INTERBANK INTEREST RATES AS TERM STRUCTURE INDICATORS

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Abstract

Interbank fixed income claims are a rich but neglected source of information on the term structure of interest rates and interest rate expectations. The first half of this paper describes the information content of two types of over-the-counter interest rate derivatives, forward rate agreements and interest rate swaps. Interbank interest rates and derivatives lend themselves well to a particular technique for fitting zero-coupon curves. The second half of this paper present this technique, along with some examples of how it can be used to gain insights into market views on interest rates and the stance of monetary policy.

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The term structure of interest rates is determined in part by expectations of future short-term interest rates, exchange rates, inflation, and the real economy, and therefore provides information on these expectations. Unfortunately, most of the term structure of interest rates is unobservable, a startling contrast to prices of most assets, such as spot exchange rates or stock-index futures, prices of which are directly observable.

The term structure is difficult to describe because fixed-income investments differ widely in the structure of their cash flows. Any one issuer will have debt outstanding for only a relative handful of maturities. Likewise, observable forward interest rates pertain to only a handful of settlement and maturity date pairs. Also, most bonds with original maturities longer than a year or two are coupon bonds, so their yields are affected not only by the underlying term structure of interest rates, but by the accident of coupon size.

To compensate for these gaps and distortions, one can try to build a standard representation of the term structure using observable interest rates. This is typically done in terms of prices and interest rates of **discount bonds**, fixed-income investments with only one payment at maturity, and **spot interest rates**, or interest rates on notional discount bonds of different maturities. The function relating spot rates to maturity is called the **spot curve** or **zero-coupon curve**, denoted $r(t,T)$ or $r(t,t+\tau)$, where t is today's date and $\tau \equiv T-t$ the time to maturity in years.

Another way of representing the term structure is by **forward interest rates**, interest rates applicable to fixed-income investments with a given time to maturity commencing on a given date in the future, called the settlement or effective date. The function relating forward rates to the time to settlement is called the **forward curve**, denoted $f(t,T_1,T_2)$ or $f(t,t+\tau_1,t+\tau_1+\tau_2)$, where $\tau_1 \equiv T_1 - t$ is the time to settlement and $\tau_2 \equiv T_2 - T_1$ the time to maturity. The three-month forward rate six months hence at time t would be denoted $f(t,t+\frac{1}{2},t+\frac{3}{4})$.[1]

Central bankers and market analysts often find the forward curve of greater interest than the spot curve, since it states explicitly market participants' expectations of future rates. However, the

[1] We will avoid worrying about compounding intervals by focusing on continuously compounded spot and forward curves. Malz (1997), the companion paper to this article, discusses the relationship between these and spot and forward rates with discrete compounding intervals.

information content of the two curves is identical.

This article shows how some well-known financial instruments, **forward-rate agreements** (FRAs) and **plain vanilla interest-rate swaps**, can serve as indicators of the term structure. It also describes a simple method for estimating zero-coupon curves, introduced by Nelson & Siegel (1987) and refined by Svensson (1994), and applies it using data on interbank credits such as Eurodeposit rates and interest-rate swaps, rather than government debt instruments. Many central banks have adopted some or all of these indicators as aids to market analysis and policy-making.[2]

1 FRAs and swaps as expectational indicators

Market analysts typically view prices of Eurodeposit futures as the market consensus about future short-term rates and the near-term policy stance of the currency's monetary authority. They rely on benchmark government bonds as the preferred way of ascertaining the level of long-term interest rates. The prices of FRAs and plain vanilla interest-rate swaps contain the same information in a cleaner and easier to interpret form.

1.1 FRAs keep a constant forecast horizon

In a FRA, one party agrees to pay a specific interest rate on a Eurodeposit of a specified currency, maturity, and amount, beginning on a specified date in the future. FRA prices are expressed as the rate the buyer pays on the notional deposit, that is, as a forward interest rate. FRAs are cash-settled, by the present value of the difference between the realized short-term rate and the FRA price, so no deposit will actually be made.

For example, Bank A might agree on January 1 to pay Bank B 3.25 percent on a three-month DM 10 million deposit starting 2 months later. A FRA settling in two month and maturing in five months is called a 2×5 (spoken "2 by 5"). If the contract were settled by delivery, and the cash three-month Euromark rate were 3.5 percent on March 1, Bank B would be obliged to pay $(0.035 - 0.0325) \times 10{,}000{,}000 \times \frac{90}{360} = 6250$ marks to Bank A when the deposit matures on June

[2]Söderlind & Svensson (1997) provides an excellent summary of the more general topic of using market prices to assess market expectations of future interest and exchange rates.

1. Instead, Bank B pays Bank A the present value of that amount on March 1. With a discount factor of $1 + 0.035 \times \frac{90}{360} = 1.00875$, that comes to $\frac{6250}{1.00875} = 6195.79$ marks.

Settlement and maturity dates of most FRAs are standardized, though FRAs with precisely tailored settlement and maturity dates can be obtained at a somewhat higher bid-offer spread. FRA quotes for a range of times to settlement and maturity are available from information services such as Bloomberg and Reuters. For example, for the U.S. dollar, Reuters updates indicative levels for 37 different FRAs with maturities from three to twelve months and settling between one and eighteen months hence, while for the French franc, 21 FRA rates are available.

FRAs are very similar to Eurodeposit futures in their information content. The quote on a FRA is the risk neutral market estimate of the future money market rate. For example, the mark 2×5 FRA price is the rate at which market participants can currently lay off or take on an exposure to the three-month Euromark rate prevailing two months from now. The forecast horizon equals the time to settlement, two months.

FRAs differ from Eurodeposit futures in having a fixed time span to settlement instead of a fixed settlement date. At the time of writing, for example, a 3×6 FRA quote gives an implied three-month rate prevailing three months hence, in mid-September 1997. In one month, the then-current 3×6 FRA price will give information about the three-month rate the market expects in mid-October. Alternatively, we can draw each day on price of the September 1997 Eurodollar futures for an implied three-month rate prevailing in the third week of September 1997. Next month, the price of that futures contract will still contain information on the three-month rate expected in September 1997.

Since futures settle only quarterly, FRAs may fill in intermediate months, providing additional information. FRA rates for intermediate months do not necessarily lie on or close to a line between adjacent futures contracts. Straight-line interpolation can be particularly misleading when there is a good deal of curvature in the term structure, or when significant events such as a central bank governors' meeting are scheduled between futures expiration dates.

Figures 1 and 2 give two illustrations of how the information in FRA rates can be tapped.

Figure 1 plots, for four currencies, prices of FRAs with three-month maturities and different times to settlement, that is, the 1×4, 2×5, 3×6, etc., against the time to settlement. The cash three-month Eurodeposit rate is displayed as a three-month rate with zero time to settlement.

The graph can be interpreted as the implied future path of three-month Eurodeposit rates. For all four currencies, the path has a rising tendency, but for the French franc, the slope is slight, implying subdued concern about near-term monetary tightening. One interpretation is that the Banque de France maintains a steady very-short term interest-rate differential vis-à-vis Germany in pursuit of the *franc fort* policy, but that the markets expect some narrowing of that differential as long as the franc is actually strong, as in recent years.

Figure 3 displays the cash three-month U.S. dollar and German mark Eurodeposit and 1×4 FRA rates from the beginning of 1996 to mid-1997, chronicling changes in sentiment about the monetary postures of the Fed and the Bundesbank. The cash-forward differential is more often than not positive for both currencies, as it contains a generally positive but variable term premium: a spread of zero would likely indicate expectations of a decline in three-month cash rates. The spread tends to rise when cash rates are rising.

It can be misleading to compare cash and futures rates over a long period, as the forecast horizon will shrink over the period as the futures rate converges to the cash rate. For example, if Figure 3 displayed September 1997 futures rather than FRA prices, the forecast horizon would be 21 months at the beginning and three months at the end of the graph. FRA prices avoid this difficulty by maintaining a constant forecast horizon.

The rate implied by the price of a FRA should be very close to the rate implied by the price of a futures expiring on the same day as the FRA settles. However, the two rates will not, in general, be identical, since FRAs are forward contracts, not over-the-counter futures. Because of the daily revaluation of futures contracts, their values are affected by the expected volatility of short-term interest rates, and may differ for that reason by a few basis points per month to expiry from congruent forward interest rates.

To summarize, futures prices indicate interest rates expected to prevail at a *specific date* in the future, the maturity date of the contract. FRAs indicate expected future interest rates at a *specific horizon.* If we want to monitor market expectations of the interest rate on or around a specific date, e.g. a particular FOMC meeting, and if Eurodeposit futures contracts happen to expire around that date, the futures contract is more useful. If we want to monitor expectations of the pace of, say, interest rate changes over the next six months, FRAs are preferable.

1.2 Swaps avoid the coupon effect

The medium- and long-term portion of the term structure is often represented by government bonds yields. Analysts prefer government debt because in most countries the government is the only issuer with a wide maturity spectrum of adequate liquidity, and because government bonds are viewed as virtually free of credit risk. However:

- In most countries, "specialness" is a persistent feature of government bond markets. The reasons for specialness have to do with the way dealer and investor inventories of bonds are financed and managed, and with the need of some investors for coupon instruments of high credit quality. Specialness is typically most pronounced for "on the run" issues, and it lowers the yield to maturity of government bonds in an uneven way across the maturity spectrum and over time.[3]

- With the exception of rare "tap" issues, government bonds are not issued daily, but rather at discrete intervals, often on a fixed auction calendar. Particular issues may be preferred by some investors because of the timing of their coupon payments. Insurance companies in some countries, for example, prefer coupon payments which occur shortly before the end of a fiscal quarter or year, so the cash appears on their published balance sheets but do not remain uninvested for long. Such bonds may have slightly lower yields.

- The income from government bonds often receives favorable tax treatment, which generally can be most readily enjoyed by nationals of the issuing country. In other cases, non-residents

[3]See Jordan & Jordan (1997) for examples of the impact of specialness on U.S. Treasury bond prices.

are exempt from taxes on government bond income. These features render otherwise identical bonds issued by different countries imperfect substitutes and influence the yields of government issues in a way which is difficult to adjust for accurately.

- In general, not all government bonds of a given maturity range are deliver into the corresponding futures contract. Deliverability depends on the configuration of current market interest rates and the futures contract specifications. Bonds which are deliverable may be considered more desirable and have somewhat lower yields.

- Perhaps most importantly, the **coupon effect** renders yield an ambiguous indicator of the term structure, since yield can be affected not only by the underlying term structure, but the size and timing of the payments. Bonds with high coupons tend to have lower yields to maturity than bonds with low coupons. To illustrate, compare three-year bonds with annual coupons of 4 and 8 percent. If the one-, two- and three-year spot rates are, say, 5.5, 6.5, and 7.25 percent, the yield of the 4 percent bond is 7.47 percent, while the yield of the 8 percent bond is only 7.43 percent. The coupon effect is most pronounced when the term structure is sharply sloped.

