Time-Consistent Fiscal Policy and Heterogeneous Agents

Roc Armenter*
Federal Reserve Bank of New York

First Draft: April 2004
This Draft: June 2006

Abstract

This paper characterizes the time-consistency properties of the set of Pareto efficient (or second best) fiscal policies, in a two-class, stochastic economy similar to that in Judd (1985). The key finding is that the continuation of any Pareto efficient policy is always Pareto efficient. Hence, to require any policy revision to be approved by unanimity safeguards the time consistency of efficient fiscal policy. I also show that any Pareto efficient policy from a timeless perspective can be rendered time consistent by a policymaker whose objective function is given by a utilitarian social welfare function with precise welfare weights. These results link the policymaker's equity considerations with the credibility of efficient fiscal policy.

J.E.L. Codes: E61, E62.

1 Introduction

In the representative agent model, the optimal fiscal policy is usually not time-consistent. Hence its implementation in a rational expectations equilibrium requires a commitment technology, as pointed out in Kydland and Prescott (1977) and Fischer (1980). However, fiscal

*International Research Department, Federal Reserve Bank of New York, 33 Liberty Street, New York, NY 10045. E-Mail: roc.armenter@ny.frb.org. I would like to thank first my advisors Lawrence Christiano and Gadi Barlevy. For helpful comments I am primarily indebted to Martin Bodenstein as well as Stefania Albanesi, Martin Eichenbaum, Per Krusell, Sergio Rebelo, and Şevin Yeltekin, and seminar participants at Northwestern University, the Federal Reserve Bank of Chicago, and the SED 2005 meetings. Financial support from Fundación Rafael del Pino is gratefully acknowledged. The views expressed here are those of the author and do not necessarily reflect the position of the Federal Reserve Bank of New York or the Federal Reserve System.

policy is rarely coded in constitutions—an obvious mechanism device. There is a large literature exploring alternative mechanisms that render the optimal fiscal policy time-consistent. In the absence of asset taxation, Lucas and Stokey (1983) show how one can restructure government debt holdings to ensure that the optimal fiscal policy is time-consistent. Chari and Kehoe (1990) and Stokey (1989) note that history-dependent equilibria can make the optimal fiscal policy sustainable in equilibrium.

In this paper I analyze the set of Pareto efficient (or second best) fiscal policies in the context of heterogeneous agents. I analyze a two-class, stochastic economy drawn from Judd (1985). There are two types of households who differ in the factor endowments, correspondingly labelled capitalists and workers. The fiscal policy instruments available are linear tax rates on labor and capital, as well as riskless government securities. Markets are complete, but worker households are excluded from trading any asset. In a very similar environment, Judd (1985) shows that an asymptotic positive capital tax is never a Pareto efficient policy.[1] Assuming isoelastic preferences for the capitalist household, I show that every Pareto efficient policy features a zero capital tax from date $t = 1$ onwards.[2] There is a surprising lack of diversity within the set of Pareto efficient policies.

I show that equity considerations, while known not to shape Pareto efficient policies, are a key determinant of their time-consistency properties. The key finding is that, at any date, the continuation of any Pareto efficient policy is Pareto efficient. In other words, given any Pareto efficient policy, at any date there is no policy revision capable of improving the welfare of every household. A stark implication is that a "unanimous approval" requirement is sufficient to ensure the time-consistency of efficient fiscal policy.

The intuition is simple. The time inconsistency in fiscal policy arises from the eventual inelasticity of the capital supply. Once investment decisions are done, a capital tax mimics lump sum taxation. Thus, an ex-post increase in capital taxes allows a reduction in the inefficiency associated with distortionary labor taxation. There is a clear redistributive pattern associated with such a policy revision, as the fiscal burden is shifted toward capitalist households. Indeed, the capitalist household will be worse off as capital taxes were zero everywhere under the original Pareto efficient policy. Hence, the efficiency gains cannot map into Pareto superior allocations and no policy revision will achieve unanimity.

Since no deviation from a Pareto efficient policy is Pareto efficient, the time-consistency of Pareto efficient policies is compatible with Paretian social welfare functions. I explore the class of utilitarian social welfare functions. I map the entire class into a set of policymaker types who differ on the constant weight assigned to the capitalist household in the welfare function. A unconstrained version of the policy problem is introduced, whose solution is dynamically consistent. Hence, whenever the constrained and unconstrained solutions coincide, the former is "time consistent." Using this device to characterize time consistent solutions, I

[1] The same result is mentioned in Chamley (1986).

[2] This result is an extension of the Chari and Kehoe (1998) analysis of the optimal fiscal policy in a representative agent economy.

show that there exists a policymaker type who does not find it optimal to deviate from her date $t = 0$ policy choice at any posterior date.

