optimal price-setting decisions, when prices are sticky, as in Calvo (1983) and suppliers are in monopolistic competition. The parameter $\kappa$ depends on the speed of price adjustment and $y_t^n$ represents the *natural* rate of output, i.e., the equilibrium rate of output under perfectly flexible prices.

[4] Derivations of the structural equations from first principles can be found in Woodford (2003a, chap. 3).

This natural rate of output is a composite exogenous variable that depends on a variety of perturbations such as productivity shocks, shifts in labor supply, fluctuations in government expenditures and shifts in preferences. We also allow for exogenous time variation in the degree of inefficiency of the natural rate of output, $y_t^e - y_t^n$, where $y_t^e$ is the rate of output that would maximize the representative household's welfare in the absence of distortions.[5]

As we will evaluate monetary policy in terms of deviations of output from the *efficient* rate, it will be convenient to define the welfare-relevant "output gap" as $x_t \equiv y_t - y_t^e$, and rewrite (1)–(2) as

$$x_t = \mathrm{E}_t x_{t+1} - \sigma^{-1}\left(i_t - \mathrm{E}_t \pi_{t+1} - r_t^e\right) \tag{3}$$

$$\pi_t = \kappa x_t + \beta \mathrm{E}_t \pi_{t+1} + u_t, \tag{4}$$

where we now have two composite exogenous variables $r_t^e \equiv \sigma \mathrm{E}_t\left[\left(y_{t+1}^e - y_t^e\right) - \left(g_{t+1} - g_t\right)\right]$ denoting the "efficient" rate of interest, i.e., the equilibrium real interest rate that would prevail in the absence of distortions, and a "cost-push" term $u_t \equiv \kappa\left(y_t^e - y_t^n\right)$. Finally, by definition, the log of the price level $p_t$, satisfies

$$p_t = \pi_t + p_{t-1}. \tag{5}$$

## 2.2 Shock processes

We think of the composite shocks $r_t^e$ and $u_t$ as being functions of a potentially large number of underlying disturbances, with each of the underlying disturbance having a different degree of persistence. We assume that the central bank knows perfectly the shocks that have hit the economy until the present, but may not be able to assess the realization of all possible future shocks. Following Giannoni and Woodford (2003a), we let $r_t^e$ and $u_t$ follow the processes

$$r_t^e = \sum_{j=0}^{\infty} \rho_r^j \varepsilon_{r,t-j} + \sum_{m=0}^{\infty}\sum_{j=0}^{\infty} \hat{\rho}_m^j \hat{\varepsilon}_{r,m,t-j} \tag{6}$$

[5] Fluctuations in $y_t^e - y_t^n$ could be due, e.g., to exogenous variation in the degree of market power of firms or in distortionary taxation.

$$u_t = \sum_{j=0}^{\infty} \rho_u^j \varepsilon_{u,t-j} + \sum_{m=0}^{\infty} \sum_{j=0}^{\infty} \hat{\rho}_m^j \hat{\varepsilon}_{u,m,t-j} \tag{7}$$

where $\varepsilon_{k,t}$, and $\hat{\varepsilon}_{k,m,t}$ are iid, mean-zero random variables, for all $k \in \{r, u\}$ and $m, t \geq 0$, but where the innovations $\hat{\varepsilon}_{k,m,t}$ have a distribution with a large atom at zero and $\rho_k, \hat{\rho}_k \in [0, 1)$. The shocks $r_t^e$ and $u_t$ are thus affected in each period by typical innovations $\varepsilon_{r,t}$ and $\varepsilon_{u,t}$, with a persistence given by $\rho_r, \rho_u$, and they may be infrequently affected by a large number of other types of unforecastable disturbances, $\hat{\varepsilon}_{r,m,t}$ and $\hat{\varepsilon}_{u,m,t}$ each of which may have a different degree of persistence. To simplify the analysis, we furthermore assume that such infrequent innovations have not been observed in the past, up to the date 0 at which the policymaker sets policy, so that the historical exogenous processes can be characterized by

$$r_t^e = \rho_r r_{t-1}^e + \varepsilon_{rt} \tag{8}$$

$$u_t = \rho_u u_{t-1} + \varepsilon_{ut}. \tag{9}$$

up to date 0, and the conditional forecasts are given by $\mathrm{E}_0 r_t^e = \rho_r^t r_0^e$ and $\mathrm{E}_0 u_t = \rho_u^t u_0$. Since it is impractical for the central bank to catalog all of the possible disturbances $\hat{\varepsilon}_{r,m,t}$, $\hat{\varepsilon}_{r,m,t}$ before they are realized, and since the policymaker cannot reject the hypothesis that the past shocks and conditional forecasts of future shocks are described by (8)–(9), we assume that the central bank wants to choose at date 0 a rule that would be optimal (at least within the class of rules that it considers) under the assumption that the shock processes are given by (8)–(9).[6]

## 2.3 Policy objective

We assume that the policymaker seeks to minimize the expected loss criterion

$$\mathrm{E}[L] = (1 - \beta)\, \mathrm{E} \sum_{t=0}^{\infty} \beta^t \left[ \pi_t^2 + \lambda_x \left( x_t - x^* \right)^2 + \lambda_i \left( i_t - i^* \right)^2 \right], \tag{10}$$

[6]The central bank does not need to regard it as certain that (8)–(9) are correct. However, we assume that it will only consider rules that would be optimal in the case that (8)–(9) were correct. Subject to that requirement, we assume that it would also like its rule to be as robust as possible to alternative shock processes within the family (6)–(7).

with weights $\lambda_x, \lambda_i > 0$, $\beta \in (0,1)$, and where $x^* \geq 0$ and $i^*$ represent some optimal levels of the output gap and the nominal interest rate. The expectation $\mathrm{E}[\cdot]$ is conditional on the state of the economy at the time that the policy is evaluated, which we assume takes place before the realization of the shocks at that date. This loss criterion can be viewed as a second-order Taylor approximation to the lifetime utility function of the representative household in the underlying model (see Woodford, 2003a, chap. 6). The concern for interest rate variability in (10) reflects both welfare costs of transactions and an approximation to the zero lower bound on nominal interest rates. The approximation of the utility function allows us also to determine the relative weights $\lambda_x, \lambda_i$, and the parameters $x^*, i^*$ in terms of the parameters of the underlying model.[7]

The inefficient supply shock is responsible for a trade-off between the stabilization of inflation on one hand and the output gap on the other hand: in the face of an increase in $u_t$, the policymaker could not keep both inflation and the output gap constant. In the absence of inefficient supply shocks, however, both inflation and the output gap could be completely stabilized by letting the interest rate track the path of the efficient rate of interest, $r_t^e$. But when $\lambda_i > 0$ in (10), a tension appears between stabilization of inflation and the output gap on one hand and stabilization of the nominal interest rate on the other hand.

## 2.4 Calibration

While we characterize optimal monetary policy for arbitrary positive values of the parameters, we will however at times focus on a particular parametrization of the model, using the parameter values estimated by Rotemberg and Woodford (1997) for the U.S. economy, and summarized in Table 1. The weights $\lambda_x$ and $\lambda_i$ are calibrated as in Woodford (2003a). Rotemberg and Woodford (1997) provide estimated time-series for the disturbances $y_t^n$ and $g_t$ but do not split $y_t^n$ into an efficient component $y_t^e$ and an inefficient component. We calibrate the variance of $r_t^e$ by assuming that all supply shifts are efficient, so that the variance of the efficient rate of interest is the same as the variance of the natural rate of interest re-

[7] Woodford (2003a, chap. 6)'s derivation of the loss criterion accounts for transaction frictions and the approximation of the lower bound on interest rates, but abstracts from inefficients supply shocks $u_t$. The approximation remains valid in the presence of inefficient supply shocks.

ported in Woodford (2003a). In our benchmark calibration, we set $\text{var}(u_t)$ to its upper bound $\kappa^2 \text{var}(y_t^n)$, assuming that all supply shifts are inefficient, but we verify that our conclusions remain unchanged for lower values of $\text{var}(u_t)$.

Table 1: "Calibrated" parameter values

| Structural parameters | | | Shock processes | | | | Loss function | |
|---|---|---|---|---|---|---|---|---|
| $\beta$ | $\sigma$ | $\kappa$ | $\rho_r$ | $\rho_u$ | $\text{var}(r_t^e)$ | $\text{var}(u_t)$ | $\lambda_x$ | $\lambda_i$ |
| 0.99 | 0.1571 | 0.0238 | 0.35 | 0.35 | 13.8266 | 0.1665 | 0.048 | 0.236 |

# 3 Optimal Plan

Before evaluating alternative policies below, it will be useful to consider as a benchmark the optimal state-contingent plan, i.e., the processes $\{\pi_t, x_t, i_t\}$ that minimize the unconditional expectation of the loss criterion (10) subject to the constraints (3) and (4) imposed by the private sector's behavior at all dates, assuming that the policymaker can commit to the plan for the entire future.[8] The first-order necessary conditions with respect to $\pi_t, x_t$, and $i_t$ are

$$\pi_t - (\beta\sigma)^{-1} \phi_{1t-1} + \phi_{2t} - \phi_{2t-1} = 0 \tag{11}$$

$$\lambda_x (x_t - x^*) + \phi_{1t} - \beta^{-1}\phi_{1t-1} - \kappa\phi_{2t} = 0 \tag{12}$$

$$\lambda_i (i_t - i^*) + \sigma^{-1}\phi_{1t} = 0 \tag{13}$$

at each date $t \geq 0$, and for each possible state, where $\phi_{1t}$ and $\phi_{2t}$ refer to Lagrange multipliers associated respectively with the constraint (3) and (4). In addition, we have the initial conditions

$$\phi_{1,-1} = \phi_{2,-1} = 0 \tag{14}$$

indicating that the policymaker has no previous commitment at time 0.

The optimal plan is a bounded solution $\{\pi_t, x_t, i_t, \phi_{1t}, \phi_{2t}\}_{t=0}^{\infty}$ to the system of equations (3), (4), (11) – (13) at each date $t \geq 0$, together with the initial conditions (14). A bounded solution exists, it is unique. In this solution, the endogenous variables depend not only upon

[8] This section generalizes slightly the results of Clarida *et al.* (1999) and Woodford (2003a, 2003b) by considering the optimal plan with both inefficient supply shocks and a concern for interest-rate stabilization ($\lambda_i > 0$).

expected future values of the disturbances, but also upon the predetermined variables $\hat{\phi}_{1t-1}$, $\hat{\phi}_{2t-1}$, so that optimal policy is generally history dependent.

# 4 Commitment to an Optimal Taylor Rule

We next consider an optimal policy problem in the case that the policymaker restricts its policy by setting the interest rate according to the standard "Taylor rule"

$$i_t = \psi_\pi \pi_t + \psi_x x_t + \psi_0, \tag{15}$$

at all dates $t \geq 0$, where $\psi_\pi, \psi_x$, and $\psi_0$ are policy coefficients to be chosen optimally, i.e., so as to maximize the expected welfare (10), subject to the structural equations (3)–(4) and the shock processes (8)–(9). Such a simple rule, while restrictive hence not fully optimal, is known to perform well in a wide range of models. We focus here on the simplest Taylor rule without inertia, but we discuss an extension including the lagged interest rate in section 6.2.

To determine the optimal policy coefficients, we first characterize the optimal equilibrium that is consistent with the given rule, and next we determine policy coefficients that correspond to that equilibrium. Using (15) to substitute for the interest rate in (3) and (4), we observe that inflation and the output gap must satisfy a system of difference equations of the form

$$\mathrm{E}_t z_{t+1} = A\ z_t + a\ e_t \tag{16}$$

where $z_t \equiv [\pi_t, x_t, 1]'$, and $e_t \equiv [r_t^e, u_t]'$. Given that $z_t$ does not involve any predetermined variable in this case, the resulting equilibrium, if it exists, must be non inertial. The evolution of the endogenous variables can then be described by

$$\pi_t = \pi^{ni} + \pi_r r_t^e + \pi_u u_t, \qquad x_t = x^{ni} + x_r r_t^e + x_u u_t, \qquad i_t = i^{ni} + i_r r_t^e + i_u u_t, \tag{17}$$

where $\pi^{ni}, x^{ni}, i^{ni}$ are the steady-state values of the respective variables in this equilibrium, and $\pi_r, \pi_u$, and so on, are the equilibrium response coefficients to fluctuations in $r_t^e$ and $u_t$.