Swap rates do not have these drawbacks. A swap is a contract in which two counterparties agree to exchange income streams based on two different fixed-income investments with a common notional principal. Swaps are a pure "inside" asset: the net supply is always zero and the volume of outstanding swaps rises and falls as the number of counterparties wishing to contract with each other at prevailing interest rates changes. Swaps, therefore, unlike bonds or other commodities of which at any moment there is a well-defined stock, cannot be "squeezed."

Like FRAs, fresh swaps with standard maturities are issued daily, so ten-year swap rates on two different dates refer to fixed-income investments with precisely the same maturity. Government bonds are issued in accordance with a fixed calendar, so "ten-year" bond yields on two different dates, or "ten-year" bond yields for different countries on the same date, generally refer to fixed-income investments with somewhat different maturities. Swap rates therefore make possible unambiguous longer-term interest rate comparisons over time or across currency denominations.

The most liquid variety is the **plain-vanilla swap**, in which one counterparty pays the cash flows of a notional coupon bond in exchange for those of a notional floating-rate bond. The market sets the coupon rate of the notional coupon bond so that the present values of the two sets of cash flows are equal. A freshly-issued floating-rate bond always trades at par, so the notional coupon bond must also trade at par on issuance. Thus the quoted swap price can be treated algebraically as the coupon rate of a notional par bond.[4]

Even plain-vanilla swap pricing can be complicated by some unessential features, such as the choice of floating-rate index and the annual frequency with which payments are exchanged. One of the more common setups is a swap in which a coupon payment is exchanged for six-month LIBOR every six months. Swap quotes are also available from the standard information services. For most G-7 currencies, Reuters carries quotes on swaps of fixed against six-month LIBOR with two to ten years to maturity. For a few currencies, quotes on longer-term swaps and swaps with a three-month reset period are also provided.

Figure 2 displays quoted swap rates for the dollar, mark, franc and yen. The rates displayed are comparable across maturities, countries and over time, since they are all expressed as par bond yields, and thus give an undistorted reading on medium- and long-term rates.

2 Estimating the term structure

FRA and swap rates provide only partial information on the term structure. As we have seen, FRA rates with different times to settlement but the same time to maturity plot a shorter-term forward curve. Swap rates with different times to maturity can be plotted to provide information on the medium- to long-term structure. Analysts and central bankers find it particularly useful to combine short- and long-term, spot and forward rate data to obtain a simple graphical representation of the entire curve using estimates of the term structure.

[4]Swap prices are also frequently quoted as a spread over the yield on a Treasury security of like maturity. This is misleading in two ways: First, the maturities of the two instruments rarely match up exactly. Second, the yields on the two instruments are incommensurable because they have different coupons–the coupon effect again.

The estimated term structure can generate a variety of indicators of market sentiment, including term spreads, e.g. the overnight-to-2 year spread, which is an indicator of market expectations regarding future money market rates, and international comparisons, e.g. the U.S. dollar-German mark 10-year spread, which indicate sentiment regarding future currency values. Some of these spreads can in turn be used as indicators of inflation expectations and of expectations regarding the business cycle.

Another indicator is the **forward swap rate**, the rate applicable to a plain-vanilla swap settling at some future date. Forward swap rates have been widely used as indicators of sentiment regarding the likelihood of countries' becoming initial participants in European Monetary Union.

Interpolation of zero-coupon curves is usually done by "bootstrapping" or by fitting of spline functions. Both techniques suffer from the defect that the forward curve implied by the estimate may be implausible. In particular, it may be jagged or wavy in ways which imply that short-term rates are expected to move sharply up and down over short time intervals, or may imply that the market has implausibly detailed expectations about variations in short-term rates many years in the future.[5]

An alternative technique, first proposed by Nelson & Siegel (1987), is to represent the zero-coupon curve by a mathematical function which can take on a wide variety of typical yield-curve shapes depending on the values of a small number of parameters, and find the parameter values which best fit the interest-rate data. The approach approximates that the instantaneous-maturity forward curve—the limit of $f(t, t+\tau_1, t+\tau_1+\tau_2)$ as τ_2 tends to zero—by the function:[6]

$$\phi(\tau, \beta_0, \beta_1, \beta_2, \theta) = \beta_0 + \beta_1 e^{-\frac{\tau}{\theta}} + \beta_2 \frac{\tau}{\theta} e^{-\frac{\tau}{\theta}}. \tag{1}$$

The spot rate $\rho(\tau, \beta_0, \beta_1, \beta_2, \theta)$ for a given time to maturity is an "average" of the instantaneous forward rates for settlement dates from now to the maturity date of the discount bond:

[5] Shiller & McCulloch (1990) survey term structure fitting techniques.

[6] The times to maturity and settlement of instantaneous forward rates are identical, both represented by a subscriptless τ.

$$\rho(\tau,\beta_0,\beta_1,\beta_2,\theta) = \frac{1}{\tau}\int_0^\tau \phi(s,\beta_0,\beta_1,\beta_2,\theta)ds = \beta_0 + (\beta_1+\beta_2)\frac{1-e^{-\frac{\tau}{\theta}}}{\frac{\tau}{\theta}} - \beta_2 e^{-\frac{\tau}{\theta}}. \tag{2}$$

The first parameter, β_0, is a constant to which the forward rate tends as the settlement date $t+\tau$ grows large. The second term describes a monotonic convergence path to the long-run short rate. As τ approaches zero, that is, the maturity becomes very short, the estimated spot rate tends toward $\beta_0+\beta_1$, so β_1 is the difference between the "overnight" rate and the long-term spot rate. To better mimic typically observed yield curve patterns, the third term overlays an additional hump-shaped or U-shaped time pattern on the spot and forward curves. The parameter θ governs the speed with which the monotonic convergence occurs and the hump component decays. For some values of β_2 and θ, the hump-shaped pattern can appear monotone. Figure 4 shows how the individual components of the function contribute to its overall shape.

Svensson (1994) proposed an extension of Nelson & Siegel (1987)'s approach which often fits the data better. It approximates the forward curve by the somewhat more complex function

$$\phi(\tau,\beta_0,\beta_1,\beta_2,\beta_3,\theta_1,\theta_2) = \beta_0 + \beta_1 e^{-\frac{\tau}{\theta_1}} + \beta_2\frac{\tau}{\theta_1}e^{-\frac{\tau}{\theta_1}} + \beta_3\frac{\tau}{\theta_2}e^{-\frac{t}{\theta_2}}. \tag{3}$$

in which the terms β_3 and θ_2 parameterize an additional hump-shaped component with its own rate of decay. The spot rate, by integration, is

$$\rho(\tau,\beta_0,\beta_1,\beta_2,\beta_3,\theta_1,\theta_2) = \beta_0 + (\beta_1+\beta_2)\frac{1-e^{-\frac{\tau}{\theta_1}}}{\frac{\tau}{\theta_1}} - \beta_2 e^{-\frac{\tau}{\theta_1}} - \beta_3 e^{-\frac{\tau}{\theta_2}}. \tag{4}$$

The Nelson-Siegel-Svensson approach to term-structure fitting is not a *theory* of the term structure. It does not attempt to explain the typical features of the term structure or how it changes over time based on underlying features of the economy or how expectations are formed, as does the Cox, Ingersoll & Ross (1985) model. Nor is it a *model* of the term structure, attempting to represent derive complex real-world term structures from simpler, tractable stochastic processes for short-term rates, as does the Black, Derman & Toy (1990) model. Rather, it merely tries to find a close representation of the term structure at a point in time, based on the interest-rate data as we find them.

2.1 Estimating the parameters

Term structure estimation is usually carried out using government bond data. The evolution of the Euromarkets now make available another source of data, interest rates on interbank credits, for which specialness is not a factor in determining price and yield and the tax treatment of which (namely none) is uniform across currencies and geographical locations. The technique described here uses Eurodeposits, FRA prices and swap rates.[7]

Eurodeposits are time deposits by banks outside a currency's country of issuance. Their interest rates are mathematically similar to those of discount instruments like U.S. Treasury bills, since there is only one payment of principal plus interest earned at maturity. A τ-year Eurodeposit rate $r_e(t, t+\tau)$ can be converted to a spot rate via the relationship

$$r(t, t+\tau) = \frac{\ln[1 + r_e(t, t+\tau)]}{\tau}. \tag{5}$$

The FRA rate settling at time $T_1 \equiv t + \tau_1$ and maturing at time $T_2 \equiv t + \tau_1 + \tau_2$ is related to Eurodeposit rates by a no-arbitrage relationship

$$r_{FRA}(t, T_1, T_2) = \frac{1}{\tau_2}\left[\frac{1 + r_e(t, t+\tau_1+\tau_2)(\tau_1+\tau_2)}{1 + r_e(t, t+\tau_1)\tau_1} - 1\right], \tag{6}$$

which can be stated in terms of spot rates as

$$\ln[1 + r_{FRA}(t, T_1, T_2)\tau_2] = r(t, t+\tau_1+\tau_2)(\tau_1+\tau_2) - r(t, t+\tau_1)\tau_1. \tag{7}$$

[7] Details on the interest-rate calculations below are presented in Malz (1997).

Swaps are also closely related to spot rates. For a swap rate with six-month reset intervals to be consistent with the term structure of interest rates, we must have

$$1 = \frac{r_s(t, T, \frac{1}{2})}{2} \sum_{j=1}^{2\tau} e^{-r\left(t, t+\frac{j}{2}\right)\frac{j}{2}} + e^{-r(t, t+\tau)\tau}. \tag{8}$$

The left-hand side of equation (8) is set to unity, because the notional coupon rate of a freshly created swap is set so the fixed-rate side trades at par, to equal the value of the floating-rate side. The right-hand side of equation (8) prices each part of the notional coupon bond—the $2 \cdot \tau$ coupon payments and the principal payment at maturity—using the spot rate of appropriate maturity.

Eurodeposit futures prices could also be added to the mix. For U.S. dollar deposits, futures contracts with delivery dates ten years in the future are available. However, for deposits denominated in many other key currencies, futures are available only one or two years out, if at all. More importantly, futures are priced differently from forwards, incorporating not only the expectations of the underlying asset price at maturity, but also fluctuations in the short-term financing rate.

The potential disadvantages of interbank data, namely credit risk, liquidity, and the availability of actual transactions prices, are less serious than might be thought. Credit risk is probably a small component, relative to specialness, of the spread between interbank rates and rates on comparable government credits. Moreover, these instruments have similar credit risk properties across currencies, which cannot be guaranteed for government paper. One does, however, need to be aware that credit risk is embedded in these interest rates, and that it may vary with time to maturity.

