

SELF-VALIDATING OPTIMUM CURRENCY AREAS*

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First Draft: December 2001

Revised: May 2002

Abstract

A currency area can be a self-validating optimal policy regime, even when monetary unification does not foster real economic integration and intra-industry trade. In our model, firms choose the optimal degree of exchange rate pass-through to export prices while accounting for expected monetary policies, and monetary authorities choose optimal policy rules while taking firms' pass-through as given. We show that there exist two equilibria, each of which defines a self-validating currency regime. In the first, firms preset prices in domestic currency and let prices in foreign currency be determined by the law of one price. Optimal policy rules then target the domestic output gap, and floating exchange rates support the flex-price allocation. In the second equilibrium, firms preset prices in consumer currency, and a monetary union is the optimal policy choice for all countries. Although a common currency helps synchronize business cycles across countries, flexible exchange rates deliver a superior welfare outcome.

JEL classification: E5, F4

Keywords: optimum currency areas, monetary union, optimal cyclical monetary policy, nominal rigidities, exchange rate pass-through.

*We thank Luca Dedola, Rudi Dornbusch, Petra Geraats, Maury Obstfeld, Ken Rogoff, Cédric Tille and seminar participants at the 2002 AEA Meetings in Atlanta, the 2002 Summer Symposium for Central Bank Researchers at Gerzensee, the Board of Governors of the Federal Reserve System, the University of Virginia, and the Johns Hopkins SAIS for many helpful comments. We also thank Raymond Guiteras for excellent research assistance. Corsetti's work on this paper is part of a research network on "The Analysis of International Capital Markets: Understanding Europe's Role in the Global Economy," funded by the European Commission under the Research Training Network Programme (Contract No. HPRN-CT-1999-00067). The views expressed here are those of the authors, and do not necessarily reflect the position of the International Monetary Fund, the Federal Reserve Bank of New York, the Federal Reserve System, or any other institution with which the authors are affiliated.

I Introduction

The theory of optimum currency areas (OCAs), from the classic contributions by Mundell [1961], McKinnon [1963], Kenen [1969] and Ingram [1973] to its modern applications and revisions,¹ stresses that asymmetric, country-specific shocks represent a key element in the choice of exchange rate regime. Specifically, asymmetric real shocks weaken the case for a common currency, as member states of a monetary union lose their ability to use domestic exchange rate and interest rate policies for stabilization purposes. Conversely, business cycle synchronization and macroeconomic convergence make a currency area an optimal monetary arrangement, other things being equal, by reducing the scope for asymmetric policy responses to the disturbances hitting the union-wide economy.

Monetary union may nonetheless become optimal *ex post*, even though the individual countries that join it do not meet the optimality criteria *ex ante*. In the vast literature on optimum currency areas, the argument for an *endogenous* optimal currency area emphasizes that the monetary union itself could act as a catalyst of business cycle synchronization. An influential contribution by Frankel and Rose [1997, 1998] suggests that the reduction of foreign exchange transaction costs promotes trade integration across countries: to the extent that the process of integration enhances intra-industry trade rather than product specialization, national business cycles become more synchronized, since sectoral demand shocks and productivity innovations affect all countries at the same time. Higher national output correlation then reduces the need for exchange rate adjustments to stabilize national employment and prices, and minimizes the welfare costs of giving up national currencies. Not everyone agrees with this argument: for instance Eichengreen [1992] and Krugman [1993] stress that monetary integration could lead to greater specialization in production, thus lowering output correlation and making regions more vulnerable to local shocks. On an empirical basis, however, the evidence presented by Frankel and Rose [1997, 1998] supports the view that trade links raise income correlations.

¹Those include, among others, Eichengreen [1990, 1992], Dowd and Greenaway [1993], Tavlas [1993], Bayoumi and Eichengreen [1994], Melitz [1996], Bayoumi [1997], and Alesina and Barro [2002]. See Buitier, Corsetti and Pesenti [1998], ch.10, for a critical survey of the literature.

Moreover, Rose and Engel [2000] show that membership in a common currency area increases international business cycle correlations by a significant amount.

But what if monetary integration fails to boost economic convergence and intra-industry trade? Would it still be possible for a monetary union to satisfy *ex post* the OCA criterion when monetary and real integration fail to move in tandem? In this paper, we offer a distinct argument for a theory of endogenous optimal currency areas, with conceptual roots in the Lucas critique. We show that — independent of economic integration — the adoption of a common monetary policy can be self-validating: when the private sector chooses pricing strategies that are optimal in a monetary union, such strategies make a currency area the optimal monetary regime from the vantage point of the national policymakers as well. In other words, there is no incentive for monetary authorities to pursue independent strategies of national output stabilization. As a result, even if there is no structural change in fundamentals (e.g., no increase in intra-industry trade), national outputs become more correlated.

We analyze endogenous optimal monetary unions within a general-equilibrium two-country, choice-theoretic stochastic setting where national welfare is measured by the utility of each country’s representative household.² The key elements of our approach are imperfect competition in production, nominal rigidities in the goods markets, and forward-looking price-setting by firms. Drawing on Corsetti and Pesenti [2001a], our setup allows for imperfect pass-through of exchange rate onto export prices. In what follows, the degree of pass-through is endogenously chosen *ex ante* by exporters, in the form of a rule of limited price flexibility contingent on exchange rate movements.

To distinguish our theory from the previous argument for endogenous optimal currency areas, we rule out *a priori* structural change: countries are perfectly specialized in the production of one type of good before and after the monetary union. Firms optimally choose prices and pass-through elasticities based on information on shocks and policy rules. Taking firms’ pricing and pass-through strategies as given, monetary authorities choose optimal state-contingent policy rules. In a world equilibrium, both the degree of pass-through

²Other contributions in the recent literature include Obstfeld and Rogoff [2000, 2002], Devereux and Engel [2000], Corsetti and Pesenti [2001a], Benigno [2001], Clarida, Gali and Gertler [2001], Canzoneri, Cumby and Diba [2002]. On the welfare effects of a currency area when political factors influence monetary policy see Neumeyer [1998].

and monetary policy are jointly determined by optimizing agents.

We show that there are two equilibria, which can be Pareto ranked. While exporters could in principle choose any intermediate level of pass-through, in equilibrium pass-through is either 100 percent or zero as producers optimally choose ‘corner’ pricing strategies to prevent their markups from being affected by exchange rate fluctuations.³ There is one equilibrium in which firms choose to preset prices in domestic currency, and let the foreign price adjust according to the law of one price. With complete pass-through, monetary policies are fully inward-looking: they implement stabilization rules that close national output gaps completely in every period. This equilibrium is inconsistent with fixed exchange rates, and implies low correlation among output levels — depending on the cross-country correlation of fundamental shocks. The exchange rate plays the role stressed by Friedman [1953]: it brings about the required relative price adjustments that are hindered by the presence of nominal price rigidities.

But there is another equilibrium in which firms preset prices in consumer currency. With zero pass-through in the world economy, monetary policies are perfectly symmetric across countries, that is, they both respond to the same average of national shocks. This equilibrium is consistent with an OCA: there is no cost in giving up monetary sovereignty because, even if national monetary authorities remained independent, they would still choose to implement the same policy rules, moving interest rates in tandem and responding symmetrically to world-wide shocks. National outputs are perfectly correlated even when shocks are asymmetric.

In welfare terms, the optimal monetary union is Pareto-inferior to the Friedman-style arrangement in the first equilibrium. Although the private and the public sector ‘do the right thing’ — in terms of policy and pricing strategies — once the equilibrium with a monetary union is selected, there is still room for welfare improvement by creating conditions for relative price adjustment via changes of the exchange rate. A move toward more volatile rates and less synchronized business cycles would bring about the appropriate change in firms’ pricing and pass-through strategies, which in turn would validate the floating regime as optimal.

³Related literature focuses on the choice of pricing strategies where monetary authorities are assumed to implement non-optimizing, *noisy* policies (as in the work by Bacchetta and Van Wincoop [2001] and Devereux and Engel [2001]) rather than optimal rules.

This paper is organized as follows. Section 2 develops the model. Section 3 studies price-setters' optimal behavior and endogenous pass-through strategies for given monetary policies. Section 4 focuses instead on optimal monetary policies given firms' pricing strategies. The previous two pieces of analysis are brought together in sections 5 and 6, which characterize the equilibrium of the economy. A final section discusses our main results.

II The model

A Preferences, technology and budget constraints

We model a world economy with two countries, H (Home) and F (Foreign), each specialized in one *type* of traded good. Each good is produced in a number of *brands* defined over a continuum of unit mass. Brands are indexed by h in the Home country and f in the Foreign country. Each country is populated by households defined over a continuum of unit mass. Households are indexed by j in the Home country and j^* in the Foreign country.

Home agent j 's lifetime expected utility \mathcal{W} is defined as:

$$(1) \quad \mathcal{W}_{t-1}(j) \equiv E_{t-1} \sum_{\tau=t}^{\infty} \beta^{\tau-t} [\ln C_{\tau}(j) - \kappa \ell_{\tau}(j)] \quad 0 < \beta < 1, \kappa > 0$$

where β is the discount rate. The instantaneous utility is a positive function of the consumption index $C(j)$ and a negative function of labor effort $\ell(j)$. Foreign agents' preferences are similarly defined: the discount rate is the same as in the Home country, while κ^* in the Foreign country need not coincide with κ in the Home country.