Next I explore how delegation within Paretian policymakers can solve the time-inconsistency problem. I show that any Pareto efficient policy from a timeless perspective can be rendered time-consistent by appointing a precise policymaker type.[3] The welfare weights of the delegate policymaker type do not need to coincide with the welfare weights that index the choice of policy among the Pareto efficient set at date $t = 0$. In line with the previously discussed intuition, I show that equity considerations must be biased toward the capitalist household in order to achieve the time-consistency of Pareto efficient policy.

Why only policy from a timeless perspective can be delegated? The answer is that the timeless component of any Pareto efficient policy is fully determined by efficiency considerations: there is no room for redistribution in the long run. All policymaker types value efficiency so twisting the equity considerations of the policymaker has no other impact that ensuring the time consistency of the policy. What delegation cannot do is implement the ideal level of redistribution along the transition path. Interestingly, delegation presents a novel redistribution-efficiency trade-off: society may choose to forgo equity goals in order to ensure long term efficiency, even if distortion-free redistribution is available in the form of contemporaneous capital taxation.[4]

Krusell (2002) analyzes time-consistent redistribution in a similar two-class economy. His focus is on the computation of time-consistent equilibria under an utilitarian policymaker with a negligible weight on capitalists. Krusell (2002) shows that capital income tax rates are substantially lower than 100%, yet they are usually different from the Ramsey policy—the symmetric Pareto efficient policy in the terminology of this paper. In contrast, I characterize the time-consistency properties of the whole set of Pareto efficient policies. Accordingly, delegation can be considered as a solution to the time-consistency problem.

There has been previous work relating redistribution and the credibility of public debt. Dixit and Londregan (2000), presenting an explicit political model, show that if political power and government bond holdings are positively correlated, then government debt repayment is credible. They illustrate the argument with an example where human capital formation is the alternative use for wealth. Sleet and Yeltekin (2002) argue that if debt market participants possess sufficient political influence, they would block debt default, increasing the set of sustainable debt policies. This paper undertakes a more complete model of fiscal policy. It also emphasizes that the relevant redistribution considerations are along the factor endowment distribution.

Bassetto (1999) and Albanesi (2002) also deal with the time-inconsistency problem of fiscal policy with heterogeneous agents, but do not consider direct asset taxation. They show that by manipulating the distribution and maturity structure of government assets,

[3]Section 5 includes the definition of Pareto efficient policy from a timeless perspective.

[4]Building on this paper, Armenter (2004) shows how a simple model of representative democracy with rational voters can sustain Pareto efficient policy. A society in which workers predominate may elect a capitalist-friendly policymaker, forgoing redistribution in order to preserve efficient fiscal policy.

the optimal fiscal policy can be made time-consistent. These results relate to the work of Lucas and Stokey (1983) and Persson and Svensson (1986), and direct asset taxation is never included. Another important distinction is the instrument necessary to implement the optimal fiscal policy. In Bassetto (1999) and in Albanesi (2002), the government must be able to influence the distribution of public debt across households. In this paper, time-consistency is achieved by shifting the policymaker's redistributive goals. The latter contributes to our understanding and design of political institutions.

The delegation results have a resonance with the famous "conservative central banker" of Rogoff (1985).[5] However, there is a key difference in the class of policymakers considered. In Rogoff (1985), policymakers differ in the relative weights assigned to the inflation and output gap. Appointing a conservative central banker is then equivalent to "*[giving] the central bank concrete incentives to achieve an intermediate monetary target.*"[6] The theory calls for handing a contract and independence to the central bank. In contrast, the class of policymakers I consider suits much better the desired outcome of a political process. Delegation occurs within policymakers whose objective functions are social welfare functions with desirable properties: they are Paretian and complete as they can provide a welfare ranking for any public choice problem. It is interesting to note that, despite the success of the movement toward central bank independence, no similar call has been heard for the management of fiscal policy.

Persson and Tabellini (1994) show that an once-and-for-all representative democracy election can ensure the time-consistency of optimal fiscal policy. The mechanism at work is closely related to the delegation results featured here. However, Persson and Tabellini (1994) limit their analysis to a two-period, deterministic economy and to a particular policy out of the Pareto efficient set. This paper states the conditions such that delegation can effectively implement any Pareto efficient policy in an infinite-horizon, stochastic economy. This is no trivial extension. As discussed in Armenter (2004), an implication is that delegation can be implemented with sequential elections.

The rest of the paper is organized as follows. Section 2 describes the economy and introduces the private-sector competitive equilibrium. The set of Pareto efficient policies is characterized in Section 3. I move to the time-consistency properties of fiscal policy in Section 4. Delegation and its possibilities are explored in Section 5. Finally, Section 6 concludes.

[5]However, I do not claim that the results presented here extend to monetary policy.

[6]Rogoff (1985), page 1170.