The liquidity of most of interbank instruments, particularly swaps, is quite high, and in some cases better than that of government debt. The liquidity of government issues is divided among a great many securities, of which only a few trade actively at any one time.

We estimate the term structure by fitting the spot curve, finding the values of the parameters of the Nelson & Siegel or Svensson function which best fit the data using nonlinear least squares. Each observed interest rate contributes a equation based on equation (5), (7), or (8) to the least squares exercise. In each, the Nelson & Siegel or Svensson function of the appropriate maturity is substituted for the unobservable spot rate, and the equation is assumed to hold up to a random error u_t^i.

For t-year Eurodeposit rates, the equations take the form

$$\frac{\ln[1 + r_e(t, t+\tau)]}{\tau} = \rho(\tau, \beta_0, \beta_1, \beta_2, \beta_3, \theta_1, \theta_2) + u_t^i, \tag{9}$$

For most currencies, Eurodeposit rate quotes with times to maturity one week and one, two, three, six, nine and twelve months are readily available, providing seven equations.

For FRA rates the equations takes the form

$$\begin{aligned} &\ln[1 + r_{FRA}(t, T_1, T_2)\tau_2] \\ &\quad = \rho((\tau_1 + \tau_2), \beta_0, \beta_1, \beta_2, \beta_3, \theta_1, \theta_2)(\tau_1 + \tau_2) - \rho(\tau_1, \beta_0, \beta_1, \beta_2, \beta_3, \theta_1, \theta_2)\tau_1 + u_t^i. \end{aligned} \tag{10}$$

The number of FRA contracts for which data are readily available vary from currency to currency, with most available for the U.S. dollar, providing 37 equations.[8]

For τ-year plain-vanilla swaps with semiannual resets, the equations take the form

$$1 = \frac{r_s(t, T, \frac{1}{2})}{2} \sum_{j=1}^{2\tau} e^{-\rho\left(\frac{j}{2}, \beta_0, \beta_1, \beta_2, \beta_3, \theta_1, \theta_2\right)\frac{j}{2}} + e^{-\rho(\tau, \beta_0, \beta_1, \beta_2, \beta_3, \theta_1, \theta_2)\tau} + u_t^i. \tag{11}$$

For most G-7 currencies, swap rate quotes with times to maturity two, three, four, five, seven, and ten years are available, providing an additional six equations. We estimate the curve out to ten years, since our data incorporates unobserved spot rates with maturities up to ten years.[9]

[8]Some of the FRA data adds in principle no information to that in the corresponding Eurodeposit rates. For example, bid and ask rates for 3×6 FRA rates can be constructed by no-arbitrage arguments from bid and ask rates on 3- and 6-month Eurodeposits. It is preferable to retain the redundant information because of data errors and market frictions.

[9]For a handful of currencies, 15-year swap quotes are readily available, permitting the estimation of the zero-coupon curve out to 15 years. I have not done so, since there is rarely any interesting curvature beyond ten years worth studying, and because few international comparisons are possible.

As noted earlier, the Nelson & Siegel and Svensson functions tend toward $\beta_0+\beta_1$ as the maturity τ approaches zero. It generally helps to reduce the squared errors to "anchor" the estimated curve to the overnight rate $r(t, t+\frac{1}{360})$, by setting $\beta_1 = r(t, t+\frac{1}{360}) - \beta_0$, eliminating the parameter β_1.[10]

To better capture the market's perception of very short-term rates in the estimated term structure, it is preferable to set $\beta_0 + \beta_1$ equal, not to the realized, but to the *targeted* overnight rate. In some countries, such as France, central bank tactics and the structure of the money market enable the monetary authorities to virtually set the overnight rate, while in other countries, such as Germany and the United States, although the central bank targets the overnight rate as a tactical matter, the rate still fluctuates from time to time. However, even in these cases, except on quarter- or year-end dates, it can be argued that the targeted rate is closer than the realized to what the market perceives as "the" price of overnight funds, or to the overnight rate market participants expect to prevail the next day, which may on any given day be considerably higher or lower than the central bank target for very transient reasons.

The parameter estimates for each date are chosen as the solution to

$$\underset{\{\beta_0,\beta_2,\beta_3,\theta_1,\theta_2\}}{\operatorname{argmin}} \sum_{i=1}^{n} (u_t^i)^2. \tag{12}$$

Computer code for solving such numerical minimization problems is widely available, for example as part of mathematical software packages such as MATLAB and Mathematica. I did the estimates reported here using Mathcad's *Minerr* function.

Table 1 and Figures 5 through 9 present estimates of the term structure on May 26 and June 17, 1997, for the U.S. dollar, Japanese yen, German mark, and French franc. All the estimates were obtained using the Svensson functional representation. Even for such apparently simple term structures as that of the U.S. dollar, the Svensson function has a better fit than the Nelson & Siegel function. In the U.S. dollar case, the curve flattens out quite rapidly beyond 2 years, and the extra parameters help capture that angularity more accurately. When the more complex version

[10]See Svensson (1994) for details. Note that since the estimation procedure I am presenting employs interbank data throughout, anchoring to the overnight rate, which is also an interbank rate, introduces no data heterogeneity.

has nothing useful to add, the procedure either will not converge for any reasonable starting values, because the parameter pair (β_2, θ_1) is indistinguishable from (β_3, θ_2), or one of the parameters β_2 or β_3 is indistinguishable from zero.[11]

The usefulness of a graphical summary of the entire term structure can best be illustrated by discussing the mark and franc curves in more detail. Figures 6 and 7 display the estimated spot and forward curves for the mark and franc on the two dates.[12] For both currencies, interest rates fell along the entire curve, and rates with maturities under one year and long-term rates fell more than medium-term rates, possibly indicating reduced anxiety about monetary tightening in both France and Germany.

The contrasts between the franc and the mark are particularly interesting. For the mark, there has been a pronounced flattening of the curve, leading to a greater drop in longer-term rates. Also, short-term franc rates have dropped considerably more than mark rates. These contrasts can be best summarized graphically, as in Figure 8, in the term structure of interest-rate *differentials*. The very short end of the differential term structure is quite steep both on May 26 and June 17. This has been a persistent feature of the mark-franc relationship and appears to reflect the *franc fort* policy: the franc's strength has been sustained by the excess return over German money-market rates. The excess return required to sustain the mark-French franc exchange rate has diminished over time.

However, the implied path of the differential over the coming year drops significantly. One interpretation of the decline in the short-term differential is that exchange-market tensions eased from May 26 to June 17, lowering the interest-rate differential the market believes required for the Banque de France to maintain a strong franc over the next year.

We can calculate the rate on a τ_2-year forward swap with semiannual resets settling in τ_1 years by substituting estimated spot rates $\rho(\tau, \beta_0, \beta_1, \beta_2, \beta_3, \theta_1, \theta_2)$ for the $r(t, \tau)$ in the relationship

[11]For the dollar, the targeted rate is the Fed funds target, for the mark, the two-week repo rate, for the franc, the *taux de jour*, and for the yen, I have chosen a value one basis point below the call money rate.

[12]The forward curves trace the implied path of overnight rates but could be easily recalculated to display forward rates of any maturity.

$$1 = f_s(t, T_1, T_2, h)h \sum_{i=1}^{\frac{\tau_2}{h}} e^{-[r(t,\tau_1+hi)(\tau_1+hi)-r(t,T_1)\tau_1]} + e^{-[r(t,T_2)(\tau_1+\tau_2)-r(t,T_1)\tau_1]}, \tag{13}$$

where $f_s(t, T_1, T_2, h)$ is the forward swap rate. Resurgent optimism on France's participation in European Monetary Union on January 1, 1999 may be reflected in two-year forward two-year swap rates, which on May 26 was 12 basis point lower for the franc than the mark, but 22 basis points lower on June 17.

In the short to medium term, the news is good for France, but the good news erodes in the long term. At the longer end of the differential term structure, another important change takes place: franc forward rates, which earlier were below mark rates as far as the eye could see, now rise well above mark yields for longer-term maturities. The rise in the long-term forward-rate differential might indicate greater gains for the long-term credibility of German than for French monetary policy from May 26 to June 17, which would become relevant in the event of a delayed or derailed EMU.

For policy makers and market analysts, the advantage of the Nelson-Siegel-Svensson approach over alternative techniques is its simplicity. It is easy to implement, since it uses a straightforward functional form to capture the term structure and is easy to set up computationally. The technique is designed to highlight interpretable differences in the term structure across countries and time, but does not identify the implied stochastic process of the short-term interest rate.

The Nelson & Siegel and Svensson functions tend to fit the data somewhat more loosely than spline approximations, which try to mimic every wrinkle traced by the data, eliminating instead some of the analytically uninteresting variation in the fitted curve. For these reasons, it the not an appropriate technique for pricing interest rate derivatives, nor for identifying rich and cheap securities.

2.2 Accuracy

We assume that $\rho(\tau, \beta_0, \beta_1, \beta_2, \beta_3, \theta_1, \theta_2)$ is a good representation of the unobservable spot curve $r(t, t+\tau)$, so that once we have chosen the parameters $\beta_0, \beta_1, \beta_2, \beta_3, \theta_1$ and θ_2 for time t,

$$\rho(\tau, \beta_0, \beta_1, \beta_2, \beta_3, \theta_1, \theta_2) - r(t, t+\tau)$$

is a small random error. However, we cannot identify most of these deviations individually, since the spot rates which determine FRA and swap rates enter into those rates in combination.

We can, however, use the estimated swap curve to see how close the u_t^i have been driven to zero. For Eurodeposit rates, this is equivalent to measuring $\rho(\tau, \beta_0, \beta_1, \beta_2, \beta_3, \theta_1, \theta_2) - r(t, t+\tau)$. For FRA rates, it measures the deviation between the FRA rate and the linear combination of spot rates which determines it. For swap rates, the u_t^i measure the percent deviation from par of a bond with a coupon equal to the quoted swap rate and priced using the estimated spot curve. If the fit is good, the Eurodeposit and FRA rates will differ by only a few basis points from the spot rates on which they depend, the prices of the fixed-rate portion of the swaps will differ by only a fraction of a percent from unity, and the signs of the u_t^i will fluctuate from positive to negative rather than mostly having the same sign.

Table 1 reports the largest absolute errors for the eight estimates. The largest rate error is generally on the order of 10 basis points. In all cases the second largest error was considerably smaller, about 5 basis points. Most of the rate errors were under one basis point. The largest swap errors are on the order of one-tenth of one percent. To put that in perspective, it corresponds to an error in swap rates of one or two basis points.