As we show in Appendix A.1, $i_r$ and $i_u$ are positive for any positive weights $\lambda_i, \lambda_x$. Thus

the optimal non-inertial plan involves an adjustment of the nominal interest rate in the direction of the perturbations. Furthermore, the response coefficients $\pi_r$, $x_r$ are positive if and only if

$$\frac{\sigma}{\kappa} > \frac{\rho_r}{(1-\beta\rho_r)(1-\rho_r)}, \tag{18}$$

that is, whenever the fluctuations in the efficient rate are not too persistent (relative to the ratio $\frac{\sigma}{\kappa}$). When (18) holds, a positive shock to the efficient rate $r_t^e$ stimulates aggregate demand, raising the output gap and inflation.

For the Taylor rule to be consistent with an optimal equilibrium of the form (17), we show in Appendix A.1 that the policy coefficients must satisfy

$$\psi_\pi = \frac{x_u i_r - i_u x_r}{x_u \pi_r - \pi_u x_r}, \qquad \psi_x = \frac{\pi_r i_u - i_r \pi_u}{x_u \pi_r - \pi_u x_r}, \tag{19}$$

Substituting the coefficients $\pi_r, x_r, ...$ with their values characterizing the optimal non-inertial equilibrium, yields the coefficients of the optimal Taylor rule as functions of the underlying structural coefficients. However, for the optimal Taylor rule to implement the optimal non-inertial equilibrium, it must guarantee that the dynamic system involving (3), (4), and (15) admits a unique bounded solution. It is well known (see, e.g., Woodford, 2003a, chap. 4) that if we restrict our attention to the case in which $\psi_\pi, \psi_x \geq 0$, then the policy rule (15) results in a determinate equilibrium if and only if

$$\psi_\pi + \frac{1-\beta}{\kappa}\psi_x > 1. \tag{20}$$

## 4.1 Optimal Taylor rule and sensitivity to shock processes

To get some intuition about the optimal Taylor rule, let us consider the special case in which both perturbations have the same degree of persistence, i.e., $\rho_r = \rho_u \equiv \rho$. In this case, the optimal Taylor rule coefficients reduce to

$$\psi_\pi = \frac{\kappa}{\lambda_i\left(\sigma(1-\rho)(1-\beta\rho) - \rho\kappa\right)}, \qquad \psi_x = \frac{\lambda_x(1-\beta\rho)}{\lambda_i\left(\sigma(1-\rho)(1-\beta\rho) - \rho\kappa\right)}. \tag{21}$$

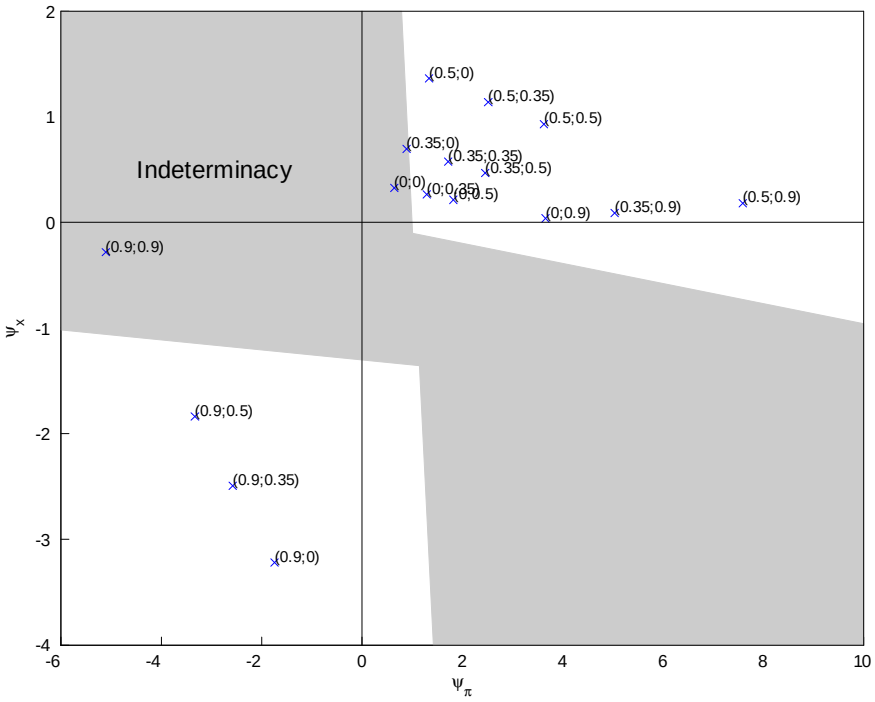


Figure 1: Optimal Taylor rules $(\psi_\pi, \psi_x)$ for different degrees of shock persistence $(\rho_r; \rho_u)$.

They are both positive when (18) holds. The optimal coefficient on inflation, $\psi_\pi$, increases with the slope to the aggregate supply, $\kappa$, to prevent a given output gap from creating more inflation. Similarly the optimal coefficient on output gap, $\psi_x$, increases when $\lambda_x$ increases as the policymaker is more willing to stabilize the output gap. In addition, the optimal Taylor rule becomes more responsive to both inflation and output gap fluctuations when $\lambda_i$ decreases as the policymaker is willing to let the interest rate vary more, and when the IS curve becomes flatter ($\sigma$ is smaller) as shocks to the efficient rate of interest have a larger impact on the output gap and inflation.

These expressions reveal that the optimal Taylor coefficients are particularly sensitive to the assumed degree of persistence of the shocks, as shown in Figure 1. As $\rho$ increases to approach the bound (18), corresponding to $\rho \simeq 0.68$ in our calibration, the optimal Taylor rule coefficients become in fact unboundedly large, and become negative when the inequality in (18) is reversed, i.e., when $\rho > 0.68$. Table 2 reports the optimal coefficients (given by (A.41) and (A.42) in Appendix A.1) for different degrees of persistence of the perturbations.

While the white region of Figure 1 indicates the set of Taylor rules that result in a

unique bounded equilibrium, the gray region indicates combinations $(\psi_\pi, \psi_x)$ that result in indeterminacy of the equilibrium. Figure 1 reveals for example that when both shocks are purely transitory ($\rho_r = \rho_u = 0$), the "optimal" Taylor rule lies in the region of indeterminacy, as they are not large enough to satisfy (20). Committing to an "optimal" Taylor rule that lies in the region of indeterminacy can thus result in a large set of bounded equilibria, including some that involve an arbitrarily large value of the loss criterion (10). Note from Figure 1 that the problem of indeterminacy arises not only when $\rho_r = \rho_u = 0$, but also in some cases when the disturbances are more persistent (e.g., when $\rho_r = 0.35$ and $\rho_u = 0$, or when $\rho_r = \rho_u = 0.9$).

## 4.2 Desirability of history dependence

Even if we abstract from equilibrium indeterminacy, the optimal Taylor rule may yield substantially higher welfare losses than the first best. Table 2 reports the policymaker's loss, $\mathrm{E}[L]$, in addition to the following measure of variability

$$V[z] \equiv \mathrm{E}\left[(1-\beta)\sum_{t=0}^{\infty}\beta^t \hat{z}_t^2\right]$$

for the four endogenous variables, $\pi, x, i$, and $p$, so that $\mathrm{E}[L]$ is a weighted sum of $V[\pi]$, $V[x]$, and $V[i]$ with weights being the ones of the loss function (10).[9] When $\rho_r = \rho_u = 0.35$, as in the baseline calibration, the loss is 1.28 in the optimal plan, while it is 2.63 when committing to an optimal Taylor rule. The welfare losses of generated by this simple policy rule stem primarily from a higher variability of inflation and of interest rates.

To understand better the source of the welfare losses under the simple Taylor rule, we show in Figure 2 the response of endogenous variables to an unexpected disturbance to the efficient rate of interest, assuming for illustrative purposes no shock persistence ($\rho_r = 0$).[10]

[9] The table reports the statistics in the case in which $x^* = i^* = 0$, so that the steady state is the same for each plan (and is zero for each variable). The statistics measure therefore the variability of each variable around its steady state, and $\mathrm{E}[L]$ indicates the loss due to temporary disturbances in excess of the steady-state loss. All statistics in Table 2 are reported in annual terms: $\mathrm{V}[\pi]$, $\mathrm{V}[i]$, and $\mathrm{E}[L]$ are multiplied by 16. The weight $\lambda_x$ in Table 1 is also multiplied by 16 to represent the weight attributed to the output gap variability (in annual terms) relative to that of annualized inflation and annualized interest rate.

[10] The responses of $\hat{\imath}_t$ and $\hat{\pi}_t$ in Figures 2 and 3 are multiplied by 4 so that the responses of all variables are reported in annual terms.