2 The Economy and Private-Sector Competitive Equilibrium

Let $\{s_t\}_{t=0}^{\infty}$ be an exogenous first-order Markov process with finite possible realizations $s \in S$ and transitions probabilities given by Π. Let $s^t = \{s_0, s_1, ..., s_t\}$ denote the history up to date $t \geq 0$. The set of possible histories at date j that are continuation of some history s^t, $j \geq t$, is denoted $S^j(s^t)$. However, I will omit the dependence on s^t if there is no confusion possible.

The economy is populated by a representative firm, a policymaker, and a continuum of households. A measure ψ of households are type $i = 1$, labelled **capitalist households**. They own all of the capital stock $k(s^t)$ and government debt holdings $b(s^t)$ in the private-sector. They have no time endowment and hence no labor income. At every node s^t, capitalist households have access to a complete array of one-period state contingent bonds.

Households of type $i = 2$ are **worker households**. They have an endowment of one unit of time at every node s^t to split between labor supply $n(s^t)$ and leisure $1 - n(s^t)$. They do not have access to any savings means, neither physical capital nor bonds. I normalize the measure of worker households to one.

Both types of agents view the fiscal plan τ,

$$\tau = \left\{\tau^k\left(s^t\right), \tau^n\left(s^t\right), b^g\left(s^{t+1}\right) | \forall s^t \in S^t, t \geq 0\right\}$$

and prices p,

$$p = \left\{r^k\left(s^t\right), w\left(s^t\right), \left\{q\left(s^t, s'\right)\right\}_{s' \in S} | \forall s^t \in S^t, t \geq 0\right\}$$

as given. The fiscal plan consists of linear tax rates on capital and labor, τ^k and τ^n, respectively. Each tax rate is contingent on the history s^t. The government debt holdings are denoted by b^g. Prices p include the pre-tax capital and wage rates, r^k and w, as well as the discount price of one-period state contingent bonds.

As all households within each type are identical and I will consider only symmetric equilibria, it is convenient to consider an economy populated with just two representative households, a capitalist and a worker.

The capitalist household's preferences over consumption plan c_1 after history s^t are given by

$$U^1\left(c_1, s^t\right) = \sum_{j=t}^{\infty} \sum_{s^j \in S^j} \beta^{j-t} \pi\left(s^j | s_t\right) u^1\left(c_1\left(s^j\right)\right)$$

with $0 < \beta < 1$ and $c_1 = \{c_1(s^t) | \forall s^t \in S^t, t \geq 0\}$. I will further assume that the capitalist's preferences are given by the isoelastic utility function

$$u^1(c_{1t}) = \frac{c_{1t}^{1-\sigma}}{1-\sigma}$$

for $\sigma \geq 0$, with $u^1(c_{1t}) = \log(c_{1t})$ for $\sigma = 1$.

The **capitalist household problem** at node s^t consists of choosing allocation plans c_1, k and b given fiscal plan τ and prices p, to solve

$$\max U^1\left(c_1, s^t\right) \tag{\textit{Cap.}-HH}$$

subject to

$$c\left(s^j\right) + \sum_{s_{j+1}} q\left(s^j, s_{j+1}\right) b\left(s^{j+1}\right) + k\left(s^j\right) \leq a\left(s^j\right),$$

$$a\left(s^j\right) = \left(\left(1 - \tau^k\left(s^j\right)\right) r^k\left(s^j\right) + 1 - \delta\right) k\left(s^{j-1}\right) + b\left(s^j\right)$$

and

$$\begin{aligned} 0 &\leq c_1\left(s^j\right), \\ 0 &< k\left(s^j\right), \\ -B &\leq b\left(s^j\right) \end{aligned}$$

for all $s^j \in S^j, j \geq t$, with $k\left(s^{t-1}\right)$ and $b\left(s^t\right)$ given. In order to avoid the possibility of a zero gross return to capital, I assume the capital tax is bounded above by one, $\tau^k\left(s^j\right) \leq 1$, and the depreciation rate is non-negative and strictly less than one, $0 \leq \delta < 1$.

The worker household's preferences over $c_2 = \{c_2\left(s^t\right) | \forall s^t \in S^t, t \geq 0\}$ and $n = \{n\left(s^t\right) | \forall s^t \in S^t, t \geq 0$ after history s^t are given by

$$U^2\left(c_2, n, s^t\right) = \sum_{j=t}^{\infty} \sum_{s^j \in S^j} \beta^{j-t} \pi\left(s^j | s_t\right) u^2\left(c_2\left(s^j\right), n\left(s^j\right)\right)$$

where u is a standard utility function, i.e., differentiable, strictly increasing and concave, with the usual Inada conditions in place.