One drawback of the approach is that the minimization problem equation (12) does not weight the deviations of observed from estimated spot rates at different points on the curve equally, but rather weights each equation equally. Since the FRA equations each contain two spot rates and the swap equations between four and twenty spot rates, the weight of the deviation of a given spot rate from its observed counterpart is somewhat obscure. Generally, the shorter-term spot rates are weighted more heavily, since they enter into more of the minimization conditions.

3 The information content of forward rates

The **expectations hypothesis** of the term structure states that longer-term interest rates contain the market's forecasts of future shorter-term rates plus, possibly, a well-behaved (preferably constant) term premium. It suggests using the term structure as a source of information on market expectations about future interest rates.

As is the case with any indicators of market sentiment generated by financial asset prices, the term structure reveals only **risk-neutral expectations** about future interest and exchange rates. Risk-neutral expectations may reflect risk factors as well as expectations. The term "risk neutral" is somewhat misleading, since it means that if we do not assume that market participants are in fact indifferent to risk, we must instead assume that asset prices may contain risk premia.

For example, a forward money-market rate may comprise a term premium as well as the market consensus regarding the future money-market rate. The forward rate is the price at which market participants can take on or lay off an exposure to the future money-market rate. If the desire to reduce such exposures predominates, a positive term premium will emerge. If the forward curve steepens, the analyst cannot determine exactly to what extent it is due to expectations of more rapidly rising short-term rates rather than greater anxiety about rising rates, but can only state that the prospect of rising rates looms larger in market participants' plans than before.

How good a *predictor* of short-term rates is the term structure? The answer from empirical research, sadly, is that it is not a very good predictor at all. One implication of the expectations hypothesis, that long-term rates should rise when the term structure becomes more positively sloped, is not borne out at all. There is some weak support for another implication, that short-term rates should rise when the term structure becomes more positively sloped, but a strategy of rolling over short-term securities when the term structure steepens is quite risky.[13]

One interpretation of empirical research on the expectations hypothesis is that the market can at times predict changes in short-term rates with some accuracy, particularly when short rates have

[13] See Shiller & McCulloch (1990), Campbell (1995) and Campbell, Lo & MacKinlay (1997) for recent surveys of term structure theories and their empirical performance.

been moving in the same direction for some time. Often, however, the market systematically over- or underpredicts changes in short-term rates, and fails particularly to predict turning points in the interest rate cycle. A period in which the market expectation of the future short-term rate is higher or lower than the realized short-term rate may last for several months before it is corrected. These stylized facts are borne out, particularly for the U.S. dollar, by Figure 3.

However, the issue of predictive power needs to be carefully distinguished from that of information content. The fact that forward rates are poor predictors of future short rates does not render them useless as indicators of current market sentiment. They must rather be used with an awareness that expectational and risk factors are commingled in them. One common use of the expectations hypothesis, both in central banks and among market analysts, is to take the average difference between the forward rate and future realized rate as an estimate of the term premium.

However, if both term premiums and expectational errors can fluctuate widely and unpredictably, then such approaches to capturing the information content of forward rates are too mechanical. As one former Fed official (Blinder (1997, p. 17)) has commented,

> ...everyone—and here I mean analysts, market participants and central bankers alike—continues to "read" the market's expectations of future short rates from the yield curve, as if doing so made sense. I find it hard to explain why everyone is doing what everyone knows to be wrong. Yet it happens all the time.

In the current state of lack of knowledge about what drives term premiums and the distribution of prediction errors, analysts can defend themselves against Blinder's critique but still make use of the information in the term structure by seeking aids to judgment rather than mechanical calculation methods. Such aids to judgment include option prices, which are closely related to the term premium, since both tend to rise when uncertainty about future short-term rates increases or market participants become less willing to commit funds long-term.

As an example of how option prices may help narrow down expectations, consider the 7 basis point in the 1×4 U.S. dollar FRA rate from the end of May to mid-June (see Figure 3). The implied volatilities of options on September 1997 Eurodollar futures during that period fell from about 8.8 to 7.4 percent, indicating that short-term premiums are unlikely to have risen during that time and may have fallen a bit. Thus the fall in the expected three-month rate one month hence will likely have been at most 7 basis points, and possibly a bit less.

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Table 1: Term structure estimates

PARAMETER ESTIMATES

	USD		DEM		FRF		JPY	
	May 26	June 17	May 26	June 17	May 26	June 17	May 26	June 17
β_0	0.075	0.071	0.077	0.071	0.076	0.074	0.046	0.040
β_2	-1.2E-05	-6.3E-07	2.7E-06	0.84	-0.05	-0.05	7.0E-06	-0.20
β_3	0.76	0.33	-8.99	63.0	1.59	0.72	-3.0E-03	0.017
θ_1	2.59	2.10	1.60	1.02	1.57	1.65	3.78	1.49
θ_2	1.12	1.02	0.23	1.06	0.29	0.32	0.45	1.44

TARGETED OVERNIGHT RATE $(\beta_0 + \beta_1) \cdot 10^2$

USD		DEM		FRF		JPY	
May 26	June 17	May 26	June 17	May 26	June 17	May 26	June 17
5.50	5.50	3.00	3.00	3.19	3.19	0.49	0.49

LARGEST $|u_t^i|$ among Eurodeposit and FRA rates (in bp) and location

USD		DEM		FRF		JPY	
May 26	June 17	May 26	June 17	May 26	June 17	May 26	June 17
10.7	10.7	11.5	8.5	8.7	6.5	8.2	9.9
12×18	15×24	1-wk.	3×15	1-wk.	3×6	1-yr.	6-mo.

LARGEST $|u_t^i|$ among swaps (in %) and location

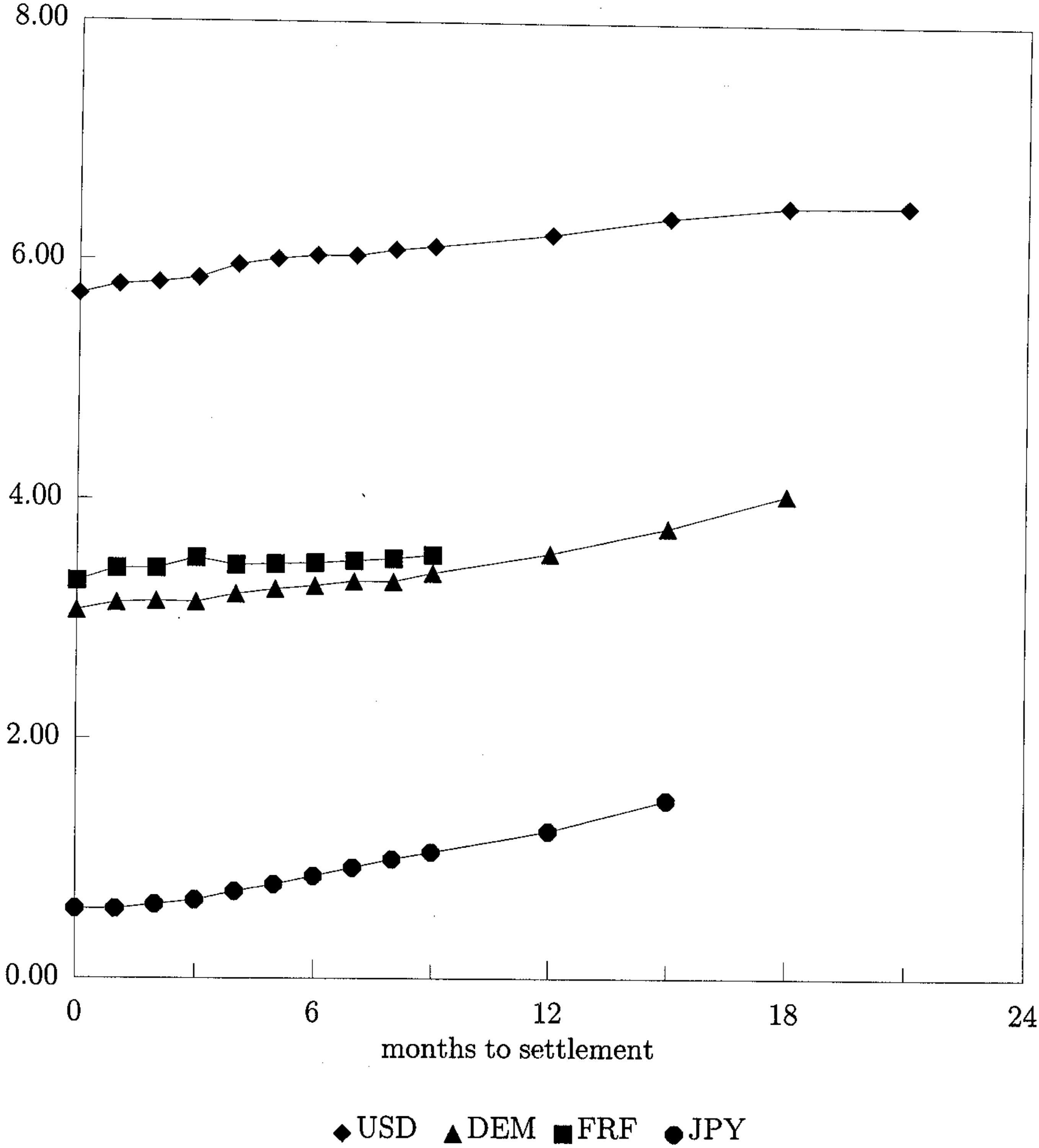
USD		DEM		FRF		JPY	
May 26	June 17	May 26	June 17	May 26	June 17	May 26	June 17
0.13	0.14	0.12	0.13	0.09	0.08	0.04	0.06
3-yr.	2-yr.	2-yr.	2-yr.	2-yr.	5-yr.	3-yr.	5-yr.