For each agent j in the Home country, the consumption indexes of Home and Foreign brands are defined as:

$$(2) \quad C_{H,t}(j) \equiv \left[\int_0^1 C_t(h,j)^{\frac{\theta-1}{\theta}} dh \right]^{\frac{\theta}{\theta-1}}, \quad C_{F,t}(j) \equiv \left[\int_0^1 C_t(f,j)^{\frac{\theta^*-1}{\theta^*}} df \right]^{\frac{\theta^*}{\theta^*-1}} \quad \theta, \theta^* > 1.$$

where $C_t(h,j)$ and $C_t(f,j)$ are respectively consumption of Home brand h and Foreign brand f by Home agent j at time t . Each Home brand is an imperfect substitute for all other Home brands, with constant elasticity of substitution θ . Similarly the elasticity of substitution among Foreign brands is θ^* . The con-

sumption indexes in the Foreign country, $C_{H,t}^*(j^*)$ and $C_{F,t}^*(j^*)$, are analogously defined.

We assume that the elasticity of substitution between Home and Foreign types is one, thus smaller than the elasticity of substitution between brands of the same type of good produced in each country, θ and θ^* . Under this assumption the consumption baskets of individuals j and j^* can be written as a Cobb-Douglas function of the Home and Foreign consumption indexes:

$$(3) \quad C_t(j) \equiv C_{H,t}(j)^\gamma C_{F,t}(j)^{1-\gamma}, \quad C_t^*(j^*) \equiv C_{H,t}^*(j^*)^\gamma C_{F,t}^*(j^*)^{1-\gamma} \quad 0 < \gamma < 1$$

where the weights γ and $1 - \gamma$ are identical across countries.

We denote the prices of brands h and f in the Home market (thus expressed in the Home currency) as $p(h)$ and $p(f)$, and the prices in the Foreign market (in Foreign currency) as $p^*(h)$ and $p^*(f)$. Given the prices of brands, we can derive the utility-based price indexes P_H , P_F , P and their Foreign analogs.⁴ In particular, the utility-based CPIs are:

$$(4) \quad P_t = \frac{P_{H,t}^\gamma P_{F,t}^{1-\gamma}}{\gamma_W}, \quad P_t^* = \frac{(P_{H,t}^*)^\gamma (P_{F,t}^*)^{1-\gamma}}{\gamma_W} \quad \gamma_W \equiv \gamma^\gamma (1-\gamma)^{1-\gamma}.$$

Each brand h is produced by a single Home agent and sold in both countries under conditions of monopolistic competition. Output is denoted Y . Technology is linear in household's h labor, $\ell(h)$. The resource constraint for brand h is:

$$(5) \quad Y_t(h) = \frac{\ell_t(h)}{\alpha_t} \geq \int_0^1 C_t(h, j) dj + \int_0^1 C_t^*(h, j^*) dj^*$$

where α is a country-specific productivity shock. The resource constraint for Foreign brand f is similarly defined as a function of the productivity shock in the Foreign country, α^* .

Home agent j receives a revenue $R(j)$ from the sale of the good she produces. Since each agent is the sole producer of a specific brand, we associate individual j with brand h . The sales revenue of agent j , expressed in Home currency, is:

$$(6) \quad R_t(j) \equiv p_t(h) \int_0^1 C_t(h, j) dj + \mathcal{E}_t p_t^*(h) \int_0^1 C_t(h, j^*) dj^*$$

⁴For instance, the utility-based price index $P_{H,t}$ is defined as the minimum expenditure required to buy one unit of the composite good $C_{H,t}$ and is derived as $P_{H,t} = \left[\int_0^1 p_t(h)^{1-\theta} dh \right]^{\frac{1}{1-\theta}}$.

where \mathcal{E} is the nominal exchange rate, defined as Home currency per unit of Foreign currency.

Home agents hold two international bonds, B_H and B_F , denominated in Home and Foreign currency, respectively.⁵ Both international bonds are in zero net supply. The individual flow budget constraint for agent j in the Home country is:

$$(7) \quad B_{H,t+1}(j) + \mathcal{E}_t B_{F,t+1}(j) \leq (1 + i_t) B_{H,t}(j) + (1 + i_t^*) \mathcal{E}_t B_{F,t}(j) \\ + R_t(j) - \int_0^1 p_t(h) C_t(h, j) dh - \int_0^1 p_t(f) C_t(f, j) df$$

In the expression above the short-term nominal rates i_t and i_t^* are paid at the beginning of period t and are known at time $t - 1$. The two short-term rates are directly controlled by the national governments.⁶ Similar expressions hold for the Foreign country, after associating household j^* with brand f .

Taking prices as given, Home agent j maximizes (1) subject to (7) with respect to consumption and asset holdings. A similar optimization problem is solved by Foreign agent j^* . In our analysis below we focus on symmetric equilibria in which at some initial point in time $t = 0$ all agents have zero net financial wealth. As shown in Corsetti and Pesenti [2001a, b], in equilibrium both wealth and the current account are endogenously zero at *any* subsequent point in time: Home imports from Foreign are always equal in value to Foreign imports from Home. Since agents are equal within countries (though not necessarily symmetric across countries) we can drop the indexes j and j^* and interpret all variables in per-capita (or aggregate) terms. As trade and the current account are always balanced, countries consume precisely their aggregate sales revenue:

$$(8) \quad R_t - P_t C_t = 0, \quad R_t^* - P_t^* C_t^* = 0.$$

⁵We adopt the notation of Obstfeld and Rogoff [1996, ch.10]. Specifically, our timing convention has $B_{H,t}(j)$ and $B_{F,t}(j)$ as agent j 's nominal bonds accumulated during period $t - 1$ and carried over into period t .

⁶The adoption of a cash-less economy framework is merely motivated in terms of analytical convenience. All the results of this model go through when money demand is explicitly considered. Liquidity demand could easily be introduced in the model through a cash-in-advance constraint of the type $M_t(j) = P_t C_t(j)$ and $M_t^*(j) = P_t^* C_t^*(j^*)$, or through a money-in-utility-function framework. In both cases the money stock would be residually determined in equilibrium as a function of nominal interest rates.

B Nominal rigidities, exchange rate pass-through and price setting

It is assumed that individual firms set the nominal price of their product one period in advance, and stand ready to meet demand at given prices for one period. In terms of our notation, Home firms selling in the Home market choose $p_t(h)$ at time $t - 1$ by maximizing (1) subject to (7) with respect to $p_t(h)$, accounting for (5) and (6).⁷ In an environment in which national agents are symmetric (i.e. all prices $p_t(h)$ are equal and $p_t(h) = P_{H,t}$), Home firms optimally set domestic prices equal to expected nominal marginal costs MC_t (equal to the nominal wage rates $\kappa\alpha_t P_t C_t$ in our model), augmented by the equilibrium markup $\theta/(\theta - 1)$:

$$(9) \quad p_t(h) = P_{H,t} = \frac{\theta}{\theta - 1} E_{t-1} [\kappa\alpha_t P_t C_t] = \frac{\theta}{\theta - 1} E_{t-1} [MC_t]$$

Home firms selling abroad also set nominal prices one period in advance. Different from most models in the literature, however, we do not impose *a priori* the restriction that export prices are set in Home currency, implying that all unexpected fluctuations in the exchange rate are ‘passed through’ one-to-one onto export prices in Foreign currency (in the literature this scenario is referred to as ‘Producer Currency Pricing’ or PCP). At the same time, we do not impose *a priori* the opposite restriction that export prices are set in Foreign currency, implying that Foreign-currency prices of Home goods do not respond at all to unexpected exchange rate fluctuations (i.e. the case of ‘Local Currency Pricing’ or LCP). We consider instead the more general case in which Home firms preset export prices in Foreign currency but are able to modify them after observing

⁷Standard intratemporal optimization yields agent j ’s demand for brand h as a function of the relative price of h and total consumption of Home goods:

$$C_t(h, j) = (p_t(h)/P_{H,t})^{-\theta} C_{H,t}(j)$$

Similar expressions can be derived for the other brands. Accounting for the demand functions above, we can rewrite the resource constraint for agents h in the Home country as:

$$\ell(h)/\alpha_t \geq p_t(h)^{-\theta} P_{H,t}^\theta C_{H,t} + p_t^*(h)^{-\theta} (P_{H,t}^*)^\theta C_{H,t}^*$$

and sales revenue as:

$$R_t(j) = p_t(h)^{1-\theta} P_{H,t}^\theta C_{H,t} + \varepsilon_t p_t^*(h)^{1-\theta} (P_{H,t}^*)^\theta C_{H,t}^*$$

exchange rate changes. In our setup, the extent to which the Foreign-currency prices of Home exports adjust — contingent on the realization of the exchange rate — is a choice variable, determined by Home firms at time $t - 1$. In other words, the elasticity of exchange rate pass-through can endogenously be zero (as in the LCP case), one (as in the PCP case), or any intermediate number.

Formally, for any given elasticity of pass-through in the Foreign market η_t^* , Foreign-currency prices of Home brands are:

$$(10) \quad p_t^*(h) \equiv \frac{\tilde{p}_t(h)}{\mathcal{E}_t^{\eta_t^*}} \quad 0 \leq \eta_t^* \leq 1$$

where $\tilde{p}_t(h)$ is the predetermined component of the Foreign-currency price of good h that is not adjusted to variations of the exchange rate during period t .⁸ Home firms choose $\tilde{p}_t(h)$ and η_t^* one period in advance at time $t - 1$ in order to maximize (1), while the actual $p_t^*(h)$ depends on the realization of the exchange rate at time t . Clearly, depending on firms' choice, the degree of pass-through may vary over time.⁹

In equilibrium we obtain:

$$(11) \quad p_t^*(h) = P_{H,t}^* = \frac{\tilde{p}_t(h)}{\mathcal{E}_t^{\eta_t^*}} = \frac{\theta}{\theta - 1} \frac{E_{t-1} \left[MC_t / \mathcal{E}_t^{1-\eta_t^*} \right]}{\mathcal{E}_t^{\eta_t^*}}$$

Interpreting (11), domestic firms set $\tilde{p}_t(h)$ such that at the margin the expected disutility from an increase in labor effort is equal to the expected utility from consumption financed by additional sales revenue.¹⁰

Analogous expressions can be derived for the prices set by Foreign firms in the Foreign and the Home market. In the case of Foreign exports the notation

⁸For instance, if $\eta^* = 1$, pass-through in the Foreign country is complete — as in the PCP case. If $\eta^* = 0$, we have $p_t^*(h) = \tilde{p}_t(h)$ which coincides with the price chosen by the Home producer in the LCP case.