## Table 2: Statistics and Optimal Policy Rules

| | | Statistics | | | | | Coefficients of optimal policy rule | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | $V[\pi]$ | $V[x]$ | $V[i]$ | $V[p]$ | $E[L]$ | $p_t$ | $\pi_t$ | $x_t$ | $x_{t-1}$ | $i_{t-1}$ | $i_{t-2}$ |
| ***Optimal Plan / Optimal Rule*** | | | | | | | | | | | | |
| $\rho_r = 0$ | $\rho_u = 0$ | 0.157 | 10.215 | 0.983 | 0.938 | 0.883 | -- | 0.641 | 0.325 | -0.325 | 2.163 | -1.010 |
| | $\rho_u = 0.35$ | 0.192 | 10.994 | 0.983 | 0.951 | 0.956 | -- | 0.641 | 0.325 | -0.325 | 2.163 | -1.010 |
| | $\rho_u = 0.9$ | 0.195 | 20.057 | 0.983 | 1.098 | 1.397 | -- | 0.641 | 0.325 | -0.325 | 2.163 | -1.010 |
| $\rho_r = 0.35$ | $\rho_u = 0$ | 0.217 | 11.056 | 1.922 | 2.727 | 1.206 | -- | 0.641 | 0.325 | -0.325 | 2.163 | -1.010 |
| | $\rho_u = 0.35$ | 0.252 | 11.835 | 1.922 | 2.740 | 1.279 | -- | 0.641 | 0.325 | -0.325 | 2.163 | -1.010 |
| | $\rho_u = 0.9$ | 0.255 | 20.898 | 1.922 | 2.887 | 1.720 | -- | 0.641 | 0.325 | -0.325 | 2.163 | -1.010 |
| $\rho_r = 0.9$ | $\rho_u = 0$ | 0.487 | 5.196 | 6.765 | 46.597 | 2.337 | -- | 0.641 | 0.325 | -0.325 | 2.163 | -1.010 |
| | $\rho_u = 0.35$ | 0.522 | 5.975 | 6.765 | 46.610 | 2.410 | -- | 0.641 | 0.325 | -0.325 | 2.163 | -1.010 |
| | $\rho_u = 0.9$ | 0.525 | 15.038 | 6.766 | 46.757 | 2.851 | -- | 0.641 | 0.325 | -0.325 | 2.163 | -1.010 |
| ***Optimal Non-Inertial Plan / Optimal Taylor Rules*** | | | | | | | | | | | | |
| $\rho_r = 0$ | $\rho_u = 0$ | 0.269 | 13.495 | 2.030 | 1.680 | 1.401 | -- | 0.641 | 0.325 | -- | -- | -- |
| | $\rho_u = 0.35$ | 0.391 | 13.957 | 2.233 | 4.224 | 1.593 | -- | 1.291 | 0.263 | -- | -- | -- |
| | $\rho_u = 0.9$ | 0.144 | 30.354 | 2.159 | 3.217 | 2.122 | -- | 3.658 | 0.038 | -- | -- | -- |
| $\rho_r = 0.35$ | $\rho_u = 0$ | 0.358 | 9.989 | 6.747 | 3.631 | 2.435 | -- | 0.888 | 0.694 | -- | -- | -- |
| | $\rho_u = 0.35$ | 0.479 | 10.451 | 6.949 | 6.175 | 2.627 | -- | 1.724 | 0.572 | -- | -- | -- |
| | $\rho_u = 0.9$ | 0.233 | 26.848 | 6.876 | 5.168 | 3.156 | -- | 5.041 | 0.089 | -- | -- | -- |
| $\rho_r = 0.9$ | $\rho_u = 0$ | 0.500 | 0.529 | 10.437 | 39.175 | 2.993 | -- | -1.743 | -3.222 | -- | -- | -- |
| | $\rho_u = 0.35$ | 0.622 | 0.991 | 10.640 | 41.719 | 3.185 | -- | -2.575 | -2.495 | -- | -- | -- |
| | $\rho_u = 0.9$ | 0.375 | 17.388 | 10.566 | 40.712 | 3.714 | -- | -5.108 | -0.283 | -- | -- | -- |
| ***Optimal Wicksellian Rules*** | | | | | | | | | | | | |
| $\rho_r = 0$ | $\rho_u = 0$ | 0.122 | 13.923 | 1.237 | 0.008 | 1.087 | 1.997 | -- | 0.182 | -- | -- | -- |
| | $\rho_u = 0.35$ | 0.146 | 15.191 | 1.294 | 0.015 | 1.186 | 1.383 | -- | 0.228 | -- | -- | -- |
| | $\rho_u = 0.9$ | 0.142 | 24.466 | 1.391 | 0.090 | 1.653 | 0.853 | -- | 0.274 | -- | -- | -- |
| $\rho_r = 0.35$ | $\rho_u = 0$ | 0.149 | 15.898 | 2.646 | 0.013 | 1.543 | 2.872 | -- | 0.139 | -- | -- | -- |
| | $\rho_u = 0.35$ | 0.161 | 17.597 | 2.778 | 0.018 | 1.669 | 2.338 | -- | 0.201 | -- | -- | -- |
| | $\rho_u = 0.9$ | 0.096 | 28.790 | 2.761 | 0.012 | 2.140 | 3.323 | -- | 0.140 | -- | -- | -- |
| $\rho_r = 0.9$ | $\rho_u = 0$ | 0.158 | 7.072 | 10.459 | 0.089 | 2.973 | 2.613 | -- | 0.134 | -- | -- | -- |
| | $\rho_u = 0.35$ | 0.166 | 8.239 | 10.699 | 0.102 | 3.093 | 2.426 | -- | 0.241 | -- | -- | -- |
| | $\rho_u = 0.9$ | 0.110 | 18.822 | 10.779 | 0.094 | 3.567 | 2.626 | -- | 0.259 | -- | -- | -- |
| ***Estimated Historical Rule*** | | | | | | | | | | | | |
| $\rho_r = 0$ | $\rho_u = 0$ | 0.178 | 10.531 | 1.313 | 0.833 | 0.997 | -- | 0.424 | 0.297 | -0.032 | 1.160 | -0.430 |
| | $\rho_u = 0.35$ | 0.330 | 10.639 | 1.351 | 3.586 | 1.164 | -- | 0.424 | 0.297 | -0.032 | 1.160 | -0.430 |
| | $\rho_u = 0.9$ | 5.182 | 13.019 | 5.078 | 555.455 | 7.012 | -- | 0.424 | 0.297 | -0.032 | 1.160 | -0.430 |
| $\rho_r = 0.35$ | $\rho_u = 0$ | 0.215 | 11.941 | 2.961 | 1.218 | 1.492 | -- | 0.424 | 0.297 | -0.032 | 1.160 | -0.430 |
| | $\rho_u = 0.35$ | 0.367 | 12.049 | 2.999 | 3.972 | 1.659 | -- | 0.424 | 0.297 | -0.032 | 1.160 | -0.430 |
| | $\rho_u = 0.9$ | 5.219 | 14.429 | 6.726 | 555.841 | 7.507 | -- | 0.424 | 0.297 | -0.032 | 1.160 | -0.430 |
| $\rho_r = 0.9$ | $\rho_u = 0$ | 3.080 | 9.485 | 21.954 | 317.172 | 8.728 | -- | 0.424 | 0.297 | -0.032 | 1.160 | -0.430 |
| | $\rho_u = 0.35$ | 3.232 | 9.592 | 21.993 | 319.926 | 8.895 | -- | 0.424 | 0.297 | -0.032 | 1.160 | -0.430 |
| | $\rho_u = 0.9$ | 8.084 | 11.973 | 25.719 | 871.794 | 14.743 | -- | 0.424 | 0.297 | -0.032 | 1.160 | -0.430 |

Notes: The gray cases indicate that the policy rule results in an indeterminate equilibrium
The estimated historical rule refers to Judd and Rudebusch (1998).

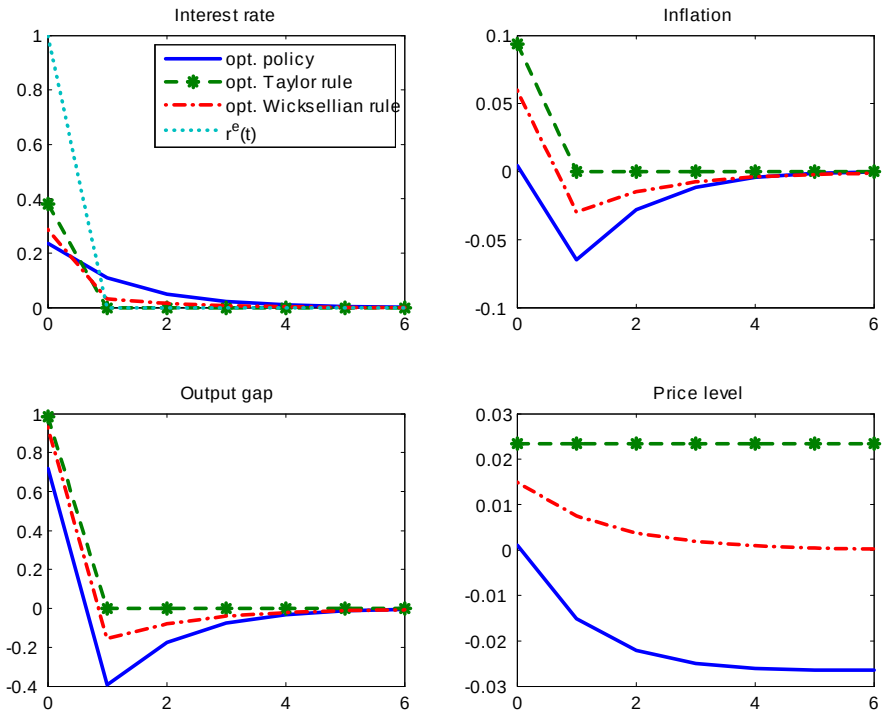


Figure 2: Impulse responses to an innovation in $r^e$ with autocorrelation of $\rho_r = 0$.

Under the optimal Taylor rule (dashed lines), the nominal interest rate increases by less than the natural rate of interest, in order to dampen the variability of the nominal interest rate. Monetary policy is therefore relatively expansionary so that inflation and the output gap increase at the time of the shock. In later periods however, these variables return to their initial steady-state as the perturbation vanishes. In contrast, in the optimal plan (solid lines), the short-term interest rate is more inertial than the efficient rate. Inertia in monetary policy is especially desirable here because it induces the private sector to expect future restrictive monetary policy, hence future negative output gaps which in turn have a disinflationary effect already when the shock hits the economy. Thus the expectation of an inertial policy response allows the policymaker to offset the inflationary impact of the shock by raising the short-term interest rate by less than with the simple Taylor rule.[11]

Similarly, in the face of an unexpected transitory inefficient supply shock $u_t$ (with $\rho_u = 0$),

[11] When shocks are more persistent, the nominal interest rate also increases modestly on impact, in the optimal plan, but is expected to be higher than the efficient rate in later periods, so that agents can expect a tight monetary policy in the future, with negative output gaps and a decline of price level then. However to achieve a similar future path of the output gap and the price level, the optimal Taylor rule needs raise the interest rate sufficiently on impact, so as to bring down inflation and the output gap already at the time of the shock. This is why the optimal Taylor rule coefficients become negative when $\rho_r$ is sufficiently large.

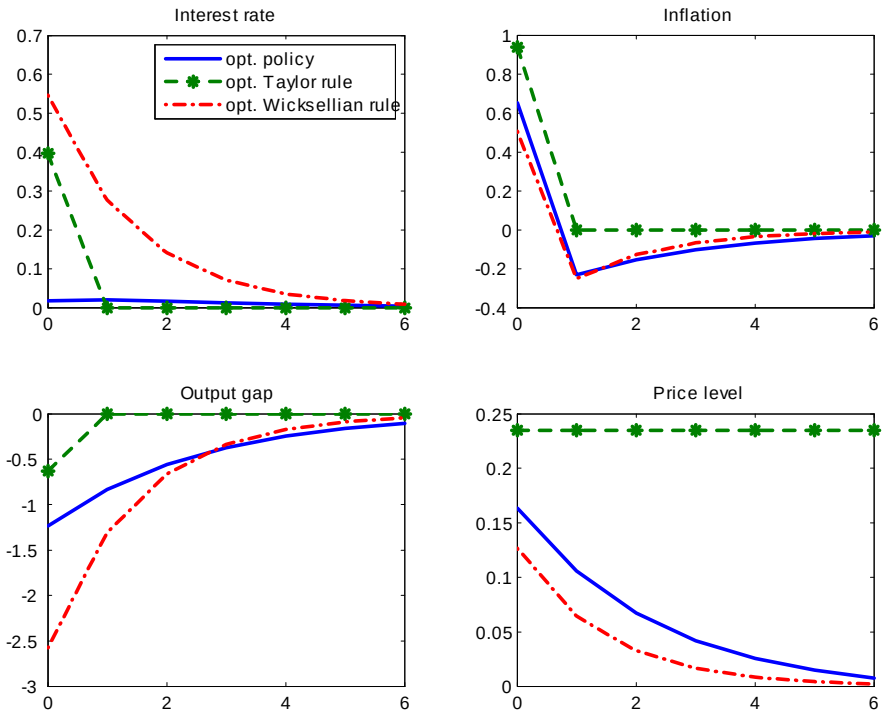


Figure 3: Impulse responses to an innovation in $u$ with autocorrelation of $\rho_u = 0$.

Figure 3 shows that the optimal Taylor rule induces the nominal interest rate to increase, so as to reduce output (gap), and therefore to mitigate inflationary pressures. In the optimal plan (solid lines), however, it is optimal to maintain the output gap below steady state even after the shock has vanished. This generates the expectation of a slight deflation in later periods and thus helps dampening the initial increase in inflation. The last panel confirms that the price level initially rises with the adverse shock but then declines back to almost return to its initial steady-state level. In fact the new steady-state price level is slightly below the initial one. The optimal interest rate hardly deviates from the steady-state, but remains above steady-state for several periods, so as to achieve the desired deflation in later periods.

Figures 2 and 3 reveal that in the optimal plan, the effects of disturbances are mitigated more effectively than with the optimal Taylor rule by being spread out over a longer period of time, through an inertial policy.