NUMBER OF EQUATIONS

USD		DEM		FRF		JPY	
May 26	June 17	May 26	June 17	May 26	June 17	May 26	June 17
50	50	46	46	34	34	35	35

Figure 1

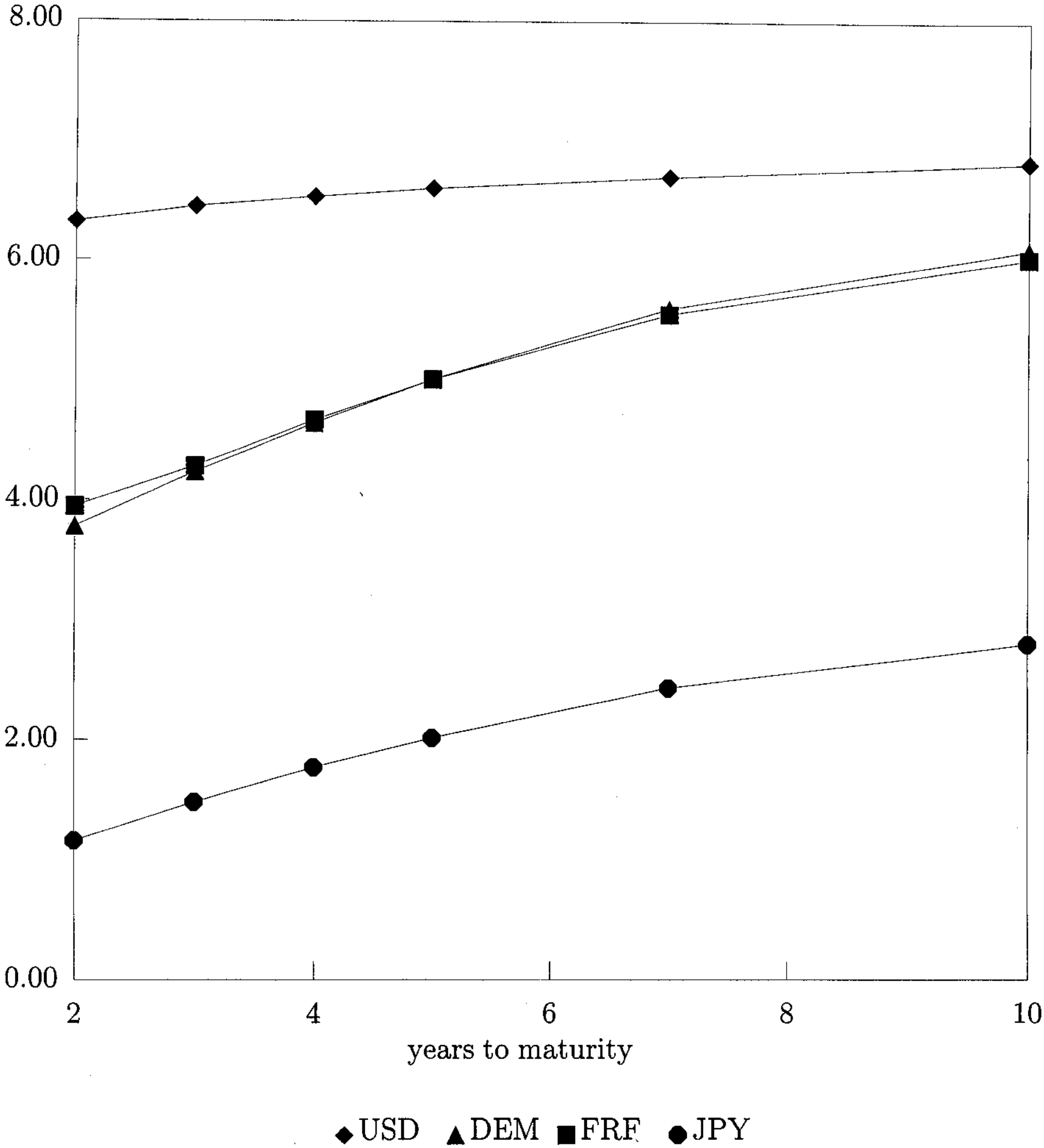
Three-month FRA curves 17June97



3-mo. Eurodeposit and FRAs with 3-mo. maturities (midpoints). Source: Reuters

Figure 2

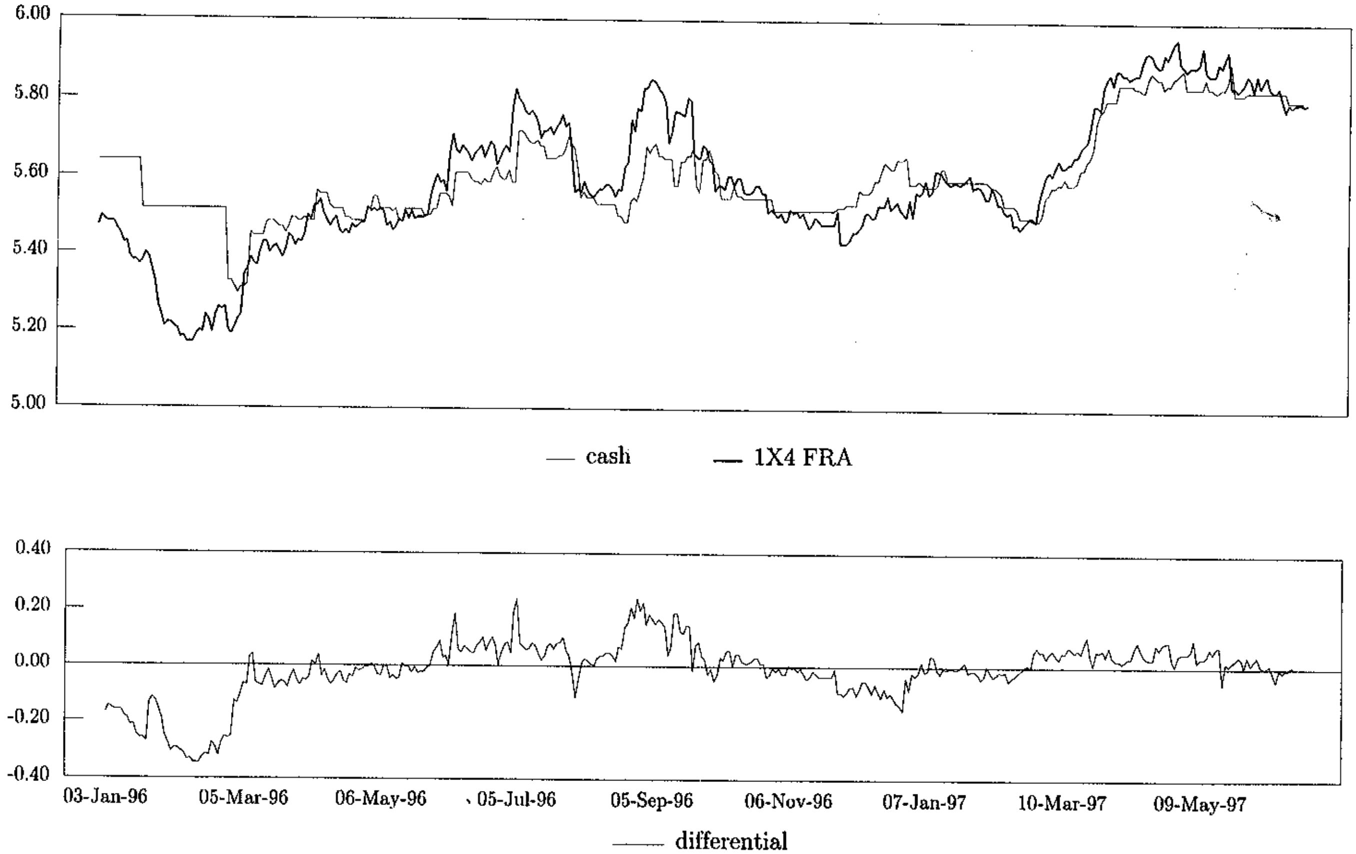
Swap curves 17June97



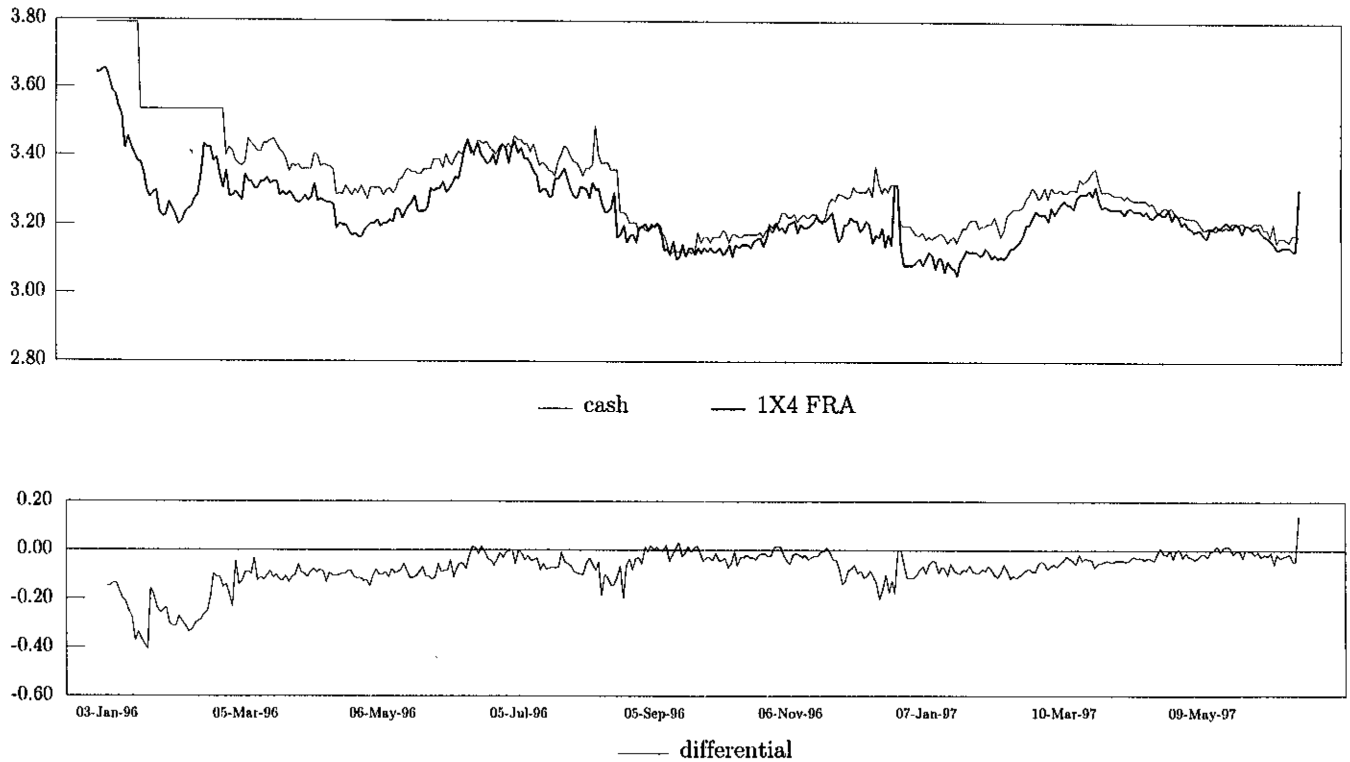
Plain-vanilla swaps, six-month reset interval (midpoints). Source: Reuters