⁹The model could be easily extended to encompass the case in which the pass-through elasticity is a non-linear function of the exchange rate (e.g., η^* is close to zero for small changes of the exchange rate \mathcal{E} but close to one for large exchange rate fluctuations). The key results of our analysis would remain unchanged.

¹⁰The optimal export price solves:

$$0 = E_{t-1} \left[-\kappa \theta p_t^*(h)^{-\theta} \tilde{p}_t(h)^{-1} (P_{H,t}^*)^\theta \alpha_t C_{H,t}^* \right. \\ \left. + (\theta - 1) p_t^*(h)^{-\theta} \mathcal{E}_t^{1-\eta_t^*} (P_{H,t}^*)^\theta C_{H,t}^* / P_t C_t \right]$$

and equilibrium conditions imply: $P_{H,t}^* C_{H,t}^* = \gamma P_t^* C_t^* = (1 - \gamma) P_t C_t / \mathcal{E}_t$.

is:

$$(12) \quad p_t(f) = \tilde{p}_t^*(f) \mathcal{E}_t^{\eta_t}, \quad 0 \leq \eta_t \leq 1$$

where the degree of pass-through in the Home country, η_t , need not be equal to that in the Foreign country, η_t^* . The optimal pricing strategy is such that:

$$(13) \quad P_{F,t}^* = \frac{\theta^*}{\theta^* - 1} E_{t-1} [MC_t^*], \quad P_{F,t} = \frac{\theta^*}{\theta^* - 1} \mathcal{E}_t^{\eta_t} E_{t-1} [MC_t^* \mathcal{E}_t^{1-\eta_t}]$$

Clearly, Home firms are willing to supply goods at given prices as long as their *ex-post* markup does not fall below one:

$$(14) \quad P_{H,t} \geq MC_t, \quad P_{H,t}^* \geq \frac{MC_t}{\mathcal{E}_t}$$

Otherwise, agents would be better off by not accommodating shocks to demand. In what follows, we restrict the set of shocks so that the ‘participation constraint’ (14) and its Foreign analog are never violated.

C Monetary policy and the closed-form solution of the model

Before deriving the general solution of the model, it is helpful to introduce an index of monetary stance μ_t such that:

$$(15) \quad \frac{1}{\mu_t} = \beta(1 + i_{t+1}) E_t \left(\frac{1}{\mu_{t+1}} \right)$$

or, integrating forward,

$$(16) \quad \frac{1}{\mu_t} = E_t \lim_{N \rightarrow \infty} \beta^N \frac{1}{\mu_{t+N}} \prod_{\tau=0}^{N-1} (1 + i_{t+\tau+1})$$

This expression shows that the index of monetary stance is a function of both current and future expected Home nominal interest rates: Home monetary easing at time t (a higher μ_t) reflects either an interest rate cut at time t (i.e., a lower i_{t+1}) or expectations of future interest rates cuts. A similar expression holds for the Foreign country.

Note that in equilibrium μ_t is equal to $P_t C_t$ (and μ_t^* is equal to $P_t^* C_t^*$): a monetary expansion delivers increased nominal spending.¹¹ A *monetary union*

¹¹This result can be obtained by comparing (15) with the Home Euler equation under logarithmic utility, i.e. $1 = \beta(1 + i_{t+1}) E_t (P_t C_t / P_{t+1} C_{t+1})$.

in our framework is defined as a regime in which $i_{t+1} = i_{t+1}^*$ for all t . If both countries adopt the same *numeraire*, this implies $\mu_t = \mu_t^*$.

Table 1 presents the general solution of the model, in which all endogenous variables (17) through (26) are expressed in closed form as functions of real shocks (α_t and α_t^*) and monetary stances (μ_t and μ_t^*).¹²

Interpreting Table 1: since the equilibrium current account is always balanced (see (8) above) and the demand for imports is proportional to nominal expenditures $P_t C_t$ and $P_t^* C_t^*$, the nominal exchange rate \mathcal{E}_t in (17) is proportional to $P_t C_t / P_t^* C_t^*$, that is, a function of the relative monetary stance.¹³ The relations (18) link marginal costs to macroeconomic shocks and monetary policy. Domestic prices of domestic goods are predetermined according to (19) and (22), while import prices vary with the exchange rate, depending on the degree of exchange rate pass-through according to (20) and (21). Equilibrium consumption is determined in (23) and (24). Finally employment and output¹⁴ levels are determined according to (25) and (26).

III Optimal exchange rate pass-through for *given* monetary policy

What is the optimal degree of exchange rate pass-through onto export prices of Home goods in the Foreign market? Taking monetary stances and policy rules as given, Home firms choose η^* as to maximize (1). In a symmetric environment

¹²Algebraic details can be found in the Appendix of Corsetti and Pesenti [2001a]. Note that the solution does not hinge upon any specific assumption or restriction on the nature of the shocks besides the participation constraints (14) and their Foreign analogues.

¹³Recall that monetary stances μ_t and μ_t^* depend on current and future short-term nominal interest rates. Hence the exchange rate, which is proportional to the ratio of μ_t to μ_t^* , is a forward-looking variable.

¹⁴Recall that national outputs Y_t and Y_t^* are respectively equal to ℓ_t / α_t and ℓ_t^* / α_t^* .

Table 1: The closed-form solution of the model

$$(17) \quad \mathcal{E}_t = \frac{1 - \gamma}{\gamma} \frac{\mu_t}{\mu_t^*}$$

$$(18) \quad MC_t = \kappa \alpha_t \mu_t \qquad MC_t^* = \kappa^* \alpha_t^* \mu_t^*$$

$$(19) \quad P_{H,t} = \frac{\theta}{\theta - 1} E_{t-1} (MC_t)$$

$$(20) \quad P_{F,t} = \frac{\theta^*}{\theta^* - 1} \mathcal{E}_t^{\eta_t} E_{t-1} [MC_t^* \mathcal{E}_t^{1-\eta_t}]$$

$$(21) \quad P_{H,t}^* = \frac{\theta}{\theta - 1} \frac{E_{t-1} [MC_t / \mathcal{E}_t^{1-\eta_t^*}]}{\mathcal{E}_t^{\eta_t^*}}$$

$$(22) \quad P_{F,t}^* = \frac{\theta^*}{\theta^* - 1} E_{t-1} (MC_t^*)$$

$$(23) \quad C_t = \frac{\gamma_W \left(\frac{\theta - 1}{\theta}\right)^\gamma \left(\frac{\theta^* - 1}{\theta^*}\right)^{1-\gamma} \mu_t \mathcal{E}_t^{-\eta_t(1-\gamma)}}{[E_{t-1} (MC_t)]^\gamma [E_{t-1} (MC_t^* \mathcal{E}_t^{1-\eta_t})]^{1-\gamma}}$$

$$(24) \quad C_t^* = \frac{\gamma_W \left(\frac{\theta - 1}{\theta}\right)^\gamma \left(\frac{\theta^* - 1}{\theta^*}\right)^{1-\gamma} \mu_t^* \mathcal{E}_t^{\eta_t^* \gamma}}{[E_{t-1} (MC_t / \mathcal{E}_t^{1-\eta_t^*})]^\gamma [E_{t-1} (MC_t^*)]^{1-\gamma}}$$

$$(25) \quad \ell_t = \left(\frac{\theta - 1}{\theta \kappa}\right) \left[\gamma \frac{MC_t}{E_{t-1} (MC_t)} + (1 - \gamma) \frac{MC_t / \mathcal{E}_t^{1-\eta_t^*}}{E_{t-1} [MC_t / \mathcal{E}_t^{1-\eta_t^*}]} \right]$$

$$(26) \quad \ell_t^* = \left(\frac{\theta^* - 1}{\theta^* \kappa^*}\right) \left[(1 - \gamma) \frac{MC_t^*}{E_{t-1} (MC_t^*)} + \gamma \frac{MC_t^* \mathcal{E}_t^{1-\eta_t}}{E_{t-1} [MC_t^* \mathcal{E}_t^{1-\eta_t}]} \right]$$

with $p^*(h) = P_H^*$ the first order condition is:¹⁵

$$(27) \quad 1 = \frac{\theta \kappa}{\theta - 1} \frac{E_{t-1} [(\alpha_t P_t^* C_t^* \ln \mathcal{E}_t) / P_{H,t}^*]}{E_{t-1} [\ln \mathcal{E}_t]} \frac{\gamma}{1 - \gamma}$$

Comparing (27) with (11), it follows that the optimal pass-through η^* is such that:

$$(28) \quad E_{t-1} [\alpha_t \mu_t^* \mathcal{E}_t^{\eta_t^*}] = \frac{E_{t-1} [\alpha_t \mu_t^* \mathcal{E}_t^{\eta_t^*} \ln \mathcal{E}_t]}{E_{t-1} [\ln \mathcal{E}_t]}$$

or, rearranging:

$$(29) \quad Cov_{t-1} [MC_t / \mathcal{E}_t^{1-\eta_t^*}, \ln \mathcal{E}_t] = 0$$

This is a key condition. At an optimum, the (reciprocal of the) markup in the export market must be uncorrelated with the (log of the) exchange rate. Trivially, if \mathcal{E} is constant or fully anticipated, *any* degree of pass-through is consistent with the previous expression. But if \mathcal{E} is not perfectly predictable, the optimal degree of pass-through will depend on the expected monetary policies and the structure of the shocks.