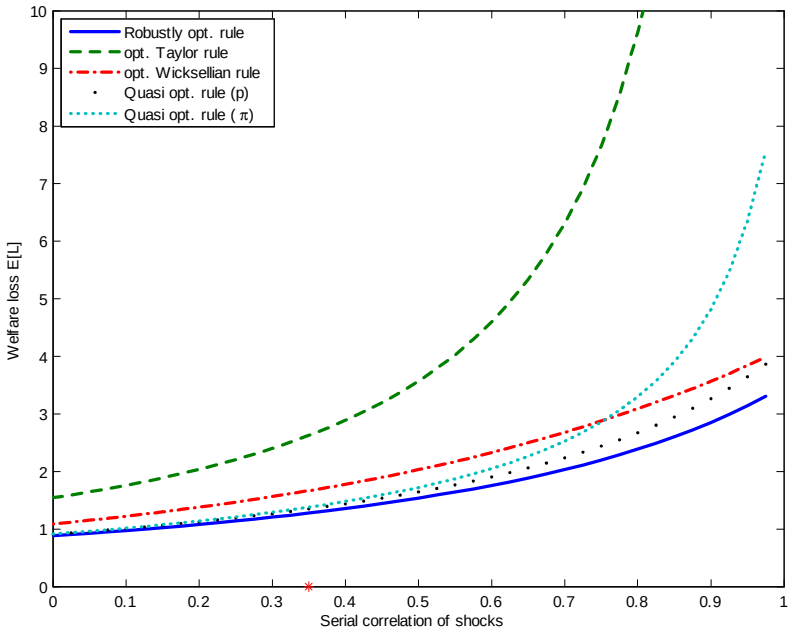


Figure 4: Welfare losses of alternative policy rules as a function of shock persistence ($\rho_r = \rho_u$). Notes: The quasi optimal rule (p) is given in (30) while the quasi optimal rule ($\pi$) is given in (31).

## 4.3 Welfare implications of alternative shock processes

We have shown above that the optimal Taylor rule coefficients are sensitive to the degree of persistence of shocks. This does not imply however that this sensitivity has important welfare implications, as two simple Taylor rules with different coefficients may in principle result in similar outcomes. To evaluate the welfare implications of alternative shock processes, we suppose that the central bank has committed to a simple Taylor rule, optimized under a correct assumption about the past shocks — i.e., that their law of motion is given by (8)–(9), with degrees of series correlation $\rho_r = \rho_u = 0.35$ — but that it now faces new disturbances $\hat{\varepsilon}_{r,m,t}$ and $\hat{\varepsilon}_{u,m,t}$ which propagate through the economy with a different persistence $\hat{\rho}$.[12]

Figure 4 plots welfare losses $E[L]$ for various policy rules as a function of the degree of persistence of the shocks $r_t^e$ and $u_t$. The dashed line represents the welfare losses implied by the commitment to the Taylor rule optimized in our benchmark calibration, i.e., with $\rho_r = \rho_u = 0.35$. Note that this policy rule which has coefficients $\psi_\pi = 1.72$, $\psi_x = 0.57$ is not

[12] We focus here on the properties of policy rules that have been optimized in a particular model, but we do not evaluate policy rules that would be robust to uncertainty about the underlying model, or uncertainty about driving shock processes, as discussed, e.g., Giannoni (2002, 2007), Hansen and Sargent (2008).

too different from the policy rule initially proposed by Taylor (1993). The figure shows again that in the benchmark case, the welfare loss under the optimal Taylor rule (2.63) is about twice as large as in the optimal plan (1.28) denoted here by the solid line, as documented in Table 2. However, as the shock persistence $\rho$ increases, the welfare performance of this simple Taylor rule deteriorates considerably with losses approaching 50, i.e., about 15 times the loss under optimal policy, as $\rho$ approaches 1.

Figure 5 shows that the welfare deterioration is due to dramatic increases in inflation and interest-rate volatility under the simple Taylor rule when the shocks become more persistent. While the Taylor rule is relatively successful at stabilizing the output gap, this does not contribute much to the overall welfare given the low value of $\lambda_x$. Figures 4 and 5 consider changes in the persistence of both shocks $r_t^e$ and $u_t$. Similar figures emerge when one considers changes in the persistence of one shock at a time.

# 5 Commitment to a Simple Wicksellian Rule

While simple Taylor rules of the form (15) lack history dependence — a key property of optimal policy in forward-looking models — and are sensitive to the degree of persistence of exogenous disturbances, we now turn to an alternative very simple rule that introduces a desirable amount of history dependence and that turns out to be less sensitive to shock persistence. It is given by

$$i_t = \psi_p \left(p_t - \bar{p}_t\right) + \psi_x x_t + \psi_0 \tag{22}$$

at all dates $t \geq 0$, where $\bar{p}_t$ is some deterministic trend for the (log of the) price-level satisfying

$$\bar{p}_t = \bar{p}_{t-1} + \bar{\pi}, \tag{23}$$

and $\bar{\pi}$ is a constant. Following Woodford (2003a, chap. 2) we call such a rule a Wicksellian rule. The price level depends by definition not only on current inflation but also on all past rates of inflation. It follows that the rule (22) introduces history dependence in monetary policy, as it forces the policymaker to compensate any shock that might have affected inflation in the past. While rules of this form are as simple as standard Taylor rules, they have received

less attention in recent studies of monetary policy. One reason may be because it is widely believed that such rules would result in a larger variability of inflation (and the output gap), as the policymaker would respond to an inflationary shock by generating inflation below target in subsequent periods. However, as we show below, this is not true when agents are forward-looking and they understand that the policymaker *commits* to a rule of the form (22). Although the policymaker and the private sector do not care about the price level *per se*, as the latter does not enter the loss criterion (10), we shall argue that a Wicksellian rule has desirable properties for the conduct of monetary policy.

To characterize the equilibrium that obtains if the policymaker commits to (22), we consider a steady state in which in which inflation, the output gap and the nominal interest rate take respectively the constant values $\pi^{wr}, x^{wr}, i^{wr}$, we define the deviations from the steady state as $\hat{\pi}_t \equiv \pi_t - \pi^{wr}$, $\hat{x}_t \equiv x_t - x^{wr}$, $\hat{\imath}_t \equiv i_t - i^{wr}$, and we let $\hat{p}_t \equiv p_t - \bar{p}_t$ be the (percentage) deviation of the price level from its trend. As discussed in Appendix A.2, using (22) to substitute for $\hat{\imath}_t$ in the intertemporal IS equation, we can rewrite (3), (4), and (5) again as in (16) but where $z_t \equiv [\hat{\pi}_t, \hat{x}_t, \hat{p}_{t-1}]'$, $e_t \equiv [r_t^e, u_t]'$. With shock processes (8) and (9), the resulting equilibrium is of the form

$$\hat{z}_t = z_r r_t^e + z_u u_t + z_p \hat{p}_{t-1} \tag{24}$$

for any variable $\hat{z}_t \in \{\hat{\pi}_t, \hat{x}_t, \hat{\imath}_t, \hat{p}_t\}$, where $z_r, z_u, z_p$ are equilibrium response coefficients to fluctuations in $r_t^e$, $u_t$, and $p_{t-1}$. As further shown in Appendix A.2, the policy coefficients $\psi_p$ and $\psi_x$ relate in turn to the equilibrium coefficients as follows

$$\psi_p = \frac{x_u i_r - i_u x_r}{x_u p_r - x_r p_u}, \qquad \psi_x = \frac{p_r i_u - i_r p_u}{x_u p_r - x_r p_u}. \tag{25}$$

The optimal equilibrium resulting from a Wicksellian rule (22) is therefore characterized by the optimal steady state and the optimal response coefficients $z_r, z_u, z_p$ in (24) that minimize the loss function (10) subject to the constraints (3), (4), (5) and (22), where $\psi_p$ and $\psi_x$ are given by (25). For the Wicksellian rule to implement the desired equilibrium, though, it must guarantee that the dynamic system admits a unique bounded solution. An analysis of the transition matrix yields the following result.

**Proposition 1** *In the model composed of (3), (4), (5), with* $\sigma, \kappa > 0$ *and* $0 < \beta < 1$, *a commitment to the Wicksellian policy rule (22) results in a unique bounded rational expectations equilibrium* $\{\pi_t, x_t, i_t, p_t\}$, *if*

$$\psi_p > 0 \quad and \quad \psi_x \geq 0. \tag{26}$$

**Proof.** See Appendix A.3. ■

Hence any Wicksellian rule with positive coefficients implies a determinate equilibrium.[13] In general, the coefficients of optimal Wicksellian rule are complicated functions of the parameters of the model. Unlike those of the optimal Taylor rule, they are also function of the variance of the shocks. Rather than trying to characterize analytically the optimal Wicksellian rule, we proceed with a numerical investigation of its properties and its implications for equilibrium inflation, output gap and the nominal interest rate.

## 5.1 A comparison of Taylor rules and Wicksellian rules

Figure 6 reports optimal coefficients of the Wicksellian rule for different degrees of shock persistence $(\rho_r; \rho_u)$. It is noteworthy that all optimal policy coefficients are positive for this wide range of shock persistence. It follows from Proposition 1 that these policy rules result in a determinate equilibrium, in contrast to some of the "optimal" Taylor rules presented in Figure 1. Furthermore, while the optimal Taylor rule coefficients vary importantly with different degrees of shock persistence, Figure 6 shows that the optimal coefficients of Wicksellian rules are concentrated in a narrower area that those of optimal Taylor rules (Figure 1). The optimal Wicksellian rules are thus less sensitive to the different assumptions about serial correlation of the disturbances.

In addition, optimal Wicksellian rules also introduce a kind of history dependence that is desirable for monetary policy. As shown in Figures 2 and 3, a commitment to an optimal Wicksellian policy allows the policymaker to achieve a response of endogenous variables that is closer to the optimal plan than is the case with the optimal Taylor rule.

One particularity of the equilibrium resulting from a Wicksellian policy, of course, is that the price level is trend-stationary. In the optimal plan, policy also eventually brings the

[13] A similar result is mentioned in Kerr and King (1996), in the case that $\psi_x$ is set to 0.

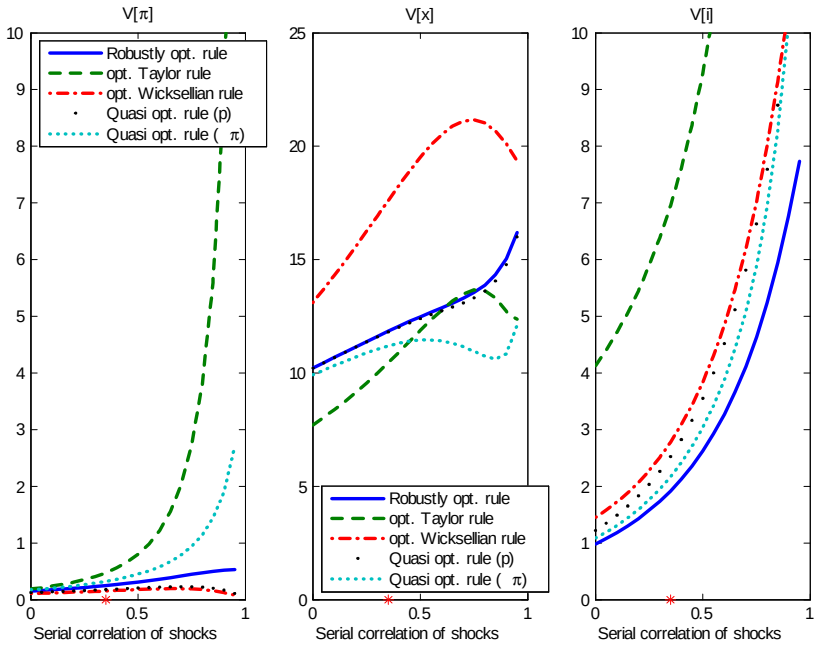


Figure 5: Volatility of key variables under alternative rules, as a function of shock persistence ($\rho_r = \rho_u$). Notes: The quasi optimal rule (p) is given in (30); the quasi optimal rule ($\pi$) is given in (31).