Figure 3

Three-month cash and forward rates: USD



Three-month cash and forward rates: DEM

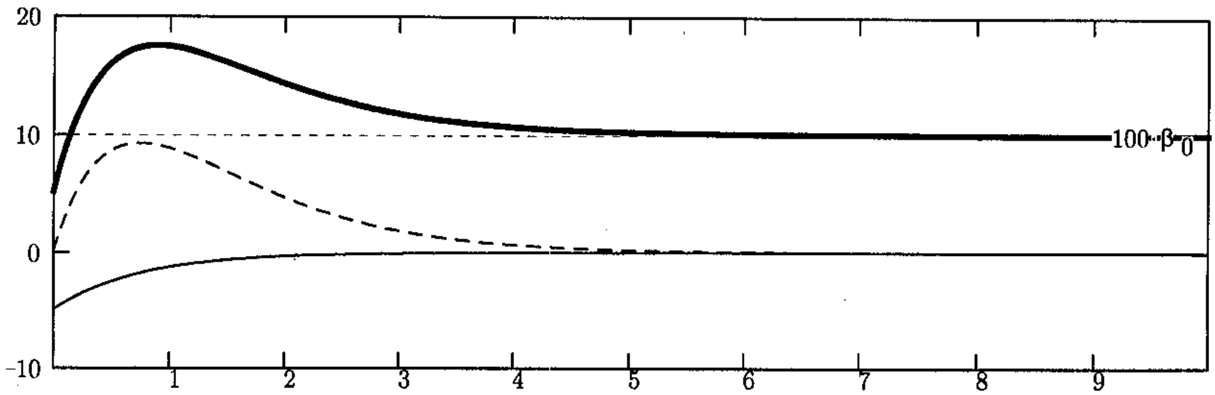


3-mo. Eurodeposit and FRAs with 3-mo. maturities (midpoints). Source: Reuters

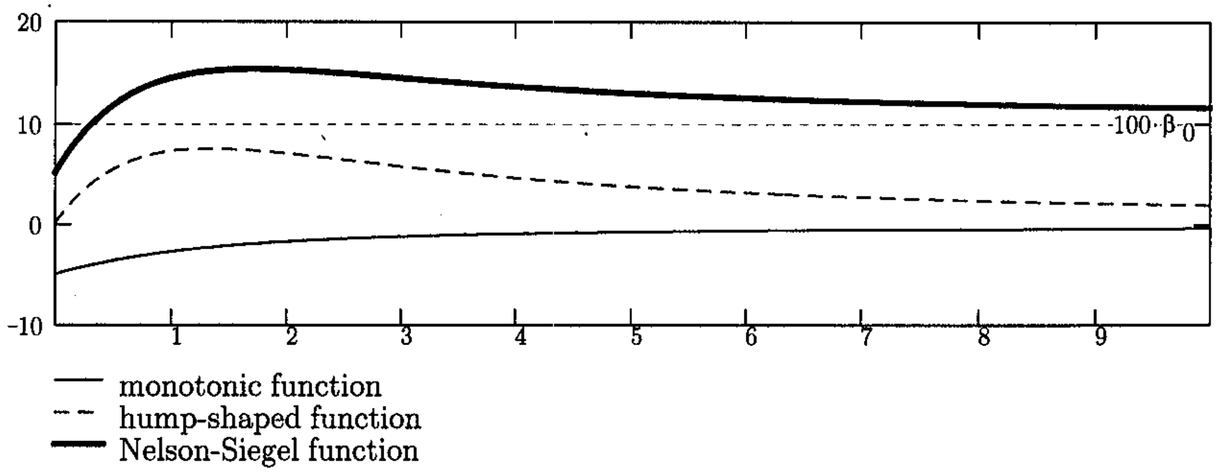
Figure 4

The Nelson-Siegel term structure function

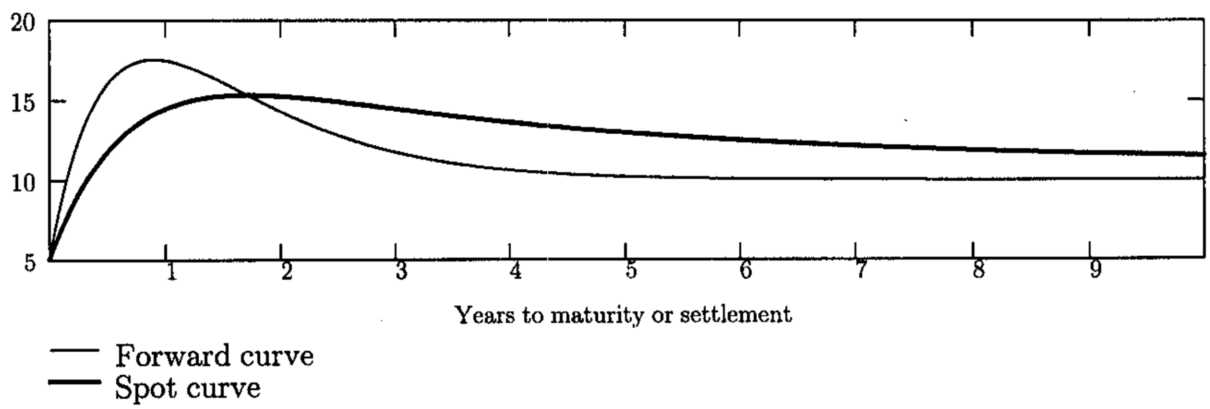
Spot curve and its components



Forward curve and its components



Spot and forward curves

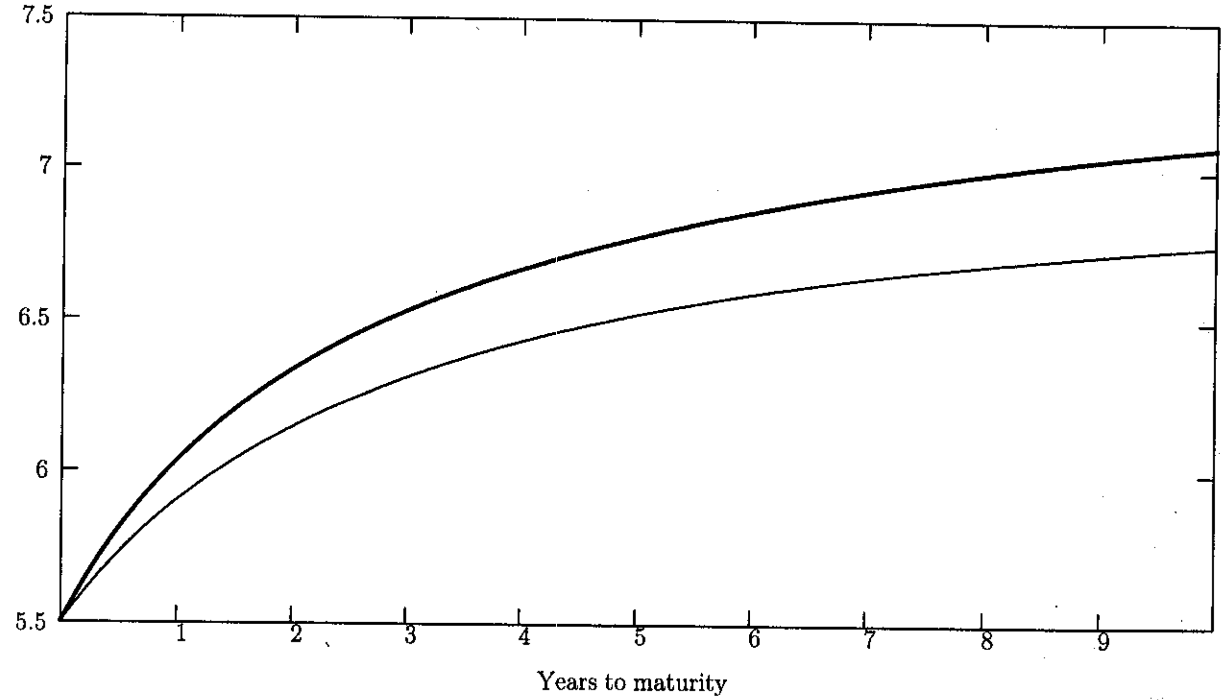


Parameter values: $\beta_0 = 0.1$, $\beta_1 = 0.05$, $\beta_2 = 0.25$, $\theta = 0.75$. Rates on *y*-axes in percent.