This is because, in equilibrium, Home *ex-post* real profits in the Foreign market¹⁶ are proportional to $\tilde{p}_t(h) - MC_t / \mathcal{E}_t^{1-\eta_t^*}$, that is, they are a concave

¹⁵The optimal pass-through solves:

$$0 = E_{t-1} \left[-\kappa \tilde{p}_t(h)^{-\theta} \ln \mathcal{E}_t \exp \{ \eta_t^* \theta \ln \mathcal{E}_t \} (P_{H,t}^*)^\theta \alpha_t C_{H,t}^* \right. \\ \left. - \mathcal{E}_t \tilde{p}_t(h)^{1-\theta} (1-\theta) \ln \mathcal{E}_t \exp \{ -\eta_t^* (1-\theta) \ln \mathcal{E}_t \} (P_{H,t}^*)^\theta C_{H,t}^* / P_t C_t \right]$$

and equilibrium conditions imply: $P_{H,t}^* C_{H,t}^* = \gamma P_t^* C_t^* = (1-\gamma) P_t C_t / \mathcal{E}_t$.

¹⁶Denoting with $\Pi_t(h)$ Home agent h 's *ex-post* real profits from selling in the Foreign market, we can write:

$$\Pi_t(h) = Q_{t-1,t} \left(\mathcal{E}_t \tilde{p}_t(h) \mathcal{E}_t^{-\eta^*} - MC_t \right) \left(\frac{\tilde{p}_t(h)}{\tilde{P}_{H,t}} \right)^{-\theta} C_{H,t}^*$$

where Q is the pricing kernel and \tilde{P}_H the predetermined export price, defined as:

$$Q_{t-1,t} \equiv \beta \frac{P_{t-1} C_{t-1}}{P_t C_t}, \quad P_{H,t}^* \equiv \tilde{P}_{H,t} \mathcal{E}_t^{-\eta^*}$$

Using the equilibrium expression for C_H^* and Q we obtain:

$$\Pi_t(h) = \beta (1-\gamma) \frac{\mu_{t-1}}{\tilde{P}_{H,t}} \left(\tilde{p}_t(h) - \kappa \alpha_t \mu_t / \mathcal{E}_t^{1-\eta^*} \right) \left(\frac{\tilde{p}_t(h)}{\tilde{P}_{H,t}} \right)^{-\theta}$$

Note that expressions 11 and 27 can be obtained by maximizing $E_{t-1} \Pi_t(h)$ with respect to $\tilde{p}_t(h)$ and η_t^* . See the discussion in Corsetti and Pesenti [2001a].

function of \mathcal{E} for $\eta^* < 1$.¹⁷ This implies that, keeping everything else constant, exchange rate shocks reduce expected profits from exports. In general, however, to assess the overall exposure of profits to exchange rate uncertainty it is crucial to know whether or not the underlying shocks make unit export revenue and marginal costs co-vary positively. In fact, if MC is high in those states of nature in which $\tilde{p}\mathcal{E}^{1-\eta^*}$ is high as well, exchange rate uncertainty has little or no effect on profits. When the firm chooses the degree of pass-through, it equates the ‘marginal costs’ of imperfect pass-through, in terms of reduced expected revenue, with the ‘marginal benefits’ of imperfect pass through, in terms of increased covariance between revenue and marginal costs. By the same token, the optimal pass-through chosen by Foreign firms selling in the Home market requires:

$$(30) \quad Cov_{t-1} \left(MC_t^* \mathcal{E}_t^{1-\eta_t^*}, \ln(1/\mathcal{E}_t) \right) = 0.$$

To build intuition, suppose that there are no productivity shocks and the only source of uncertainty is exogenous monetary volatility. If μ is the only source of uncertainty, condition (29) becomes:

$$(31) \quad Cov_{t-1} \left[\mu_t^{\eta_t^*}, \ln \mu_t \right] = 0$$

which is solved by $\eta_t^* = 0$. Home monetary shocks affect both Home marginal costs and the exchange rate: \mathcal{E} depreciates in those states of nature in which marginal costs increase. By setting $\eta_t^* = 0$ Home firms insure that export sales revenue and marginal costs move in parallel, leaving the export markup unaffected. Instead, if the only source of uncertainty is μ^* , condition (29) becomes:

$$(32) \quad Cov_{t-1} \left[(\mu_t^*)^{\eta_t^*-1}, -\ln \mu_t^* \right] = 0$$

which is solved by $\eta_t^* = 1$. Home marginal cost are uncorrelated with the exchange rate. By choosing full pass-through and letting export prices absorb exchange rate changes, Home firms can insulate their export sales revenue from currency fluctuations and avoid any uncertainty of markup and profitability in the Foreign market. Note that these examples shed light on the reason why

¹⁷This result does not rely on the linearity of labor effort disutility. Suppose the latter is nonlinear, say in the form $-\ell^v/v$. It can be shown that profits are concave in the nominal exchange rate for any $v \geq 1$.

countries with high and unpredictable monetary volatility should also exhibit a high degree of pass-through, and vice versa — a view recently expressed by Taylor [2000].¹⁸

The same intuition carries over to the case in which there is both monetary and real uncertainty. In this case, patterns of endogenous intermediate pass-through can emerge, as the following example illustrates. If the Home monetary authority adopted the policy $\mu_t = (\alpha_t^2 \mu_t^*)^{-1}$, then it would be optimal for Home firms to choose $\eta_t^* = 0.5$. Abroad, we would need $MC_t^* \mathcal{E}_t^{1-\eta_t}$ to be uncorrelated with the exchange rate. This would be the case, for instance, if $\mu_t^* = \alpha_t^4 / (\alpha_t^*)^5$ and $\eta_t = 0.6$.

IV Optimal monetary policy for *given* exchange rate pass-through

Consider now the policymakers' problem in a world Nash equilibrium where national monetary authorities are able to commit to preannounced rules, taking the pass-through coefficients as given.¹⁹ The Home monetary authority seeks to maximize the indirect utility of the Home representative consumer (1) with respect to $\{\mu_{t+\tau}\}_{\tau=0}^{\infty}$, given $\{\mu_{\tau}^*, \alpha_{\tau}, \alpha_{\tau}^*, \eta_{\tau}, \eta_{\tau}^*\}_{\tau=t}^{\infty}$. The Foreign authority faces a similar problem. Table 2 presents the closed-form reaction functions, the solution of which is the global Nash equilibrium.²⁰ To facilitate an intuitive understanding of the formulas, each reaction function is written in two ways: as a function of marginal costs and markups, or as a function of employment gaps and deviations from the law of one price.

¹⁸When monetary policy is exogenous (suboptimal) and firms are only allowed to choose between zero and 100 percent pass-through (that is, between local-currency and producer-currency pricing), the results above are consistent with the analysis of Devereux and Engel [2001] and Bacchetta and van Wincoop [2001].

¹⁹For an analysis of optimal monetary behavior under discretion see Corsetti and Pesenti [2001 a].

²⁰As well known (see e.g. Woodford [2002]), in this class of models optimal monetary rules such as the reaction functions of Table 2 do not provide a nominal anchor to pin down nominal expectations. The issue can be addressed by assuming that governments set national nominal anchors (such as target paths for P_H and P_F^*) and credibly threaten to tighten monetary policy if the price of domestic goods deviates from the target. Such threats, however, are never implemented in equilibrium as discussed in the Appendix of Corsetti and Pesenti [2001a].

Table 2: Monetary authorities' optimal reaction functions

$$\begin{aligned}
 \gamma + (1 - \gamma)(1 - \eta_t) &= \frac{\gamma MC_t}{E_{t-1}(MC_t)} + \frac{(1 - \gamma)(1 - \eta_t) MC_t^* \mathcal{E}_t^{1 - \eta_t}}{E_{t-1}(MC_t^* \mathcal{E}_t^{1 - \eta_t})} \\
 (33) \qquad \qquad \qquad &= \frac{\gamma \frac{\theta \kappa}{\theta - 1} \ell_t}{\gamma + (1 - \gamma) \frac{P_{H,t}}{\mathcal{E}_t P_{H,t}^*}} + \frac{(1 - \gamma)(1 - \eta_t) \frac{\theta^* \kappa^*}{\theta^* - 1} \ell_t^*}{\gamma + (1 - \gamma) \frac{P_{F,t}}{\mathcal{E}_t P_{F,t}^*}}
 \end{aligned}$$

$$\begin{aligned}
 1 - \gamma + \gamma(1 - \eta_t^*) &= \frac{(1 - \gamma) MC_t^*}{E_{t-1}(MC_t^*)} + \frac{\gamma(1 - \eta_t^*) MC_t / \mathcal{E}_t^{1 - \eta_t^*}}{E_{t-1}(MC_t / \mathcal{E}_t^{1 - \eta_t^*})} \\
 (34) \qquad \qquad \qquad &= \frac{(1 - \gamma) \frac{\theta^* \kappa^*}{\theta^* - 1} \ell_t^*}{1 - \gamma + \gamma \frac{\mathcal{E}_t P_{F,t}^*}{P_{F,t}}} + \frac{\gamma(1 - \eta_t^*) \frac{\theta \kappa}{\theta - 1} \ell_t}{1 - \gamma + \gamma \frac{\mathcal{E}_t P_{H,t}}{P_{H,t}}}
 \end{aligned}$$

An optimal policy requires that the Home monetary stance be eased (μ increases) in response to a positive domestic productivity shock (α falls). Absent a policy reaction, a positive productivity shock would create both an output and an employment gap. In fact, ℓ would fall below $(\theta - 1) / (\theta \kappa)$. Actual output Y would not change, but potential output, defined as the equilibrium output with fully flexible prices, would increase. In light of this, optimal monetary policy leans against the wind and moves to close the employment and output gaps.