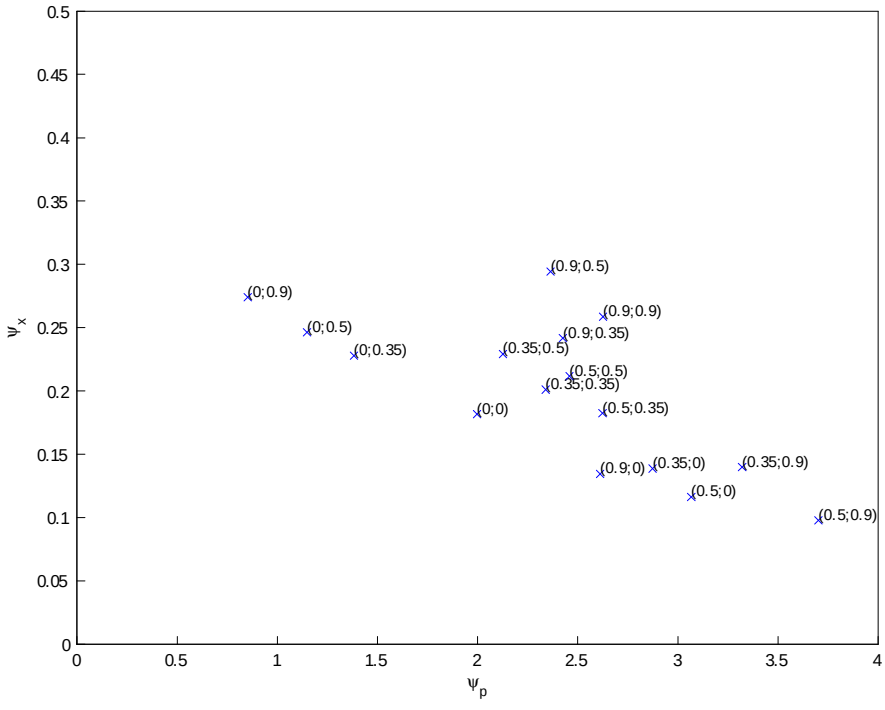


Figure 6: Optimal Wicksellian rules for different degrees of shock persistence ($\rho_r; \rho_u$).

price level to its original trend in the face of inefficient supply shocks $u_t$. However, when the economy is hit by exogenous fluctuations in $r_t^e$ and there is a concern for interest-rate stabilization ($\lambda_i > 0$), it is not optimal for the price level to be stable. In fact, in the optimal plan, Figure 2 shows that the price level is expected to end up at a slightly lower level in the future. While not fully optimal, the Wicksellian policy reduces welfare losses considerably by introducing history dependence and offsetting fluctuations in the price level. Another notable feature of optimal Wicksellian policy in Figure 3 is that the interest rate rises importantly, so that the response of inflation remains close to the optimal response. While this of creates a significant drop in output (gap) in our calibration, the welfare loss is only moderately affected by the recession, given the low weight $\lambda_x$.

A comparison of the welfare implications for both Taylor and Wicksellian rules suggests that Wicksellian rules result in general in a lower welfare loss, in the model considered here. We first show this analytically in a simple case and then proceed with a numerical investigation of the more general case.

**A special case.** To simplify the analysis, we consider the special case in which the short-term aggregate supply equation is perfectly flat so that $\kappa = 0$, and both shocks have the same degree of serial correlation $\rho$. In this case, we can solve for equilibrium inflation using (4), and we obtain

$$\pi_t = \beta \mathrm{E}_t \pi_{t+1} + u_t = \sum_{j=0}^{\infty} \beta^j \mathrm{E}_t u_{t+j} = (1 - \beta\rho)^{-1} u_t.$$

Inflation is exogenous in this case. The best the policymaker can do is therefore to minimize the variability of the output gap and the interest rate. Using (21), we note that the optimal Taylor rule reduces in this case to $\hat{\imath}_t = \lambda_x / [\lambda_i \sigma (1 - \rho)] \hat{x}_t$ and so involves no response to inflation. Since the Taylor rule is non inertial, one cannot reduce the variability of future output gaps and interest rates by responding to current fluctuations in inflation. In contrast, with a Wicksellian rule, the policymaker's response to contemporaneous price-level fluctuations and the belief that he will respond in the same way to price-level fluctuations in the future have a desirable effect on the expected future path of the output gap and the interest rate.

We can establish the following result.

**Proposition 2** *When $\kappa = 0$ and $\rho_r = \rho_u \equiv \rho > 0$, there exists a Wicksellian rule of the form (22) with $\psi_p > 0$, $\psi_x > 0$ that results in a unique bounded equilibrium, and that achieves a lower loss than the one resulting from the optimal Taylor rule.*

**Proof.** See Appendix A.4. ■

**General case: A numerical investigation.** In the more general case in which $\kappa > 0$ and we allow for arbitrary degrees of serial correlation of the shocks, the analytical characterization is substantially more complicated. However a numerical investigation suggests again that appropriate Wicksellian rules perform better than the optimal Taylor rule in terms of the loss criterion (10). Using the calibration of Table 1, and for various degrees persistence of the disturbances, Table 2 reveals that the loss is systematically lower with the optimal Wicksellian rule than it is with the optimal Taylor rule. For instance, when $\rho_r = \rho_u = .35$, the loss is 1.67 with the Wicksellian rule, compared to 2.63 with the Taylor rule, and 1.28 with the fully optimal rule.[14] This relatively good performance of the Wicksellian rules is due to the low variability of inflation and the nominal interest rate. On the other hand, the output gap is in general more volatile under the optimal Wicksellian rule. Of course the variability of the price level is higher for fully optimal rules and optimal Taylor rules, but this does not affect the loss criterion. While the results of Table 2 are based on our benchmark calibration, we still find, for alternative assumptions about the parameters $\lambda_x$, $\rho_r, \rho_u$, and the variances of the shocks, that the welfare loss $\mathrm{E}[L]$ implied by the optimal Wicksellian rule is lower than that implied by the optimal Taylor rule, and is only slightly higher than in the optimal plan.

In addition, the simple Wicksellian rules turn out to be very robust to alternative specifications of the shock processes. Looking again at Figures 4 and 5, we observe that an optimal Wicksellian rule — optimized under the assumption that the serial correlation of the shocks is $\rho_r = \rho_u = 0.35$ — performs again very well when the economy is hit by new disturbances

[14] Recall that Table 2 indicates the losses due to fluctuations around the steady state. However, since the steady states are the same for the optimal Taylor rule and the optimal Wicksellian rule, the comparison applies also to the levels of the variables, for any values $x^*, i^*$.

$\hat{\varepsilon}_{r,m,t}$ and $\hat{\varepsilon}_{u,m,t}$ which propagate through the economy with a different persistence $\hat{\rho}$. The welfare losses under the Wicksellian rule (dashed-dotted line) remain only slightly above the losses in the first best, even for very high degrees of shock persistence. This is very different from the performance of simple Taylor rules which imply very high losses when $\rho$ approaches 1, and suggests that the commitment to bringing the price level back to its original trend is an effective way of guarding against misspecifications or changes of the shock processes.

# 6 Introducing Interest-Rate Inertia

Wicksellian rules have the desirable feature of introducing history dependence while the simple Taylor rules considered so far are by assumption not inertial. One natural question is thus whether the performance of simple Taylor rules could not be dramatically improved by letting the interest rate respond also to past interest rates, as this would introduce at least some form of history dependence in policy. To answer this question, we first characterize a fully optimal rule.

## 6.1 A robustly optimal policy rule

As argued in Giannoni (2001, Chap. 1) and Giannoni and Woodford (2003a,b, 2010), it is possible under general conditions to find a policy rule that is optimal — i.e., that minimizes the welfare loss $\mathrm{E}[L]$ subject to the constraints (3) and (4) imposed by the private sector — and that is also robust to alternative specifications of the shock processes. This *robustly optimal policy rule* is obtained by combining the first-order necessary conditions (11)–(14) characterizing the optimal state-contingent plan to eliminate the Lagrange multipliers. This yields a single equation involving only target variables, that can be interpreted as an implicit policy rule. Solving (13) for $\phi_{1t}$ as a function of $i_t$, and (12) for $\phi_{2t}$ as a function of $x_t, i_t, i_{t-1}$, and using the resulting expressions to substitute for the Lagrange multipliers in (11) yields the instrument rule[15]

$$i_t = \psi_\pi \pi_t + \psi_x \Delta x_t + (1 + \psi_i)\, i_{t-1} + \psi_{\Delta i} \Delta i_{t-1} - \psi_i i^* \tag{27}$$

[15] This rule is analogous to what Svensson (2003) calls an optimal specific targeting rule.

where $\Delta x_t \equiv x_t - x_{t-1}$ denotes first differences, and the policy coefficients are given by

$$\psi_\pi = \frac{\kappa}{\lambda_i \sigma} > 0, \qquad \psi_x = \frac{\lambda_x}{\lambda_i \sigma} > 0, \qquad \psi_i = \frac{\kappa}{\beta \sigma} > 0, \qquad \psi_{\Delta i} = \beta^{-1} > 1. \tag{28}$$

This rule necessarily holds in the optimal plan in all period $t \geq 2$, for it is consistent with the first-order conditions (11)–(13) at these dates. For this policy rule to *implement* the optimal equilibrium, it must also determine a *unique* bounded equilibrium. Remarkably, the rule (27) also has this very desirable property. In fact, a commitment to (27) at all dates $t \geq 0$ implies a determinate rational-expectations equilibrium (see Giannoni and Woodford, 2003b, Proposition 1).[16] The equilibrium implied by a commitment to the time-invariant policy rule (27) at all dates $t \geq 0$ is the unique bounded solution the structural equations (3)–(4), the first-order conditions (11)–(13) at all dates $t \geq 0$, where the initial Lagrange multipliers $\phi_{1,-1}$, $\phi_{2,-1}$ are not given by (14) but depend instead on the historical values $x_{-1}$, $i_{-1}$, and $i_{-2}$, through the equations (11)–(13). Such a policy involves the same response to random shocks in periods $t \geq 0$ as in the optimal (Ramsey) plan, and is a rule that is optimal from a *timeless perspective* (see, e.g., Woodford 1999).[17]

A further interesting feature of this policy rule is that it does not involve any shock.[18] The optimal policy rule (27) has thus the desirable property of being completely *robust* to the specification of the shock processes, even if the latter are of the form specified in (6)–(7), as long as they are bounded. An implication of this is that a commitment to the optimal rule (27) with coefficients given by (28) does not only implement the optimal plan in the case of the assumed autocorrelation of the shocks, but also for any other degree of shock persistence. As a result, the policymaker would not need to reconsider its commitment and change the rule in the event that the economy would be hit by shocks that have different

[16] As further shown in Giannoni and Woodford (2010), a policy rule (or target criterion) constructed in this fashion from the first-order conditions associated with the optimal policy problem implies a locally determinate equilibrium under very general conditions, even in the context of large-scale nonlinear models.

[17] The optimal (Ramsey) plan is the bounded solution to the structural equations (3)–(4), the first-order conditions (11)–(13) at all dates $t \geq 0$, where the initial Lagrange multipliers $\phi_{1,-1}$, $\phi_{2,-1}$are given by (14). Such a plan can be implemented by the time-varying rule given by $i_0 = \psi_\pi \pi_0 + \psi_x x_0$, in period 0, $i_1 = \psi_\pi \pi_1 + \psi_x (x_1 - x_0) + (1 + \psi_{i1}) i_0$ in period 1 and (27) at all dates $t \geq 2$.