Figure 5

Term structure of U.S. dollar interest rates

Spot rates



Overnight forward rates

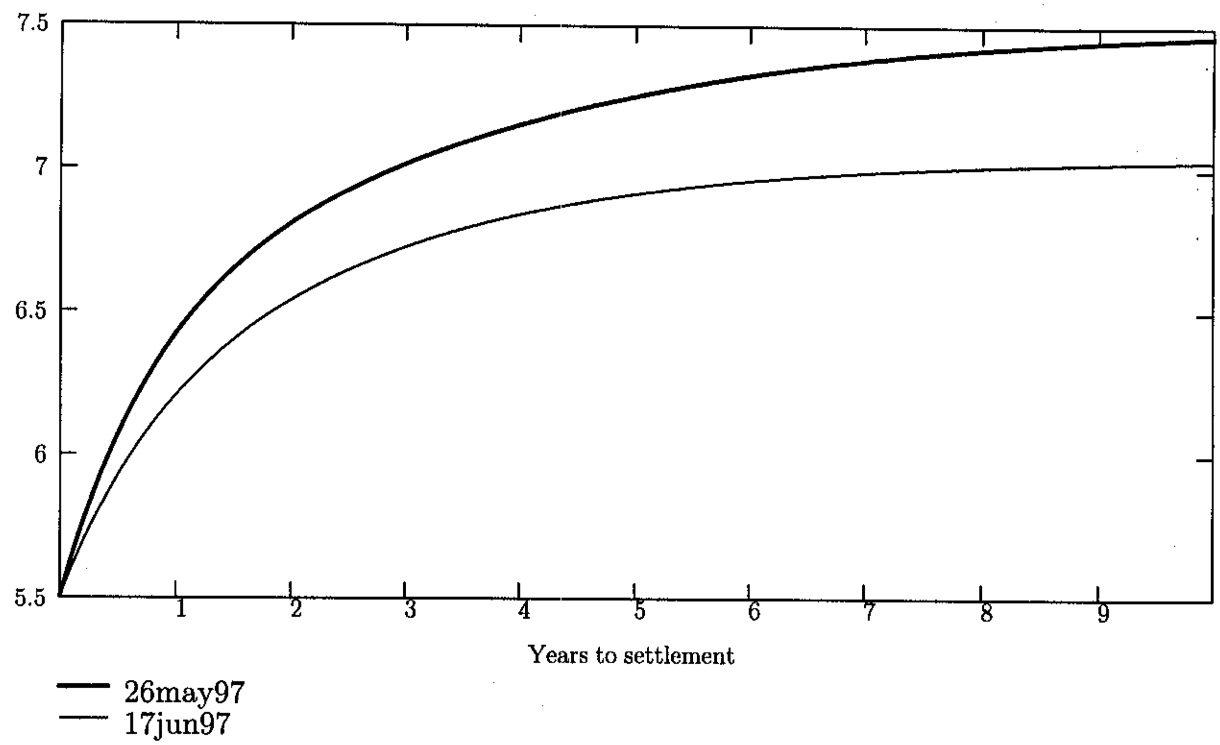


Figure 6

Term structure of German mark interest rates

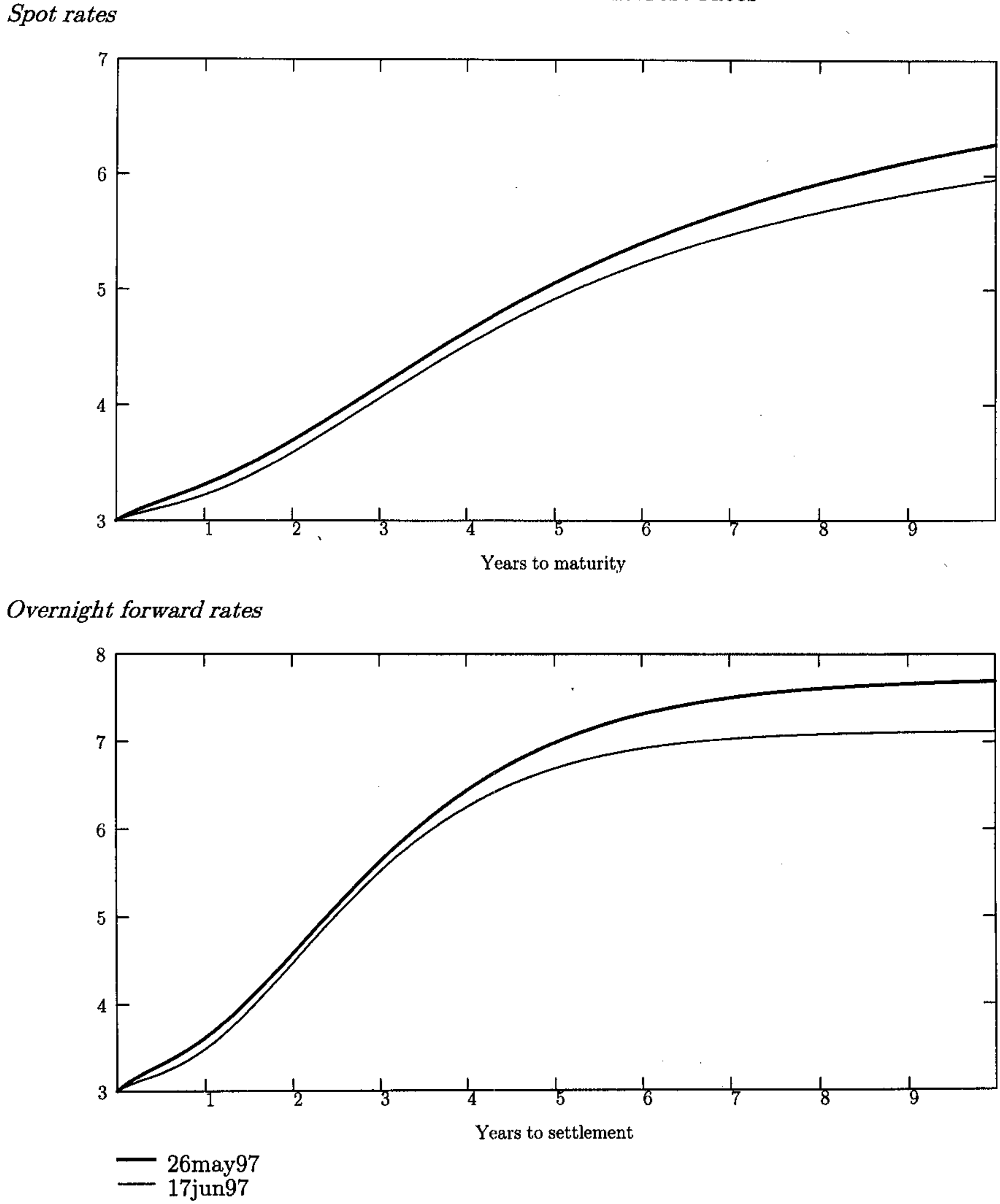
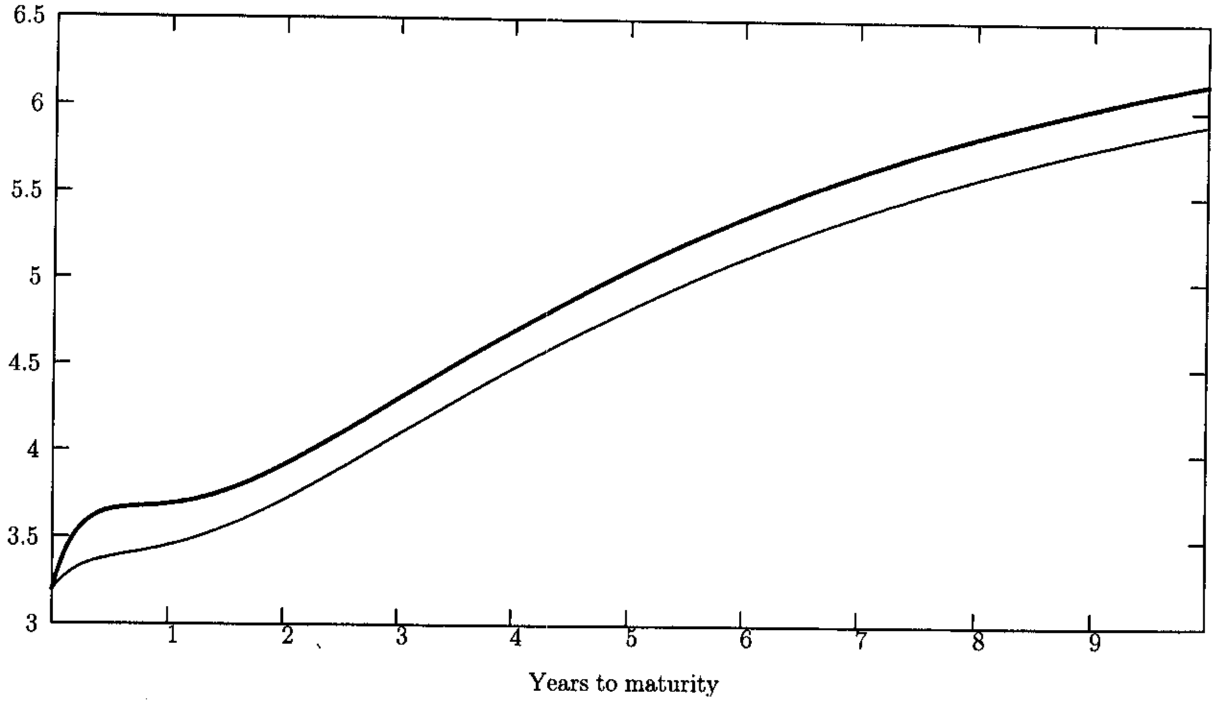


Figure 7

Term structure of French franc interest rates

Spot rates



Overnight forward rates

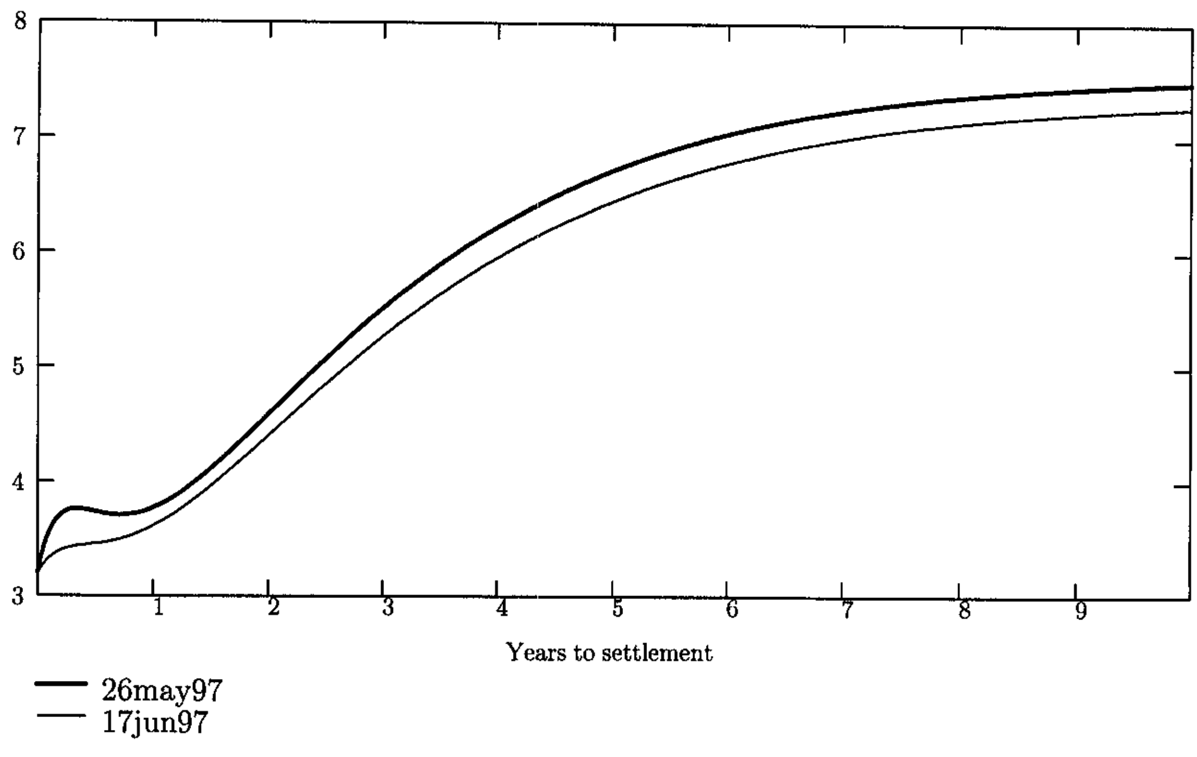
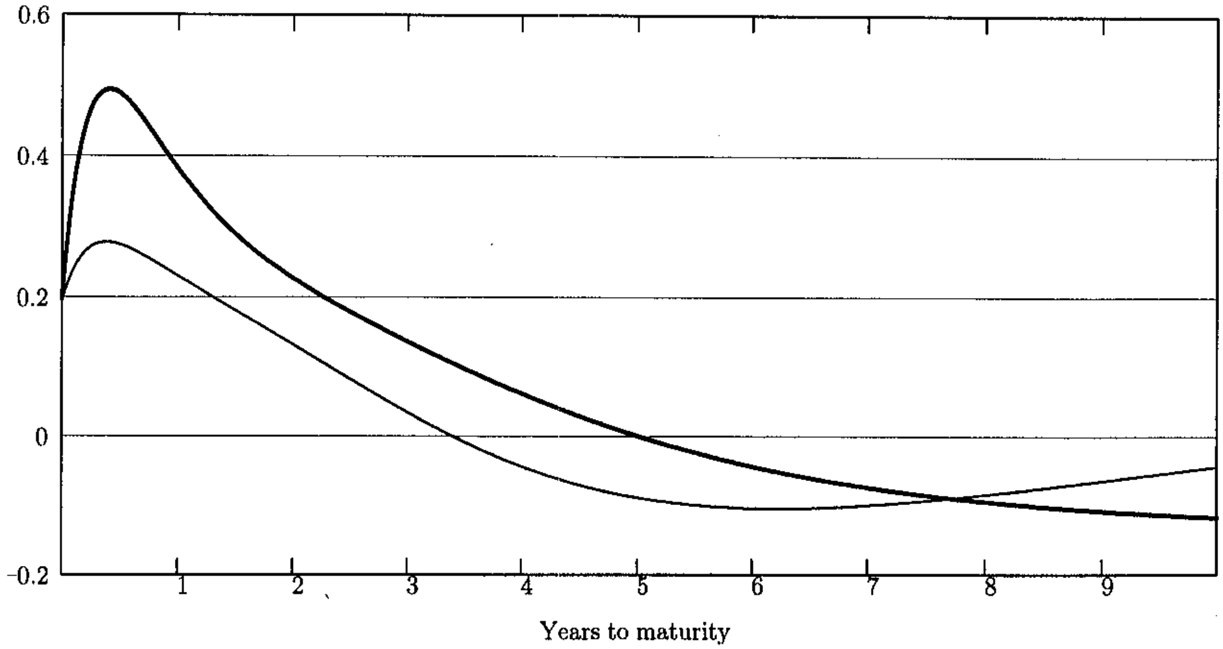