In general, however, the optimal response to a Home productivity shock will not close the output gap completely. Home stabilization policy, in fact, induces fluctuations in the exchange rate that add uncertainty to Foreign firms' sales revenue from the Home market. We have seen above that exporters' profits are a concave function of the nominal exchange rate, so that — other things

being equal — exchange rate shocks reduce Foreign firms' expected profits in the Home market. The elasticity of Foreign profits relative to the exchange rate is decreasing in $\tilde{p}^*(f)$ when pass-through in the Home market is incomplete. Then, charging a higher price $\tilde{p}^*(f)$ is a way for Foreign exporters to reduce the sensitivity of their export profits to exchange rate variability. But the higher average export prices charged by Foreign firms translate into higher average import prices in the Home country, reducing Home residents' purchasing power and welfare.

This is why the Home monetary stance required to close the domestic output gap is not optimal when pass-through is incomplete. Relative to such a stance, domestic policymakers can improve utility by adopting a policy that equates, at the margin, the benefit from exchange rate flexibility (that is, from keeping domestic output close to its potential level) with the loss from exchange rate volatility (that is, the fall in purchasing power and real wealth due to higher average import prices).

As long as η is below one, the Home monetary stance tightens when productivity worsens abroad and loosens otherwise. Rising costs abroad (an increase in α^*) lower the markup of Foreign goods sold at Home. If Home policymakers were not expected to stabilize the markup by raising rates and appreciating the exchange rate, Foreign firms would charge higher prices onto Home consumers. Only when $\eta = 1$ do Foreign firms realize that any attempt by the Home authorities to stabilize the markup is bound to fail as both P_F and the exchange rate fall in the same proportion.

With complete pass-through in both countries, the policies in a Nash equilibrium are:

$$(35) \quad \mu_t = \frac{E_{t-1}(\alpha_t \mu_t)}{\alpha_t} \quad \mu_t^* = \frac{E_{t-1}(\alpha_t^* \mu_t^*)}{\alpha_t^*}$$

Monetary authorities optimally stabilize marginal costs and markups. Output gaps are fully closed and employment remains unchanged at the potential level $(\theta - 1) / (\theta \kappa)$ or $(\theta^* - 1) / (\theta^* \kappa^*)$. Both domestic and global consumption endogenously co-move with productivity shocks. Thus, the Nash optimal monetary policy leads to the same allocation that would prevail were prices fully flexible. This result restates the case for flexible exchange rates made by Friedman [1953]: even without price flexibility, monetary authorities can engineer the right adjustment in relative prices through exchange rate movements. In our

model with PCP, expenditure-switching effects makes exchange rate and price movements perfect substitutes.

The Nash equilibrium will however *not* coincide with a flex-price equilibrium when the pass-through is less than perfect in either market. Consider, for instance, the case of LCP. In this case the optimal monetary policy in each country cannot be inward-looking, and must instead respond *symmetrically* to shocks anywhere in the world economy. The optimal monetary policy in Table 2 can in fact be written as:

$$(36) \quad \mu_t = \left[\gamma \frac{\alpha_t}{E_{t-1}(\alpha_t \mu_t)} + (1 - \gamma) \frac{\alpha_t^*}{E_{t-1}(\alpha_t^* \mu_t)} \right]^{-1}$$

$$(37) \quad \mu_t^* = \left[\gamma \frac{\alpha_t}{E_{t-1}(\alpha_t \mu_t^*)} + (1 - \gamma) \frac{\alpha_t^*}{E_{t-1}(\alpha_t^* \mu_t^*)} \right]^{-1},$$

expressions which imply $\mu = \mu^*$.

In our model, the exchange rate is a function of the relative monetary stance μ/μ^* . Our analysis then suggests that exchange rate volatility will be higher in a world economy close to purchasing power parity, and lower in a world economy where deviations from the law of one price are large.²¹ In fact, if the exposure of firms' revenue to exchange rate fluctuations is limited, inward-looking policymakers assign high priority to stabilizing domestic output and prices, with 'benign neglect' of exchange rate movements. Otherwise, policymakers 'think globally', taking into account the repercussions of exchange rate volatility on consumer prices; hence, the monetary stances in the world economy come to mimic each other, reducing currency volatility.

The characterization of *optimal monetary unions* is a simple corollary of the analysis above. We define a monetary union $\mu_t = \mu_t^*$ as optimal if the single monetary stance μ_t is optimal for both countries. It is straightforward to show that when shocks are perfectly correlated, the optimal allocation is such that $MC_t = E_{t-1}(MC_t)$ and $MC_t^* = E_{t-1}(MC_t^*)$ regardless of the degree of pass-through. Optimal monetary policies support a fixed exchange rate regime and an optimal monetary union while fully closing the national output gaps. If shocks are asymmetric, a monetary union is optimal only when both countries find it optimal to choose a symmetric monetary stance, that is, when pass-through is zero worldwide.

²¹This result was first stressed by Devereux and Engel [2000].

V Optimal exchange rate pass-through *and* monetary policy in equilibrium

To recapitulate: Home and Foreign firms choose the levels of pass-through η_t^* and η_t on the basis of their information at time $t - 1$ regarding marginal costs and exchange rates at time t , by solving (29) and (30). Home and Foreign monetary authorities take the levels of pass-through η_t^* and η_t as given and determine their optimal monetary stances by solving the conditions (33) and (34). We now consider the joint determination of μ_t , μ_t^* , η_t and η_t^* satisfying the four equations above in the non-trivial case in which the shocks α_t and α_t^* are asymmetric.

The following allocation is an equilibrium:

$$(38) \quad MC_t = E_{t-1}(MC_t), \quad MC_t^* = E_{t-1}(MC_t^*), \quad \eta_t = \eta_t^* = 1$$

Purchasing power parity holds and there is full pass-through of exchange rate changes into prices. Monetary policies fully stabilize the national economies by closing output and employment gaps. Exchange rates are highly volatile, their conditional variance being proportional to the volatility of α_t^*/α_t . We will refer to this equilibrium as an *optimal float* (OF).

The logic underlying the OF case can be understood as follows. Suppose Foreign firms selling in the Home market choose $\eta = 1$ and let Home-currency prices of Foreign goods move one-to-one with the exchange rate, stabilizing their markups. Then the Home monetary authority chooses as a rule to stabilize Home output fully, no matter the consequences for the exchange rate (the volatility of which does *not* affect Foreign exporters' profitability and therefore does not affect, on average, the price of Foreign goods paid by Home consumers). Note that when $\eta = 1$, Home output stabilization implies that MC is constant, and therefore uncorrelated with the exchange rate. Home firms, then, will optimally set their pass-through abroad and choose $\eta^* = 1$ in order to stabilize the markup on their exports to the Foreign country. Since Home firms are now fully insulated from exchange rate fluctuations, the Foreign monetary authority optimally chooses to stabilize Foreign output with benign neglect of the exchange rate, so that MC^* is a constant. But in this case Foreign firms optimally choose $\eta = 1$ as we had assumed initially: the OF case is an equilibrium.

Consider now the following allocation:

$$(39) \quad \begin{aligned} 1 &= \gamma \frac{MC_t}{E_{t-1}(MC_t)} + (1 - \gamma) \frac{MC_t^*}{E_{t-1}(MC_t^*)}, & \mathcal{E}_t &= const \\ \eta_t &= \eta_t^* = 0 \end{aligned}$$

This is the LCP scenario brought to its extreme consequences: there is no pass-through of exchange rate changes into prices, but this hardly matters since the exchange rate is fixed! Optimal national monetary policies are fully symmetric, thus cannot insulate the national economies from asymmetric shocks: it is only on average that they stabilize the national economies by closing output and employment gaps — the most apparent case of an optimal currency area.

To see why the previous scenario an equilibrium, note that if Home and Foreign firms choose $\eta = \eta^* = 0$, Home and Foreign authorities are concerned with the price-distortions of exchange rate volatility. They will optimize over the trade-off between employment stability and consumers' purchasing power. While they choose their rules independently of each other, the rules they adopt are fully symmetric, thus leading to exchange rate stability. But if the exchange rate is constant, the choice of the pass-through is no longer a concern for Home and Foreign firms: zero pass-through is as good as a choice as any other level of η and η^* . Such weak preference implies that the monetary union is an equilibrium.

Would the two allocations above still be equilibria if national authorities could commit to coordinated policies, maximizing some weighted average of expected utility of the two national representative consumers? This is an important question, as one may argue that policymakers in a monetary union would set their rules together (taking private agents' pricing and pass-through strategies as given), rather than independently. By the same token, if there were large gains from cooperation in a floating exchange rate regime, there would also be an incentive for policymakers to design the optimal float in a coordinated way. One may conjecture that, once cooperative policies are allowed for, the equilibrium allocation becomes unique.

Remarkably, it turns out that the possibility of cooperation does not modify at all the conclusions of our analysis. It can be easily shown that optimal policy rules conditional on $\eta, \eta^* = 1$ are exactly the same in a Nash equilibrium and under coordination: there are no gains from cooperation in the PCP scenario

which replicates the flex-price allocation.²² Also, as shown in Corsetti and Pesenti [2001a], optimal policy rules conditional on $\eta, \eta^* = 0$ are identical with and without cooperation: since exchange rate fluctuations are the only source of international spillover, there cannot be gains from cooperation when non-cooperative monetary rules already imply stable exchange rates!²³ While there are policy spillovers for any intermediate degree of pass-through ($0 > \eta, \eta^* > 1$), they disappear in equilibrium under the two extreme pass-through scenarios. In the only two cases relevant for our equilibrium analysis, optimal monetary policy rules are exactly the same whether or not the two national policymakers cooperate.