[18] More generally, as long as the shocks enter in an additively separable fashion in the policymaker's objective function and in the constraints imposed by the private sector, the first-order conditions to the optimal policy problem don't involve any exogenous shocks or even any properties of their driving processes.

properties than then ones observed prior to the commitment. By keeping the policy rule unchanged, it would continue to achieve the optimal equilibrium, hence the lowest possible loss for any value of $\rho$. The welfare losses associated with the rule (27)–(28) are therefore the ones associated with the optimal plan, and displayed by the solid line in Figures 4 and 5, for different values of $\rho$.[19]

Equation (27) indicates that to implement the optimal plan, the central bank should relate the interest rate positively to fluctuations in current inflation, in *changes* of the output gap, and in lagged interest rates.[20] The interest rate should not only be inertial in the sense of being positively related to past values of the interest rate, it should be *super-inertial*, as the lagged polynomial for the interest rate in (27), $1-(1+\frac{\kappa}{\beta\sigma}+\beta^{-1})\mathrm{L}+\beta^{-1}\mathrm{L}^2 = (1-z_1\mathrm{L})(1-z_2\mathrm{L})$ involves a root $z_1 > 1$ while the other root $z_2 \in (0,1)$. A reaction greater than one of the interest rate to its lagged value has been found by Rotemberg and Woodford (1999) and Woodford (2003b) to be a desirable feature of policy in models with optimizing agents. Such a *super-inertial* rule brings about the optimal responses to shocks when economic agents are forward-looking. Optimal policy requires rapidly raising the interest rate to deviations of inflation and the output gap from the target (which is 0), if such deviations are not subsequently undone. But of course, such a policy is perfectly consistent with a *stationary* rational expectations equilibrium, and in fact is the one generating the lowest overall welfare loss and a low variability of the interest rate *in equilibrium*. In fact, the interest rate does not explode in equilibrium because the current and expected future optimal levels of the interest rate counteract the effects of an initial deviation in inflation and the output gap by generating subsequent deviations with the opposite sign of these variables, as shown in Figures 2 and 3.[21]

[19] While we assume here that the policymaker knows with certainty the model of the economy, though it may face uncertainty about the shock processes, Walsh (2004) has shown that the same rule turns out to be robust to misspecifications of the structural model of the kind considered by Hansen and Sargent (2008). However, as Walsh (2004) emphasizes, while the rule is the same in the two approaches, different macroeconomic behavior would be observed, as expectations are formed differently in the two approaches.

[20] From a practical point of view, it might be an advantage to respond to changes in the output gap rather than the level as the *change* in the output gap may be known with greater precision. For example, Orphanides (2003) shows that subsequent revisions of U.S. output gap estimates have been quite large (sometimes as large as 5.6 percentage points), while revisions of estimates of the quarterly change in the output gap have been much smaller.

[21] Optimal interest-rate rules are super-inertial under general conditions, as long as the private sector is sufficiently forward-looking (see, Giannoni and Woodford, 2003a).

The coefficients of the optimal policy rule are reported in the upper right panel of Table 2, for our benchmark calibration.[22] For comparison, the last panel of Table 2 reports the coefficients derived from Judd and Rudebusch's (1998) estimation of actual Fed reaction functions between 1987:3 and 1997:4, along with the statistics that such a policy would imply if the model provided a correct description of the actual economy.[23] As shown in Table 2, the estimated historical rule in the baseline case involves only slightly smaller responses to fluctuations in inflation and the output gap than the optimal rule. However the estimated response to lagged values of the interest rate is sensibly smaller that the optimal one.

## 6.2 Simple rules and interest rate inertia

The analysis of the optimal policy rule (27) suggests that it is desirable for the current interest rate to respond strongly to movements in the past interest rate. While the policy rule (27) achieves the lowest possible loss in the model considered and remains relatively simple, recent research has given considerable attention to even simpler policy rules (see, e.g., contributions collected in Taylor, 1999). As we now show, even if we allow for considerable inertia in interest rate in the policy rule, it remains preferable to respond to fluctuations in the price level than in the inflation rate. To see this, consider a minor departure from the optimal rule (27) with coefficients given by (28), neglecting the term $\psi_i (i_{t-1} - \bar{\imath})$ and setting $\psi_{\Delta i}$ to 1 instead of $\beta^{-1}$.[24] After this simplification, and using (5) and (23), the rule (27) reduces to

$$\Delta i_t = \psi_\pi (\Delta p_t - \Delta \bar{p}_t) + \psi_x \Delta x_t + \Delta i_{t-1} \tag{29}$$

where $\psi_\pi$ and $\psi_x$ are again given by (28), and the steady-state inflation rate is given by $\bar{\pi} = \lambda_i i^*/\beta$. Assuming furthermore that at some point $t_0 - 1$ in the past, the interest rate satisfied $i_{t_0-1} = \psi_\pi (p_{t_0-1} - \bar{p}_{t_0-1}) + \psi_x x_{t_0-1} + i_{t_0-2}$ and using (23) implies that a commitment

[22] The coefficients $\psi_x$ reported here are multiplied by 4, so that the response coefficients to output gap, and to annualized inflation are expressed in the same units. (See footnote 9.)

[23] The estimated historical policy rule refers to regression A in Judd and Rudebusch (1998).

[24] Doing so prevents the rule from being super-inertial, a feature that has been criticized on the grounds that such rules lead to explosive behavior in models which involve no rational expectations and no forward-looking behavior (see, e.g., Taylor 1999b).

**Table 3: Statistics and Quasi-Optimal Policy Rules**

| | | Statistics | | | | | Coefficients of optimal policy rule | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | V[$\pi$] | V[$x$] | V[$i$] | V[$p$] | E[$L$] | $p_t$ | $\pi_t$ | $x_t$ | $x_{t-1}$ | $i_{t-1}$ | $i_{t-2}$ |
| ***Quasi-Optimal Rule (p)*** | | | | | | | | | | | | |
| $\rho_r$ = 0 | $\rho_u$ = 0 | 0.128 | 10.214 | 1.222 | 0.011 | 0.911 | 0.641 | -- | 0.325 | -- | 1.000 | -- |
| | $\rho_u$ = 0.35 | 0.163 | 10.995 | 1.222 | 0.024 | 0.984 | 0.641 | -- | 0.325 | -- | 1.000 | -- |
| | $\rho_u$ = 0.9 | 0.164 | 20.097 | 1.223 | 0.171 | 1.425 | 0.641 | -- | 0.325 | -- | 1.000 | -- |
| $\rho_r$ = 0.35 | $\rho_u$ = 0 | 0.146 | 11.025 | 2.527 | 0.015 | 1.276 | 0.641 | -- | 0.325 | -- | 1.000 | -- |
| | $\rho_u$ = 0.35 | 0.181 | 11.806 | 2.528 | 0.028 | 1.349 | 0.641 | -- | 0.325 | -- | 1.000 | -- |
| | $\rho_u$ = 0.9 | 0.182 | 20.907 | 2.528 | 0.175 | 1.790 | 0.641 | -- | 0.325 | -- | 1.000 | -- |
| $\rho_r$ = 0.9 | $\rho_u$ = 0 | 0.134 | 4.898 | 10.069 | 0.018 | 2.751 | 0.641 | -- | 0.325 | -- | 1.000 | -- |
| | $\rho_u$ = 0.35 | 0.170 | 5.679 | 10.069 | 0.030 | 2.824 | 0.641 | -- | 0.325 | -- | 1.000 | -- |
| | $\rho_u$ = 0.9 | 0.171 | 14.780 | 10.069 | 0.177 | 3.265 | 0.641 | -- | 0.325 | -- | 1.000 | -- |
| ***Quasi-Optimal Rule ($\pi$)*** | | | | | | | | | | | | |
| $\rho_r$ = 0 | $\rho_u$ = 0 | 0.179 | 9.923 | 1.089 | 1.138 | 0.916 | -- | 0.641 | 0.325 | -- | 1.000 | -- |
| | $\rho_u$ = 0.35 | 0.293 | 10.151 | 1.123 | 2.987 | 1.049 | -- | 0.641 | 0.325 | -- | 1.000 | -- |
| | $\rho_u$ = 0.9 | 1.848 | 15.403 | 2.346 | 181.241 | 3.147 | -- | 0.641 | 0.325 | -- | 1.000 | -- |
| $\rho_r$ = 0.35 | $\rho_u$ = 0 | 0.215 | 10.964 | 2.137 | 1.979 | 1.250 | -- | 0.641 | 0.325 | -- | 1.000 | -- |
| | $\rho_u$ = 0.35 | 0.329 | 11.192 | 2.171 | 3.828 | 1.383 | -- | 0.641 | 0.325 | -- | 1.000 | -- |
| | $\rho_u$ = 0.9 | 1.885 | 16.444 | 3.394 | 182.082 | 3.482 | -- | 0.641 | 0.325 | -- | 1.000 | -- |
| $\rho_r$ = 0.9 | $\rho_u$ = 0 | 0.234 | 5.354 | 8.871 | 6.154 | 2.590 | -- | 0.641 | 0.325 | -- | 1.000 | -- |
| | $\rho_u$ = 0.35 | 0.348 | 5.582 | 8.905 | 8.003 | 2.723 | -- | 0.641 | 0.325 | -- | 1.000 | -- |
| | $\rho_u$ = 0.9 | 1.903 | 10.835 | 10.129 | 186.257 | 4.821 | -- | 0.641 | 0.325 | -- | 1.000 | -- |

to (29) at all dates $t \geq t_0$ is equivalent a commitment to the rule

$$i_t = \psi_\pi \left(p_t - \bar{p}_t\right) + \psi_x x_t + i_{t-1} \tag{30}$$

at all dates $t \geq t_0$. This of course is none else than a Wicksellian rule augmented with the lagged interest rate. Given that the coefficient on the lagged interest rate is 1, this *quasi-optimal rule* specifies how changes in the interest rate $\Delta i_t$ should depend on fluctuations in the price level (in log deviations from a trend) and the output gap. As (30) reveals, a desirable policy involves even more history dependence than we had considered in the case of simple Wicksellian rules.

Table 3 quantifies the welfare losses implied by a commitment to the quasi-optimal rule (30) for different degrees of shock persistence. Figures 4 and 5 also plot the welfare losses (with black dots) as a function the shocks' autocorrelation. Importantly, this very simple rule performs remarkably well, with welfare losses appear only marginally higher than in the

fully optimal rule for a very wide range of shock persistence, given that the policy coefficients are totally invariant to the assumed properties of the shock process.

To contrast, we consider now an expanded version of the Taylor rule that allows for interest rate inertia

$$i_t = \psi_\pi (\pi_t - \bar{\pi}) + \psi_x x_t + i_{t-1}, \tag{31}$$

where $\psi_\pi$ and $\psi_x$ are again given by (28). Clearly, introducing a large amount of interest rate inertia contributes to reducing the welfare losses substantially: comparing Tables 2 and 3, we note that the welfare losses drop from 2.63 to 1.38 when introducing the lagged interest rate in the Taylor rule and $\rho_r = \rho_u = 0.35$. This is not surprising, in light of the discussion in sections 3 and 4, as the rule (31) resembles closely the simple Wicksellian rule, were it not for the response to the output gap.[25] In addition, Levin *et al.* (1999) show that rules that have a coefficient of one on the lagged interest rate tend to perform well across models, and Orphanides and Williams (2007) show that such rules are robust to potential misspecification of private sector learning, in a model in which agents have imperfect knowledge about the structure of the economy. However, as Figure 4 and 5 show, the performance of the rule (31) deteriorates also markedly as the shock persistence increases. So, while allowing for high degree of interest inertia in interest rates allows to improve the performance of simple policy rules, our results show that it remains preferable for the interest rate to respond to price-level fluctuations than to inflation fluctuations in the model considered, and the gains from price-level stabilization are larger in the face of misspecifications of the shock processes.

# 7 Conclusion

This paper has characterized the properties of various simple interest-rate rules in the context of a stylized structural forward-looking model of the economy. We have compared the performance of simple Taylor rules and simple Wicksellian rules — which determine the interest rate as a function of deviations of the *price level* from its trend and an output gap

[25] Indeed, assuming that at some point $t_0 - 1$ in the past the interest rate satisfied $i_{t_0-1} = \psi_\pi (p_{t_0-1} - \bar{p}_{t_0-1}) + \psi_x x_{t_0-1}$, a commitment to the "difference rule" $\Delta i_t = \psi_\pi (\pi_t - \bar{\pi}) + \psi_x \Delta x_t$ at all dates $t \geq t_0$ is equivalent a commitment to the Wicksellian rule $i_t = \psi_\pi (p_t - \bar{p}_t) + \psi_x x_t$ at all dates $t \geq t_0$.