Figure 8
Term structure of interest rate differentials
French franc minus German mark

Spot rates



Overnight forward rates

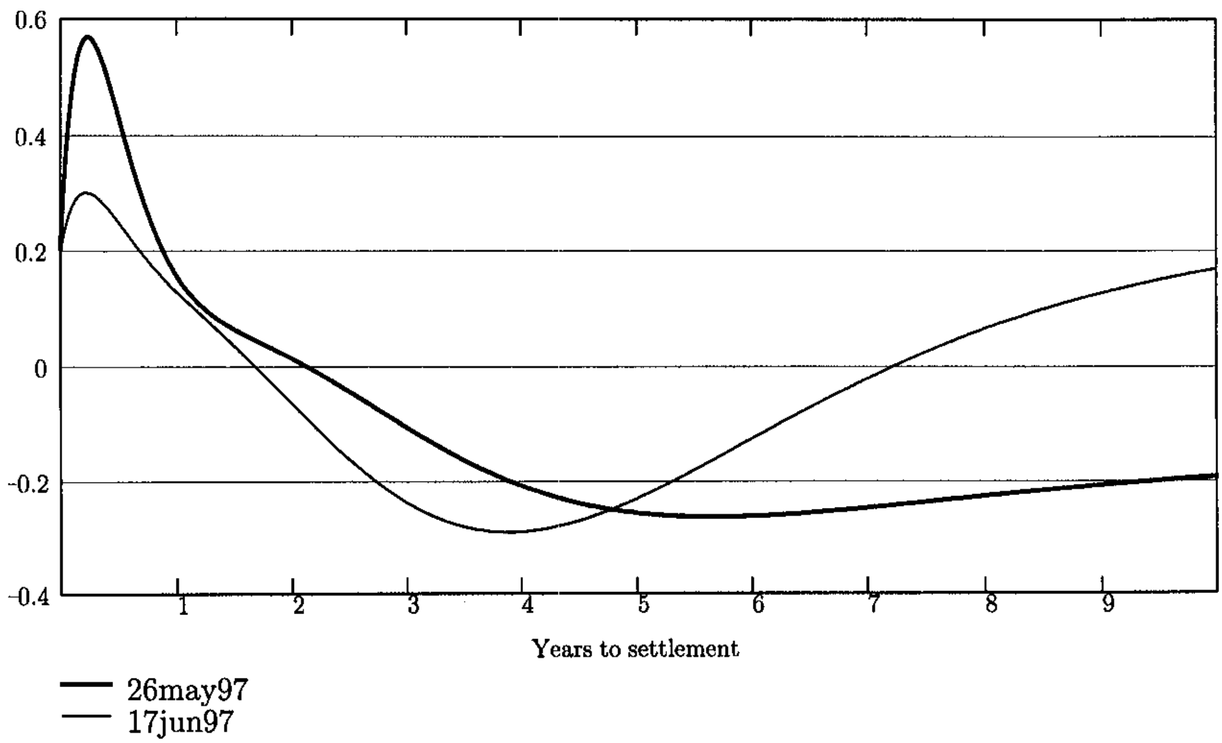
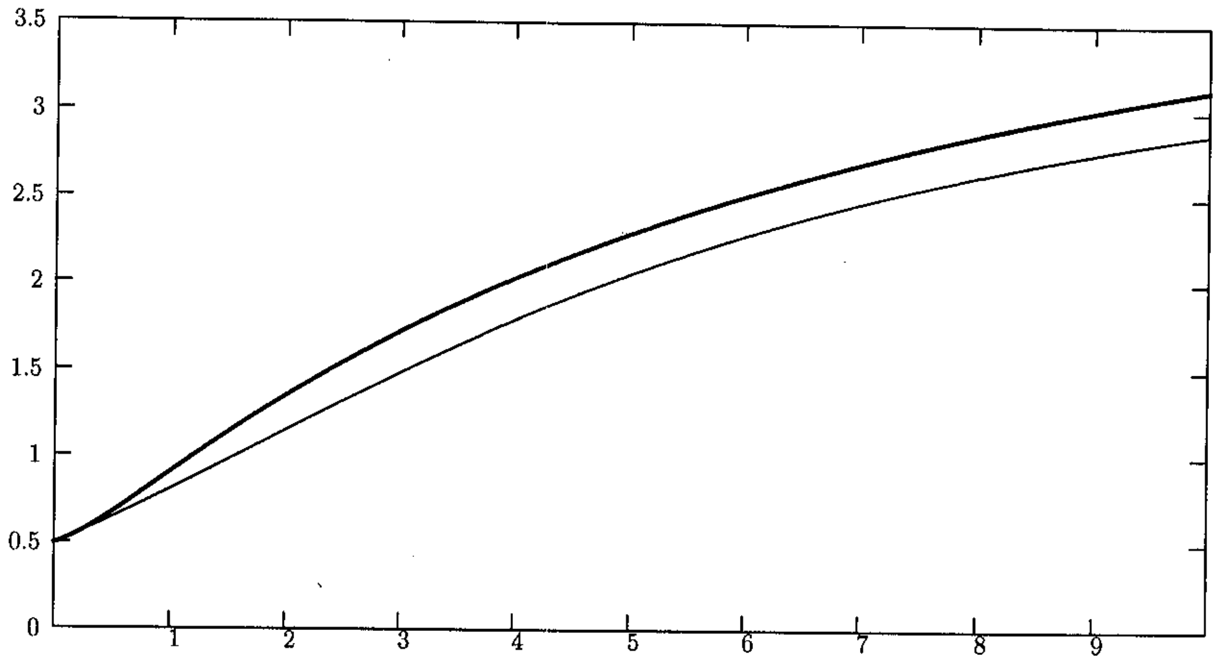


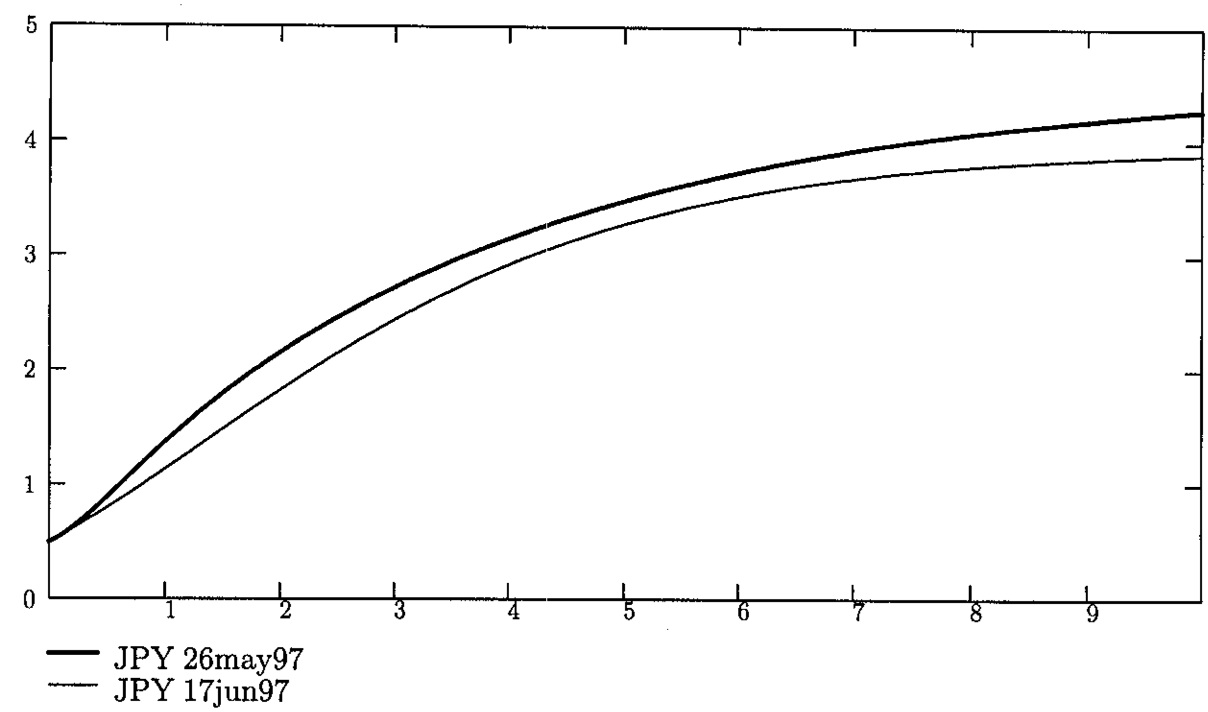
Figure 9

Term structure of Japanese yen interest rates

Spot rates



Overnight forward rates



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9801. Higgins, Matthew, and Carol Osler. "Asset Market Hangovers and Economic Growth: U.S. Housing Markets." January 1998.

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