VI Endogenous OCAs: Macroeconomics and welfare analysis

Can a monetary union be a self-validating OCA? Our model suggests yes. Policy commitment to monetary union — i.e., the adoption of the rules (36-37) — leads profit-maximizing producers to modify their pricing strategies, lowering their pass-through elasticities. Such behavioral change makes a monetary union optimal, even if macroeconomic fundamentals and the pattern of shocks (α_t and α_t^* in our framework) remain unchanged across regimes.

A crucial result is that, under an OCA, output correlation is higher than under the alternative OF equilibrium. In fact, under OF, monetary policies are such that employment in both countries is always stabilized (both *ex ante* and *ex post*) at the constant levels $\ell = (\theta - 1) / (\theta\kappa)$ and $\ell^* = (\theta^* - 1) / (\theta^*\kappa^*)$. This implies that output correlation under OF depends on the degree of asymmetry of the fundamental shocks:

$$(40) \quad Corr_t(Y_t^{OF}, Y_t^{*OF}) = Corr_t\left(\frac{\ell_t^{OF}}{\alpha_t}, \frac{\ell_t^{*OF}}{\alpha_t^*}\right) = Corr_t\left(\frac{1}{\alpha_t}, \frac{1}{\alpha_t^*}\right)$$

²²This result is stressed by Obstfeld and Rogoff [2000, 2002].

²³With LCP, expected utility at Home is identical to expected utility in the Foreign country up to a constant that does not depend on monetary policy. For any given shock, consumption increases by the same percentage everywhere in the world economy. Even if *ex post* labor moves asymmetrically (so that *ex-post* welfare is not identical in the two countries, as is the case under PCP), *ex ante* the expected disutility from labor is the same as under flexible prices.

Instead, in a monetary union, employment levels are functions of relative shocks:

$$(41) \quad \frac{\theta\kappa}{\theta-1}\ell_t^{OCA} = \frac{\alpha_t\mu_t^{OCA}}{E_{t-1}(\alpha_t\mu_t^{OCA})}, \quad \frac{\theta^*\kappa^*}{\theta^*-1}\ell_t^{*OCA} = \frac{\alpha_t^*\mu_t^{*OCA}}{E_{t-1}(\alpha_t^*\mu_t^{*OCA})}$$

where μ^{OCA} is the solution of the system (36)-(37). This implies that output levels are perfectly correlated:

$$(42) \quad Corr_t(Y_t^{OCA}, Y_t^{*OCA}) = Corr_t\left(\frac{\ell_t^{OCA}}{\alpha_t}, \frac{\ell_t^{*OCA}}{\alpha_t^*}\right) = Corr_t(\mu_t^{OCA}, \mu_t^{*OCA}) = 1$$

so that $Corr(Y^{OCA}, Y^{*OCA}) \geq Corr(Y^{OF}, Y^{*OF})$, consistent with the traditional characterization of OCAs.

It is possible to rank the OF and the OCA regimes in welfare terms. Focusing on the Home country, expected utility \mathcal{W} in (1) can be written as:

$$(43) \quad \mathcal{W}_{t-1} = \mathcal{W}_{t-1}^{flex} - \left[\gamma E_{t-1} \ln \left(\frac{E_{t-1}(\alpha_t\mu_t)}{\alpha_t\mu_t} \right) + (1-\gamma) E_{t-1} \ln \left(\frac{E_{t-1}[\alpha_t^*(\mu_t^*)^{\eta_t} \mu_t^{1-\eta_t}]}{\alpha_t^*(\mu_t^*)^{\eta_t} \mu_t^{1-\eta_t}} \right) \right]$$

where \mathcal{W}^{flex} is independent of monetary regime, and equal to the utility that consumers could expect to achieve if prices were fully flexible. By Jensen's inequality, the term in square brackets is always non-negative: expected utility with price rigidities is *never* above expected utility with flexible prices. At best, what monetary policy rules can do is to bridge the gap between the two.

Observe that under the OF equilibrium (38) the term in square bracket becomes zero and $\mathcal{W}^{OF} = \mathcal{W}^{flex}$. But this implies that $\mathcal{W}^{OF} \geq \mathcal{W}^{OCA}$, an inequality that holds with strong sign when shocks are asymmetric. It follows that an optimal currency area is always Pareto-inferior *vis-à-vis* a Friedman-style optimal flexible exchange rate arrangement.

VII Conclusion

In this paper we have shown the existence of two equilibria in a standard open-economy model where the alternative between pricing-to-market and law of one price depends on endogenous choices by firms. This result suggests that credible policy commitment to monetary union may lead to a change in pricing strategies, making a monetary union the optimal monetary arrangement in a self-validating way.

It is worth emphasizing that a common monetary policy is optimal because, for given producers' pricing strategies, the use of the exchange rate for stabilization purposes would entail excessive welfare costs, in the form of higher import prices and lower purchasing power across countries. Once a monetary union takes off and firms adapt their pricing strategies to the new environment, the best course of action for the monetary authorities is to avoid any asymmetric policy response to asymmetric shocks. As a result, even in the absence of the structural effects of monetary integration stressed by Frankel and Rose [1997, 1998] (e.g., even without an increase in intra-industry trade), the correlation of national outputs increases. In sum, the best institutional device to guarantee a credible policy commitment to a monetary union is to have the monetary union itself in place.

But our model also suggests that the argument for self-validating optimal currency areas could be used in the opposite direction, as an argument for self-validating optimal floating regimes. For a given pattern of macroeconomic disturbances, in fact, policy commitment to a floating regime may be the right choice despite the observed high synchronization of the business cycle across the countries participating in a monetary union: in equilibrium there will be an endogenous change in pricing strategies (with higher pass-through levels in all countries) which support floating rates as the optimal monetary option. In fact, the two institutional corner solutions for exchange rate regimes can be Pareto-ranked in welfare terms, leaving the optimal float the unambiguous winner.

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Appendix

Intratemporal allocation. For given consumption indices, utility-based price indices are derived as follows. P_H is the price of a consumption bundle of domestically produced goods that solves:

$$(A.1) \quad \min_{C(h,j)} \int_0^1 p(h)C(h,j)dh$$

subject to $C_H(j) = 1$. This constraint can be rewritten as:

$$(A.2) \quad [\theta / (\theta - 1)] \ln \left[\int_0^1 C(h,j)^{\frac{\theta-1}{\theta}} dh \right] = 0.$$

The solution is:

$$(A.3) \quad P_H = \left[\int_0^1 p(h)^{1-\theta} dh \right]^{\frac{1}{1-\theta}}.$$

Consider now the problem of allocating a given level of nominal expenditure \bar{Z} among domestically produced goods:

$$(A.4) \quad \max_{C(h,j)} C_H(j) \quad s.t. \quad \int_0^1 p(h)C(h,j)dh = \bar{Z}$$

Clearly, across any pair of goods h and h' , it must be true that:

$$(A.5) \quad \frac{C(h',j)}{C(h,j)} = \left(\frac{p(h')}{p(h)} \right)^{-\theta}$$

or, rearranging:

$$(A.6) \quad C(h,j)^{\frac{\theta-1}{\theta}} p(h')^{1-\theta} = C(h',j)^{\frac{\theta-1}{\theta}} p(h)^{1-\theta}$$

Integrating both sides of the previous expression we obtain:

$$(A.7) \quad \left(\int_0^1 C(h,j)^{\frac{\theta-1}{\theta}} p(h')^{1-\theta} dh' \right)^{\frac{\theta}{\theta-1}} = \left(\int_0^1 C(h',j)^{\frac{\theta-1}{\theta}} p(h)^{1-\theta} dh' \right)^{\frac{\theta}{\theta-1}}$$

so that:

$$(A.8) \quad C(h,j) \left(\int_0^1 p(h')^{1-\theta} dh' \right)^{\frac{\theta}{\theta-1}} = p(h)^{-\theta} \left(\int_0^1 C(h',j)^{\frac{\theta-1}{\theta}} dh' \right)^{\frac{\theta}{\theta-1}}$$

which can be rewritten as:

$$C(h,j) = (p(h)/P_H)^{-\theta} C_H(j)$$

Note that:

$$(A.9) \quad \int_0^1 p(h)C(h,j)dz = P_H C_H(j).$$

Intertemporal allocation. Consider the optimal allocation by Home agent j . The maximization problem can be written in terms of the following Lagrangian:

$$\begin{aligned} \mathcal{L}_t(j = h) \equiv & E_t \sum_{\tau=t}^{\infty} \beta^{\tau-t} \{ \ln C_{\tau}(j) - \kappa \ell_{\tau}(j) \\ & + \lambda_{\tau}(j) [-B_{H,\tau+1}(j) + (1 + i_{\tau})B_{H,\tau}(j) - \mathcal{E}_{\tau}B_{F,\tau+1}(j) \\ & + (1 + i_{\tau}^*)\mathcal{E}_{\tau}B_{F,\tau}(j) + R_{\tau}(j) - P_{H,\tau}C_{H,\tau}(j) - P_{F,\tau}C_{F,\tau}(j)] \} \end{aligned} \quad (\text{A.10})$$

where:

$$R_t(j) = p_t(h)^{1-\theta} P_{H,t}^{\theta} C_{H,t} + \mathcal{E}_t p_t^*(h)^{1-\theta} (P_{H,t}^*)^{\theta} C_{H,t}^* \quad (\text{A.11})$$

and:

$$\ell_t(h) = \alpha_t \left[p_t(h)^{-\theta} P_{H,t}^{\theta} C_{H,t} + p_t^*(h)^{-\theta} (P_{H,t}^*)^{\theta} C_{H,t}^* \right]. \quad (\text{A.12})$$