— to determine whether the central bank's objective function, which is assumed to depend on the volatility of inflation, output gap and interest rate, is best achieved by a policy that responds to fluctuations in inflation or the price level. We have shown that appropriate Wicksellian rules result systematically in a lower welfare loss, a lower variability of inflation and of the nominal interest rate than optimal Taylor rules, by introducing desirable history dependence in monetary policy. The coefficients of optimal Wicksellian rules have the further advantage of being less sensitive to alternative degrees of persistence in the shock processes. An implication of this is that Wicksellian rules perform better than simple Taylor rules in the face of changes in shock processes. This makes a commitment to simple Wicksellian rules more appealing as their robustness property provides little ground for a reconsideration of the commitment when the economy is affected by new kinds of disturbances. Moreover, Wicksellian rules are less prone to equilibrium indeterminacy than optimal Taylor rules.

The fact that simple Wicksellian rules perform so well in our model becomes clear when we observe a simple Wicksellian rule augmented with a large amount of interest-rate inertia (30) resembles a *robustly optimal rule* which, as argued in Giannoni and Woodford (2003a,b, 2010), implements the optimal plan and is also completely robust to the specification of exogenous shock processes. A Wicksellian rule of this form states that *changes* (and not the level) of the policy rates should depend positively on the deviations of the *price level* from trend, and the output gap. This rule remains very simple, is again fully robust to the specification of the shock processes, and introduces an even great amount of history dependence than simple Wicksellian rules, which yields a remarkable welfare performance in the model considered. Such a rule should thus be particularly appealing to policymakers who search for a simple rule, yet worry about unforeseeable circumstances (shocks) affecting the economy in the future.

Our results have been derived here in an arguably stylized model in which agents have full information about the current state of the economy and are completely rational. This has allowed us to emphasize that the history dependence generated by the price-level stabilization results in important welfare gains and has good robustness properties to the assumed shock processes. Recent research suggests that the benefits from price-level stabilization hold in different setups as well. In fact, Preston (2008) assumes that private agents are non-rational

and learn adaptively. He shows that a price-level target corrects past mistakes and yields better welfare results than inflation stabilization when the central bank cannot perfectly understand private agents' behavior. This is consistent with Orphanides and Williams (2007)'s conclusion on the desirability of "difference" interest-rate rules, which resemble our Wicksellian rules. Gaspar, Smets and Vestin (2007), using in a medium-scale model involving a number of rigidities and inertial behavior of the private sector find that the stabilization of the price-level path is a simple and effective way of implementing a desirable equilibrium. A concern raised by Mishkin and Schmidt-Hebbel (2001) is that a commitment to price-level stabilization may propagate iid measurement errors in inflation. However Gorodnichenko and Shapiro (2007) considering a forward-looking model with backward-looking features show that a rule akin to our Wicksellian rule can effectively stabilize the economy in the face of imperfectly observed shifts in potential output growth or surprises in the price level. Boivin (2009) and Woodford (2010) similarly argue that stabilizing the price level might be more desirable in the event that the price level is not perfectly observed, provided that the public is sufficiently forward looking and understands the policy regime. In addition, Eggertsson and Woodford (2003), Wolman (2005) and Billi (2008) have argued that a commitment to price-level stabilization may be an effective way of preventing deflations and exiting from deflationary traps. While these papers emphasize different desirable features of price-level stabilization, they all point to some robustness property of price-level stabilization.

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# A Appendix

## A.1 Optimal non-inertial plan and optimal Taylor rule

The coefficients in (17) need to satisfy the structural equations (3) and (4) at each date, and for every possible realization of the shocks, hence

$$\begin{aligned}
(1-\beta)\,\pi^{ni} - \kappa x^{ni} &= 0, \qquad \pi^{ni} - i^{ni} = 0 && \text{(A.32)}\\
(1-\rho_r)\,x_r + \sigma^{-1}\left(i_r - \rho_r \pi_r - 1\right) &= 0, \qquad (1-\beta\rho_r)\,\pi_r - \kappa x_r = 0 && \text{(A.33)}\\
(1-\rho_u)\,x_u + \sigma^{-1}\left(i_u - \rho_u \pi_u\right) &= 0, \qquad (1-\beta\rho_u)\,\pi_u - \kappa x_u - 1 = 0. && \text{(A.34)}
\end{aligned}$$

Substituting (17) into (10), and using $\mathrm{E}\left(r_t^e u_t\right) = 0$, we can rewrite the loss function as

$$\begin{aligned}
\mathrm{E}\left[L_0\right] &= \left[\left(\pi^{ni}\right)^2 + \lambda_x\left(x^{ni} - x^*\right)^2 + \lambda_i\left(i^{ni} - i^*\right)^2\right] + \left(\pi_r^2 + \lambda_x x_r^2 + \lambda_i i_r^2\right)\mathrm{var}\left(r_t^e\right)\\
&\quad + \left(\pi_u^2 + \lambda_x x_u^2 + \lambda_i i_u^2\right)\mathrm{var}\left(u_t\right). \qquad \text{(A.35)}
\end{aligned}$$

The optimal non-inertial plan involves equilibrium coefficients that minimize the loss $\mathrm{E}\left[L_0\right]$ subject to (A.32) – (A.34). These are given by:

$$\begin{aligned}
i^{ni} &= \pi^{ni} = \frac{(1-\beta)\,\kappa^{-1}\lambda_x x^* + \lambda_i i^*}{1 + (1-\beta)^2\,\kappa^{-2}\lambda_x + \lambda_i}, \quad x^{ni} = \frac{1-\beta}{\kappa}\frac{(1-\beta)\,\kappa^{-1}\lambda_x x^* + \lambda_i i^*}{1 + (1-\beta)^2\,\kappa^{-2}\lambda_x + \lambda_i}, && \text{(A.36)}\\
\pi_r &= \frac{\lambda_i\left(\sigma\gamma_r - \rho_r\kappa\right)\kappa}{h_r}, \qquad \pi_u = \frac{\lambda_i\sigma\left(\sigma\gamma_u - \rho_u\kappa\right)(1-\rho_u) + \lambda_x\left(1-\beta\rho_u\right)}{h_u}, && \text{(A.37)}\\
x_r &= \frac{\lambda_i\left(\sigma\gamma_r - \rho_r\kappa\right)\left(1-\beta\rho_r\right)}{h_r}, \qquad x_u = -\frac{\kappa - \rho_u\lambda_i\left(\sigma\gamma_u - \rho_u\kappa\right)}{h_u}, && \text{(A.38)}\\
i_r &= \frac{\lambda_x\left(1-\beta\rho_r\right)^2 + \kappa^2}{h_r} > 0, \qquad i_u = \frac{\sigma\kappa\left(1-\rho_u\right) + \lambda_x\left(1-\beta\rho_u\right)\rho_u}{h_u} > 0, && \text{(A.39)}
\end{aligned}$$

where $\gamma_j \equiv \left(1-\rho_j\right)\left(1-\beta\rho_j\right) > 0$, $h_j \equiv \lambda_i\left(\sigma\gamma_j - \rho_j\kappa\right)^2 + \lambda_x\left(1-\beta\rho_j\right)^2 + \kappa^2 > 0$, $j \in \{r, u\}$.

For the Taylor rule to be consistent with an equilibrium of the form (17), the policy coefficients obtained by substituting (17) into (15) must satisfy

$$i_r = \psi_\pi \pi_r + \psi_x x_r, \quad i_u = \psi_\pi \pi_u + \psi_x x_u, \quad i^{ni} = \psi_\pi \pi^{ni} + \psi_x x^{ni} + \psi_0. \qquad \text{(A.40)}$$

Solving (A.40) for $\psi_\pi, \psi_x, \psi_0$ and using (A.37)–(A.39) yields

$$\psi_\pi = \frac{\left(\kappa - \rho_u\lambda_i\left(\sigma\gamma_u - \rho_u\kappa\right)\right)\left(\xi_r\left(1-\beta\rho_r\right) + \kappa^2\right) + \left(\sigma\kappa\left(1-\rho_u\right) + \rho_u\xi_u\right)\lambda_i\left(\sigma\gamma_r - \rho_r\kappa\right)\left(1-\beta\rho_r\right)}{\lambda_i\left(\sigma\gamma_r - \rho_r\kappa\right)\left(\left(\kappa - \rho_u\lambda_i\left(\sigma\gamma_u - \rho_u\kappa\right)\right)\kappa + \left(\lambda_i\sigma\left(\sigma\gamma_u - \rho_u\kappa\right)\left(1-\rho_u\right) + \xi_u\right)\left(1-\beta\rho_r\right)\right)}$$

(A.41)

$$\psi_x = \frac{\left(\lambda_i\sigma\left(\sigma\gamma_u-\rho_u\kappa\right)\left(1-\rho_u\right)+\xi_u\right)\left(\xi_r\left(1-\beta\rho_r\right)+\kappa^2\right)-\lambda_i\left(\sigma\gamma_r-\rho_r\kappa\right)\kappa\left(\sigma\kappa\left(1-\rho_u\right)+\rho_u\xi_u\right)}{\lambda_i\left(\sigma\gamma_r-\rho_r\kappa\right)\left(\left(\kappa-\rho_u\lambda_i\left(\sigma\gamma_u-\rho_u\kappa\right)\right)\kappa+\left(\lambda_i\sigma\left(\sigma\gamma_u-\rho_u\kappa\right)\left(1-\rho_u\right)+\xi_u\right)\left(1-\beta\rho_r\right)\right)} \tag{A.42}$$

where $\xi_j \equiv \lambda_x\left(1-\beta\rho_j\right) > 0$, and $j \in \{r,u\}$, where we assume $\sigma\gamma_r - \rho_r\kappa \neq 0$.

## A.2 Optimal Wicksellian rule

We observe from (3) and (4) that in a steady state

$$i^{wr} = \pi^{wr}, \qquad x^{wr} = \frac{1-\beta}{\kappa}\pi^{wr}. \tag{A.43}$$

The structural equations (3), (4) can then be expressed in terms of the hatted variables representing deviations from the steady state, and the policy rule (22) may be written as[26]

$$\hat{\imath}_t = \psi_p\hat{p}_t + \psi_x\hat{x}_t. \tag{A.44}$$

Using (A.44) to substitute for $\hat{\imath}_t$ in (3), we can write the resulting system in matrix form as:

$$\mathrm{E}_t z_{t+1} = \hat{A}\ z_t + \hat{a}\ e_t \tag{A.45}$$

where $z_t \equiv [\hat{\pi}_t, \hat{x}_t, \hat{p}_{t-1}]'$, $e_t \equiv [r_t^e, u_t]'$, and

$$\hat{A} = \begin{bmatrix} \beta^{-1} & -\kappa\beta^{-1} & 0 \\ \sigma^{-1}\left(\psi_p - \beta^{-1}\right) & \left(1+\sigma^{-1}\left(\frac{\kappa}{\beta}+\psi_x\right)\right) & \psi_p\sigma^{-1} \\ 1 & 0 & 1 \end{bmatrix}. \tag{A.46}$$

With shock processes (8) and (9), the resulting equilibrium is of the form (24). Using (24) and noting that $\mathrm{E}\left[(1-\beta)\sum_{t=0}^{\infty}\beta^t\hat{z}_t\right] = 0$, we can write the loss criterion (10) as

$$\mathrm{E}\left[L\right] = \left[(\pi^{wr})^2 + \lambda_x\left(x^{wr}-x^*\right)^2 + \lambda_i\left(i^{wr}-i^*\right)^2\right] + \mathrm{E}[\hat{L}]. \tag{A.47}$$

where

$$\mathrm{E}[\hat{L}] \equiv \mathrm{E}\left\{(1-\beta)\sum_{t=0}^{\infty}\beta^t\left[\hat{\pi}_t^2 + \lambda_x\hat{x}_t^2 + \lambda_i\hat{\imath}_t^2\right]\right\}. \tag{A.48}$$

The optimal steady state is then found by minimizing the first term in brackets in (A.47) subject to (A.43). Since this is the same problem as the one for the optimal non-inertial plan, we have

$$\pi^{wr} = \pi^{ni}, \qquad x^{wr} = x^{ni}, \qquad \text{and} \qquad i^{wr} = i^{ni}. \tag{A.49}$$

where $\pi^{ni}, x^{ni}$, and $i^{ni}$ are given in (A.36).

[26] We normalize the steady-state value $\hat{p}_t$ to zero. It follows $\psi_0 = i^{wr} - \psi_x x^{wr}$. This normalization has no effect on the welfare analysis that follows. Note also that $\pi_t = p_t - p_{t-1} = \hat{p}_t - \hat{p}_{t-1} + \bar{\pi}$. Hence $\pi^{wr} = \bar{\pi}$.

To determine the optimal equilibrium responses to disturbances, we note that the solution (24) may only describe an equilibrium if the coefficients $z_r, z_u, z_p$ satisfy the structural equations (3) and (4) at each date, and for every possible realization of the shocks. These coefficients need therefore to satisfy the following feasibility restrictions, obtained by substituting (24) into (3), (4), (5):

$$\begin{aligned}
x_r\left(1-\rho_r\right)-x_p p_r+\sigma^{-1}\left(i_r+\left(1-p_p-\rho_r\right)p_r-1\right) &= 0 && \text{(A.50)}\\
x_u\left(1-\rho_u\right)-x_p p_u+\sigma^{-1}\left(i_u+\left(1-p_p-\rho_u\right)p_u\right) &= 0 && \text{(A.51)}\\
x_p-x_p p_p+\sigma^{-1}\left(i_p+\left(1-p_p\right)p_p\right) &= 0 && \text{(A.52)}\\
\left(\beta\rho_r+\beta p_p-1-\beta\right)p_r+\kappa x_r &= 0 && \text{(A.53)}\\
\left(\beta\rho_u+\beta p_p-1-\beta\right)p_u+\kappa x_u+1 &= 0 && \text{(A.54)}\\
\left(\beta p_p-1-\beta\right)p_p+\kappa x_p+1 &= 0. && \text{(A.55)}
\end{aligned}$$

Similarly, substituting the solution (24) into the policy rule (A.44), we have

$$i_r=\psi_p p_r+\psi_x x_r,\qquad i_u=\psi_p p_u+\psi_x x_u,\qquad i_p=\psi_p p_p+\psi_x x_p, \tag{A.56}$$

which can be solved to yield the policy coefficients

$$\psi_p=\frac{x_u i_r-i_u x_r}{x_u p_r-x_r p_u},\qquad \psi_x=\frac{p_r i_u-i_r p_u}{x_u p_r-x_r p_u}, \tag{A.57}$$

provided that $x_u p_r - x_r p_u \neq 0$. Substituting (A.57) into (A.56), we obtain

$$i_p-\frac{x_u i_r-i_u x_r}{x_u p_r-x_r p_u}p_p-\frac{p_r i_u-i_r p_u}{x_u p_r-x_r p_u}x_p=0, \tag{A.58}$$

which is an additional constraint that must be satisfied by the equilibrium coefficients.

Finally using (5), the solution (24), and (8)–(9), we can rewrite the loss (A.48) as

$$\begin{aligned}
\mathrm{E}[\hat{L}] &= \operatorname{var}\left(r_t^e\right)\left(\left(p_r^2+\lambda_x x_r^2+\lambda_i i_r^2\right)+\left(p_r\left(p_p-1\right)+\lambda_x x_r x_p+\lambda_i i_r i_p\right)\frac{2\beta\rho_r p_r}{1-\beta\rho_r p_p}\right)\\
&\quad+\operatorname{var}\left(u_t\right)\left(\left(p_u^2+\lambda_x x_u^2+\lambda_i i_u^2\right)+\left(p_u\left(p_p-1\right)+\lambda_x x_u x_p+\lambda_i i_u i_p\right)\frac{2\beta\rho_u p_u}{1-\beta\rho_u p_p}\right) \qquad \text{(A.59)}\\
&\quad+\left(\left(p_p-1\right)^2+\lambda_x x_p^2+\lambda_i i_p^2\right)\left(\operatorname{var}\left(r_t^e\right)\frac{\beta p_r^2}{1-\beta p_p^2}\frac{1+\beta\rho_r p_p}{1-\beta\rho_r p_p}+\operatorname{var}\left(u_t\right)\frac{\beta p_u^2}{1-\beta p_p^2}\frac{1+\beta\rho_u p_p}{1-\beta\rho_u p_p}\right).
\end{aligned}$$

The optimal equilibrium resulting from a Wicksellian rule (22) is therefore characterized by the optimal steady state (A.49), and the optimal response coefficients $p_r, p_u$, and so on, that minimize the loss function (A.59) subject to the constraints (A.50)–(A.55) and (A.58). The coefficients of the optimal Wicksellian rule are in turn determined by (A.57).

## A.3 Proof of Proposition 1

The model composed of (3), (4), (5), and the Wicksellian policy rule (22) can be expressed in matrix form as the dynamic system (A.45) with transition matrix (A.46). The characteristic polynomial associated to $\hat{A}$ is $P(X) = X^3 + A_2X^2 + A_1X + A_0$ where $A_0 = -\frac{\sigma+\psi_x}{\beta\sigma}$, $A_1 = \frac{\kappa+\sigma\beta+2\sigma+\psi_x\beta+\kappa\psi_p+\psi_x}{\beta\sigma}$, $A_2 = -\frac{2\sigma\beta+\sigma+\kappa+\psi_x\beta}{\beta\sigma}$. The system (A.45) results in a determinate equilibrium if and only if the characteristic polynomial $P(X)$ admits two roots outside and one root inside the unit circle. Using Proposition C.2 of Woodford (2003a), $P(X)$ has one root inside the unit circle and two roots outside if $P(1) > 0$, $P(-1) < 0$, and $|A_2| > 3$. These conditions are satisfied provided that $\psi_p$ and $\psi_x$ satisfy (26). Hence $P(X)$ has exactly 2 roots outside the unit circle, and (A.45) results in a determinate equilibrium.

## A.4 Proof of Proposition 2

We show that the loss criterion (10) for the optimal Taylor rule is higher than the one for a particular Wicksellian rule. Since the optimal steady-state is the same for both families of rules, it is sufficient to compare the loss $\mathrm{E}[\hat{L}]$ resulting from deviations from the steady-state.

**Loss for optimal Taylor rule.** Under the optimal Taylor rule, the equilibrium coefficients are given in (A.37) – (A.39). Setting $\kappa = 0$, and $\rho_r = \rho_u = \rho$, and using the resulting expressions to substitute for the equilibrium coefficients in the loss function (A.35), we obtain the loss in deviations from steady-state:

$$\mathrm{E}[\hat{L}^{tr}] = \frac{\lambda_i\lambda_x}{\lambda_i\sigma^2(1-\rho)^2+\lambda_x}\mathrm{var}(r_t^e) + \frac{\lambda_x(1+\lambda_i\rho^2)+\lambda_i\sigma^2(1-\rho)^2}{\left(\lambda_i\sigma^2(1-\rho)^2+\lambda_x\right)(1-\beta\rho)^2}\mathrm{var}(u_t). \tag{A.60}$$

**Loss for some particular Wicksellian rule.** In the case in which $\kappa = 0$ and $\rho_r = \rho_u = \rho > 0$, the restrictions (A.50)–(A.55) and (A.58) constraining the equilibrium resulting from any Wicksellian rule (A.44) can be solved in terms of $x_r, x_u$ to yield:

$$p_r = 0, \qquad p_u = \frac{1}{1-\beta\rho}, \qquad p_p = 1, \qquad x_p = \frac{x_u(1-\beta\rho)-x_r\rho}{1+x_r\sigma\rho} \tag{A.61}$$

$$i_r = 1-x_r\sigma(1-\rho), \quad i_u = \rho\frac{(1-x_r\sigma(1-\rho))(1+x_u\sigma(1-\beta\rho))}{(1+x_r\sigma\rho)(1-\beta\rho)}, \quad i_p = 0. \tag{A.62}$$

There is also a second solution which is not admissible as it involves $p_p = \beta^{-1} > 1$, hence an explosive price level. Consider now an equilibrium in which $x_r$ and $x_u$ satisfy

$$x_r = \frac{\lambda_i\sigma(1-\rho)}{\lambda_i\sigma^2(1-\rho)^2+\lambda_x}, \qquad x_u = \lambda_i\sigma\rho\frac{\left(\lambda_i\sigma^2(1-\rho)^2+\lambda_x\right)\beta(1-\beta\rho^2)-\lambda_x\rho(1-\beta)}{(1-\beta\rho)\Delta}, \tag{A.63}$$

where $\Delta \equiv \left(\lambda_i\sigma^2(1-\rho)^2+\lambda_x\right)\left(\lambda_i\sigma^2(1-\beta\rho^2)(1-\beta\rho)+\lambda_x(1+\beta\rho)\right)$, and where (A.61)–(A.62) are used to compute the remaining coefficients $x_p$, $i_r$, $i_u$, $i_p$. Expression (A.63) do in general not minimize the loss criterion (A.59), though they would do so in the case that $\mathrm{var}(u_t) = 0$. We call this equilibrium a "quasi-optimal equilibrium". One implication

of course is that the resulting loss, $\mathrm{E}[\hat{L}^{qwr}]$, cannot be smaller than the one obtained in the optimal equilibrium, $\mathrm{E}[\hat{L}^{wr}]$, so that $\mathrm{E}[\hat{L}^{qwr}] \geq \mathrm{E}[\hat{L}^{wr}]$. The Wicksellian rule (A.44) that implements this quasi-optimal equilibrium, obtained by using (A.57) yields: $\psi_p = \frac{\rho\lambda_x(1-\beta)\left(1-\beta\rho^2\right)}{(1-\rho)(\lambda_i\sigma^2(1-\beta\rho^2)(1-\beta\rho)+\lambda_x(1+\beta\rho))} > 0$, and $\psi_x = \frac{\lambda_x}{\lambda_i\sigma(1-\rho)} > 0$. It follows from Proposition 1 that this rule results in a unique bounded equilibrium.

Next, substituting the above equilibrium coefficients in the loss criterion (A.59), we obtain

$$\begin{aligned}\mathrm{E}[\hat{L}^{qwr}] &= \frac{\lambda_i\lambda_x}{\lambda_i\sigma^2\left(1-\rho\right)^2+\lambda_x}\,\mathrm{var}\left(r_t^e\right) \\ &\quad + \left(\frac{1}{\left(1-\beta\rho\right)^2} + \lambda_i\lambda_x\rho^2\frac{\lambda_i\sigma^2\beta\left(1-\rho\right)^2\left(1-\beta\rho^2\right)+\lambda_x\left(1-\beta^2\rho^2\right)}{\left(1-\beta\rho\right)^3\Delta}\right)\mathrm{var}\left(u_t\right)\end{aligned} \tag{A.64}$$

**Comparing the losses.** Comparing (A.60) and (A.64), we obtain after some algebraic manipulations: $\mathrm{E}[\hat{L}^{tr}] > \mathrm{E}[\hat{L}^{qwr}]$. Thus $\mathrm{E}[\hat{L}^{tr}] > \mathrm{E}[\hat{L}^{qwr}] \geq \mathrm{E}[\hat{L}^{wr}]$, which completes the proof. ■