The first order conditions with respect to $C_{H,t}(j)$, $C_{F,t}(j)$, $B_{H,t+1}(j)$, and $B_{F,t+1}$ are, respectively:

$$\frac{\gamma}{C_{H,t}(j)} = \lambda_t(j) P_{H,t} \quad (\text{A.13})$$

$$\frac{1-\gamma}{C_{F,t}(j)} = \lambda_t(j) P_{F,t} \quad (\text{A.14})$$

$$\lambda_t(j) = \beta E_t \lambda_{t+1}(j) (1 + i_{t+1}) \quad (\text{A.15})$$

$$\mathcal{E}_t \lambda_t(j) = \beta E_t \mathcal{E}_{t+1} \lambda_{t+1}(j) (1 + i_{t+1}^*) \quad (\text{A.16})$$

First, we solve for the multiplier $\lambda_t(j)$. Take a geometric average of (A.13) and (A.14) with weights γ and $1 - \gamma$, respectively:

$$\gamma^{\gamma} (1 - \gamma)^{1-\gamma} = \lambda_t(j) P_{H,t}^{\gamma} P_{F,t}^{1-\gamma} C_{H,t}^{\gamma}(j) C_{F,t}^{1-\gamma}(j) \quad (\text{A.17})$$

This yields:

$$\lambda_t(j) = \frac{1}{P_t C_t(j)} \quad (\text{A.18})$$

so that, at the optimum, agent j 's demand for Home and Foreign consumption goods are a constant fraction of agent j 's total consumption expenditure:

$$P_t C_t(j) = \frac{1}{\gamma} P_{H,t} C_{H,t}(j) = \frac{1}{1-\gamma} P_{F,t} C_{F,t}(j). \quad (\text{A.19})$$

Using these expressions, it is easy to verify that

$$(A.20) \quad P_t C_t(j) = P_{H,t} C_{H,t}(j) + P_{F,t} C_{F,t}(j).$$

Combining (A.13), (A.14) and (A.15) the intertemporal allocation of consumption is determined according to the Euler equation:

$$(A.21) \quad \frac{1}{C_t(j)} = \beta(1 + i_{t+1}) E_t \left(\frac{P_t}{P_{t+1}} \frac{1}{C_{t+1}(j)} \right)$$

Define now the variable $Q_{t,t+1}(j)$ as:

$$(A.22) \quad Q_{t,t+1}(j) \equiv \beta \frac{P_t C_t(j)}{P_{t+1} C_{t+1}(j)}$$

The previous expression can be interpreted as agent j 's stochastic discount rate. If markets are complete at the national level (or, if national agents are symmetric) the stochastic discount rate is identical across individuals so that:

$$(A.23) \quad Q_{t,t+1}(j) = Q_{t,t+1}.$$

Comparing (A.22) with (A.15) and (A.16) we obtain:

$$(A.24) \quad E_t Q_{t,t+1} = \frac{1}{1 + i_{t+1}}, \quad E_t \left[Q_{t,t+1} \frac{\mathcal{E}_{t+1}}{\mathcal{E}_t} \right] = \frac{1}{1 + i_{t+1}^*}$$

Using (A.24) we can write:

$$(A.25) \quad B_{H,t+1}(j) = E_t \{ Q_{t,t+1} [M_t(j) + (1 + i_{t+1}) B_{H,t+1}(j)] \}$$

and:

$$(A.26) \quad \mathcal{E}_t B_{F,t+1}(j) = E_t \{ Q_{t,t+1} (1 + i_{t+1}^*) \mathcal{E}_{t+1} B_{F,t+1}(j) \}$$

It follows that the flow budget constraint (7) can also be written as:

$$(A.27) \quad E_t \{ Q_{t,t+1} W_{t+1}(j) \} \leq W_t(j) + R_t(j) - P_t C_t(j)$$

where W_{t+1} is wealth at the beginning of period $t + 1$, defined as:

$$(A.28) \quad W_{t+1}(j) \equiv (1 + i_{t+1}) B_{H,t+1}(j) + (1 + i_{t+1}^*) \mathcal{E}_{t+1} B_{F,t+1}(j).$$

Optimization implies that households exhaust their intertemporal budget constraint: the flow budget constraint hold as equality and the transversality condition:

$$(A.29) \quad \lim_{N \rightarrow \infty} E_t [Q_{t,N} W_N(j)] = 0$$

is satisfied, where $Q_{t,N} \equiv \prod_{s=t+1}^N Q_{s-1,s}$.

Foreign constraints and optimization conditions. Similar results characterize the optimization problem of Foreign agent j^* . For convenience, we report here the key equations. The resource constraint for agent f is:

$$(A.30) \quad \frac{\ell^*(f)}{\alpha_t^*} \geq p_t(f)^{-\theta^*} P_{F,t}^{\theta^*} C_{F,t} + p_t^*(f)^{-\theta^*} (P_{F,t}^*)^{\theta^*} C_{F,t}^*$$

The flow budget constraint is:

$$(A.31) \quad \frac{B_{H,t+1}^*(j^*)}{\mathcal{E}_t} + B_{F,t+1}^*(j^*) \leq (1 + i_t) \frac{B_{H,t}^*(j^*)}{\mathcal{E}_t} + (1 + i_t^*) B_{F,t}^*(j^*) + R_t^*(j^*) - P_{H,t}^* C_{H,t}^*(j^*) - P_{F,t}^* C_{F,t}^*(j^*)$$

and, associating individual j^* with brand f , sales revenue is:

$$(A.32) \quad R^*(j^*) = \frac{1}{\mathcal{E}_t} (p_t(f))^{1-\theta^*} P_{F,t}^{\theta^*} C_{F,t} + p_t^*(f)^{1-\theta^*} (P_{F,t}^*)^{\theta^*} C_{F,t}^*.$$

First order conditions yield:

$$(A.33) \quad P_t^* C_t^*(j^*) = \frac{1}{\gamma} P_{H,t}^* C_{H,t}^*(j^*) = \frac{1}{1-\gamma} P_{F,t}^* C_{F,t}^*(j^*)$$

$$(A.34) \quad \frac{1}{C_t^*(j^*)} = \beta (1 + i_{t+1}^*) E_t \left(\frac{P_t^*}{P_{t+1}^*} \frac{1}{C_{t+1}^*(j^*)} \right),$$

$$(A.35) \quad Q_{t,t+1}^*(j^*) = Q_{t,t+1}^* = \beta \frac{P_t^* C_t^*(j^*)}{P_{t+1}^* C_{t+1}^*(j^*)} = \beta \frac{\lambda_{t+1}^*(j^*)}{\lambda_t^*(j^*)},$$

and:

$$(A.36) \quad E_t Q_{t,t+1}^* = \frac{1}{1 + i_{t+1}^*}, \quad E_t \left[Q_{t,t+1}^* \frac{\mathcal{E}_t}{\mathcal{E}_{t+1}} \right] = \frac{1}{1 + i_{t+1}}.$$

Since international bonds are in zero net supply, the following constraints hold:

$$(A.37) \quad \int_0^1 B_{H,t}(j) dj + \int_0^1 B_{H,t}^*(j^*) dj^* = 0, \quad \int_0^1 B_{F,t}(j) dj + \int_0^1 B_{F,t}^*(j^*) dj^* = 0$$

Optimal price setting. Optimal prices in the Home country are derived as follows. Maximizing the Lagrangian with respect to $p_t(h)$ yields:

$$(A.38) \quad E_{t-1} \left[-\kappa \theta p_t(h)^{-\theta-1} P_{H,t}^\theta \alpha_t C_{H,t} + (\theta - 1) p_t(h)^{-\theta} P_{H,t}^\theta \lambda_t(h) C_{H,t} \right] = 0$$

or, in a symmetric environment:

$$(A.39) \quad p_t(h) = P_{H,t} = \frac{\theta}{\theta - 1} \frac{E_{t-1} [\kappa \alpha_t C_{H,t}]}{E_{t-1} \left[\frac{C_{H,t}}{P_t C_t} \right]}$$

and, using (A.19), we can rewrite the previous expression as (9) in the main text.

Maximizing the Lagrangian with respect to $\tilde{p}_t(h)$ yields:

$$(A.40) \quad 0 = E_{t-1} \left[-\kappa \theta p_t^*(h)^{-\theta} \tilde{p}_t(h)^{-1} (P_{H,t}^*)^\theta \alpha_t C_{H,t}^* \right. \\ \left. + (\theta - 1) p_t^*(h)^{1-\theta} \mathcal{E}_t \tilde{p}_t(h)^{-1} (P_{H,t}^*)^\theta \lambda_t(h) C_{H,t}^* \right]$$

In a symmetric environment with $p_t^*(h) = P_{H,t}^*$, we can rewrite the above as:

$$(A.41) \quad 1 = \frac{\theta \kappa}{\theta - 1} \frac{E_{t-1} [\alpha_t C_{H,t}^*]}{E_{t-1} \left[\frac{P_{H,t}^* \mathcal{E}_t C_{H,t}^*}{P_t C_t} \right]}$$

and, using (A.33), we obtain:

$$(A.42) \quad 1 = \frac{\theta \kappa}{\theta - 1} \frac{E_{t-1} \left[\alpha_t \frac{P_t^* C_t^*}{P_{H,t}^*} \right]}{E_{t-1} \left[\frac{\mathcal{E}_t P_t^* C_t^*}{P_t C_t} \right]}$$

or:

$$(A.43) \quad \tilde{p}_t(h) = \frac{\theta}{\theta - 1} \frac{E_{t-1} \left[\kappa \alpha_t P_t^* C_t^* \mathcal{E}_t^{\eta^*} \right]}{E_{t-1} \left[\frac{\mathcal{E}_t P_t^* C_t^*}{P_t C_t} \right]}$$

from which we obtain (11) in the main text using (17) and (18).

Equilibrium. Given initial wealth levels W_{t_0} and $W_{t_0}^*$ (symmetric within each country), and the processes for α_t , α_t^* , i_{t+1} and i_{t+1}^* for all $t \geq t_0$, the fix-price equilibrium is a set of processes for $C_{H,t}$, $C_{F,t}$, C_t , $Q_{t,t+\tau}$, $B_{H,t+1}$, $B_{F,t+1}$, ℓ_t , R_t , W_{t+1} , $P_{H,t}$, $P_{F,t}$, P_t , $C_{H,t}^*$, $C_{F,t}^*$, $Q_{t,t+\tau}^*$, C_t^* , $B_{H,t+1}^*$, $B_{F,t+1}^*$, ℓ_t^* , R_t^* , W_{t+1}^* , $P_{H,t}^*$, $P_{F,t}^*$, P_t^* , and \mathcal{E}_t such that, for all $t \geq t_0$ and $\tau > t_0$, (i) Home consumer optimization conditions (3), (5), (6), (A.19), (A.21), (A.22), (A.24), (A.28), and their Foreign analogs hold as equalities; (ii) Home firm optimization conditions

(9), (11), (4), and their Foreign analogs hold; (iii) the Home transversality condition (A.29) and its Foreign analog hold; (iv) the markets for the international bonds clear, that is conditions (A.37) hold.

A flex-price equilibrium is similarly defined, after imposing that for all $t \geq t_0$ Home conditions (9), (11), and their Foreign analogs hold in any state of nature at time t rather than in expectation at time $t - 1$.

Equilibrium balanced current account. Aggregating the individual budget constraints and using the government budget constraint we obtain an expression for the Home current account:

$$(A.44) \quad E_t \{Q_{t,t+1} W_{t+1}\} = W_t + R_t - P_t C_t$$

and, by the definition of sales revenue:

$$(A.45) \quad R_t = P_{H,t} C_{H,t} + \mathcal{E}_t P_{H,t}^* C_{H,t}^* = \gamma (P_t C_t + \mathcal{E}_t P_t^* C_t^*).$$

Assume now that, at time t_0 , $B_{H,t_0} = B_{F,t_0} = 0$ so that initial wealth $W_{t_0} = 0$. It is easy to show that, for all $t \geq t_0$, the equilibrium conditions above are solved by the allocation:

$$(A.46) \quad R_t - P_t C_t = -\mathcal{E}_t (R_t^* - P_t^* C_t^*) = 0, \quad W_t = -\mathcal{E}_t W_t^* = 0 \quad t \geq t_0.$$

The utility gap. To derive (43) and to show its sign observe that consumption under flexible prices is:

$$(A.47) \quad C_t^{flex} = \gamma \frac{\Phi_W}{\alpha_t^\gamma (\alpha_t^*)^{1-\gamma}}$$

where

$$(A.48) \quad \Phi_W = \Phi^\gamma \Phi^{*1-\gamma}, \quad \Phi = \frac{\theta - 1}{\theta \kappa}, \quad \Phi^* = \frac{\theta^* - 1}{\theta^* \kappa^*}.$$

Consider now the following steps:

$$\begin{aligned}
& E_{t-1} \mathcal{W}_t^{flex} - E_{t-1} \mathcal{W}_t = E_{t-1} \left(\ln C_t^{flex} - \ln C_t \right) + \kappa \Phi - \kappa \Phi \\
& = E_{t-1} \ln \frac{C_t^{flex}}{C_t} = E_{t-1} \ln \frac{\{E_{t-1} [\alpha_t \mu_t]\}^\gamma \left\{ E_{t-1} \left[\alpha_t^* (\mu_t^*)^\eta \mu_t^{1-\eta} \right] \right\}^{1-\gamma}}{\alpha_t^\gamma (\alpha_t^*)^{1-\gamma} \mu_t^{1-\eta(1-\gamma)} (\mu_t^*)^{\eta(1-\gamma)}} \\
& = E_{t-1} \left[\gamma \ln \frac{E_{t-1} [\alpha_t \mu_t]}{\alpha_t \mu_t} + (1-\gamma) \ln \frac{E_{t-1} \left[\alpha_t^* (\mu_t^*)^\eta \mu_t^{1-\eta} \right]}{\alpha_t^* (\mu_t^*)^\eta \mu_t^{1-\eta}} \right] \\
& = E_{t-1} \left[\gamma (\ln E_{t-1} [\alpha_t \mu_t] - \ln \alpha_t \mu_t) + (1-\gamma) \left(\ln E_{t-1} \left[\alpha_t^* (\mu_t^*)^\eta \mu_t^{1-\eta} \right] - \ln \alpha_t^* (\mu_t^*)^\eta \mu_t^{1-\eta} \right) \right] \\
& \geq E_{t-1} \left[\gamma (E_{t-1} [\ln \alpha_t \mu_t] - \ln \alpha_t \mu_t) + (1-\gamma) \left(E_{t-1} \left[\ln \alpha_t^* (\mu_t^*)^\eta \mu_t^{1-\eta} \right] - \ln \alpha_t^* (\mu_t^*)^\eta \mu_t^{1-\eta} \right) \right] \\
& = \gamma (E_{t-1} \ln \alpha_t \mu_t - E_{t-1} \ln \alpha_t \mu_t) + (1-\gamma) \left(E_{t-1} \ln \alpha_t^* (\mu_t^*)^\eta \mu_t^{1-\eta} - E_{t-1} \ln \alpha_t^* (\mu_t^*)^\eta \mu_t^{1-\eta} \right) \\
& = 0. \quad (\text{A.49})
\end{aligned}$$

Optimal monetary rules. Recalling that:

$$(\text{A.50}) \quad \frac{\partial f [E_{t-1} (x_t \mu_t^\pi)]}{\partial \mu_t} = f' [E_{t-1} (x_t \mu_t^\pi)] x_t \pi \mu_t^{\pi-1},$$

the first order condition for maximizing (A.49) with respect to μ_t is:

$$(\text{A.51}) \quad \frac{\gamma \alpha_t}{E_{t-1} [\alpha_t \mu_t]} - \frac{\gamma}{\mu_t} + \frac{(1-\gamma)(1-\eta) \alpha_t^* (\mu_t^*)^\eta \mu_t^{-\eta}}{E_{t-1} \left[\alpha_t^* (\mu_t^*)^\eta \mu_t^{1-\eta} \right]} - \frac{(1-\gamma)(1-\eta)}{\mu_t} = 0$$

which can be written as (33) in the main text. The other rules are similarly obtained.

Nominal equilibrium. The optimal monetary rules derived in the main text do not provide a nominal anchor to pin down nominal expectations. For instance, recalling the solution for $P_{H,t}$, we can write the Home country reaction function (33) in order to emphasize the relation between μ_t and $P_{H,t}$:

$$(\text{A.52}) \quad \alpha_t \mu_t = \left[1 - \eta(1-\gamma) - \frac{(1-\gamma)(1-\eta) \alpha_t^* (\mu_t^*)^\eta \mu_t^{1-\eta}}{E_{t-1} \left(\alpha_t^* (\mu_t^*)^\eta \mu_t^{1-\eta} \right)} \right] \frac{\Phi P_{H,t}}{\gamma}$$

Now, using (19) we obtain $\Phi P_{H,t} = E_{t-1} \alpha_t \mu_t$ but taking the expectation of (A.52) we obtain $E_{t-1} \alpha_t \mu_t = \Phi P_{H,t}$ for any arbitrary choice of $P_{H,t}$. Similar considerations hold for (34) and $P_{F,t}^*$.

To address this issue, following Woodford [2002], we take μ_t and μ_t^* defined by the system (33) and (34) and define two rules $\hat{\mu}_t$ and $\hat{\mu}_t^*$ such that:

$$(A.53) \quad \hat{\mu}_t = \mu_t \left(\frac{P_{H,t}}{\bar{P}_H} \right)^\delta \quad \hat{\mu}_t^* = \mu_t^* \left(\frac{P_{F,t}^*}{\bar{P}_F^*} \right)^{\delta^*} \quad \delta, \delta^* < 0$$

where δ and δ^* are two negative constants, arbitrarily small, and \bar{P}_H and \bar{P}_F^* are the two nominal targets for the Home and Foreign monetary authorities, respectively. Under the rules above, we obtain:

$$(A.54) \quad \Phi P_{H,t} = E_{t-1} \alpha_t \hat{\mu}_t = E_{t-1} \alpha_t \mu_t \left(\frac{P_{H,t}}{\bar{P}_H} \right)^\delta = \gamma \frac{\Phi (P_{H,t})^{\delta+1}}{\gamma (\bar{P}_H)^\delta}$$

solved by $P_{H,t} = \bar{P}_H$. This implies that, in equilibrium, $\hat{\mu}_t$ is identical to μ_t once we replace $P_{H,t}$ with \bar{P}_H . Similar considerations allow to pin down $P_{F,t}^* = \bar{P}_F^*$. The government sets a nominal anchor and credibly threatens to tighten monetary policy if the price of domestic goods deviates from a given target. Such threat, however, is never implemented in equilibrium.