The **worker household problem** after history s^t is to set c_2 and n, given fiscal plan τ and prices p, to solve

$$\max U^2\left(c_2, n, s^t\right) \tag{\textit{Wor.}-HH}$$

subject to

$$c_2\left(s^j\right) \leq \left(1 - \tau^n\left(s^j\right)\right) w\left(s^j\right) n\left(s^j\right)$$

and

$$\begin{aligned} 0 &\leq c_2\left(s^j\right) \\ 0 &\leq n\left(s^j\right) \leq 1 \end{aligned}$$

for all $s^j \in S^j, j \geq t$.

The **representative firm** combines labor and capital inputs to produce final good y according to a standard constant returns to scale production function F. At every node s^t, it demands labor $n(s^t)$ and capital $k(s^{t-1})$ to maximize profits, taking factor prices $w(s^t)$ and $r^k(s^t)$ as given,

$$\max y\left(s^t\right) - r^k\left(s^t\right) k\left(s^{t-1}\right) - w\left(s^t\right) n\left(s^t\right) \tag{Firm}$$

subject to

$$y\left(s^t\right) \leq F\left(k\left(s^{t-1}\right), n\left(s^t\right), s_t\right).$$

The production function has a potential stochastic component given by state s_t.

The **government budget constraint** needs to hold at every node s^t,

$$g(s_t) + b^g\left(s^t\right) - \sum_{s^{t+1}} q\left(s^t, s^{t+1}\right) b^g\left(s^{t+1}\right) \leq \tau^n\left(s^t\right) w\left(s^t\right) n\left(s^t\right) + \tau^k\left(s^t\right) r^k\left(s^t\right) k\left(s^{t-1}\right). \tag{G.B.C.}$$

The government expenditure $g(s_t)$ is an exogenous process governed by s_t. Note that $b^g > 0$ is an obligation of the government.

Finally, the aggregate **resource constraint** is satisfied at every s^t:

$$c_1\left(s^t\right) + c_2\left(s^t\right) + g(s_t) + k\left(s^t\right) \leq y\left(s^t\right) + (1-\delta) k\left(s^{t-1}\right). \tag{R.C.}$$

By the Walras Law bond markets clear: $b(s^t) = b^g(s^t)$ for all $s^t \in S^t, t \geq 0$.

All is set to describe the competitive equilibrium. The economy's initial conditions are given by the initial stock of capital, bonds and the exogenous state of the economy, (k_0, b_0, s_0), with $k_0 > 0$. However, I define the competitive equilibrium for any state (k_t, b_t, s^t), so notation can accommodate the continuation of an equilibrium at any node s^t.

Definition 1 *A* ***private-sector competitive equilibrium*** *given* (k_t, b_t, s^t) *is a set of allocations* $x = \{c_1, k, c_2, n\}$*, prices* p*, and a fiscal plan* τ *such that:*

1. *Allocations* x *solve both household problems (Cap.-HH) and (Wor.-HH) at node* s^t *given* p*,* τ *and* k_t, b_t*.*

2. *Allocations* x *solve the firm problem (Firm) for* $\forall s^j \in S^j$*,* $j \geq t$*.*

3. *The government budget constraint (G.B.C.) is satisfied for* $\forall s^j \in S^j$*,* $j \geq t$*.*

4. *The resource constraint (R.C.) holds for* $\forall s^j \in S^j$*,* $j \geq t$*.*

Let $X(k_t, b_t, s_t)$ be the set of allocations such that there exists τ and p for which $\{x, p, \tau\}$ is a private-sector competitive equilibrium given (k_t, b_t, s_t). Let $X_0 \equiv X(k_0, b_0, s_0)$. A fiscal policy plan τ is said to be feasible given (k_t, b_t, s_t) if there exists p and x such that $\{x, p, \tau\}$ is a private-sector competitive equilibrium. Abusing notation a little bit, I will also define utility functions U^1 and U^2 over allocations x.

2.1 Competitive Equilibrium Conditions

In this subsection I completely characterize a private-sector competitive equilibrium $\{x, p, \tau\}$ and provide a lemma on the sequential nature of the private-sector competitive equilibrium.

The following first-order conditions are necessary and sufficient to characterize the solution of the capitalist household problem (*Cap.*-HH).

$$\begin{aligned} q\left(s^t, s^{t+1}\right) u_c^1\left(s^t\right) &= \beta\pi\left(s_{t+1}|s_t\right) u_c^1\left(s^{t+1}\right), \\ u_c^1\left(s^t\right) &= \beta\sum_{s_{t+1}}\pi\left(s_{t+1}|s_t\right) u_c^1\left(s^{t+1}\right)\left[\left(1-\tau^k\left(s^{t+1}\right)\right) r^k\left(s^{t+1}\right)+1-\delta\right] \end{aligned}$$

for all $s^t \in S^t, t \geq 0$, and the transversality condition

$$\lim_{t\to\infty}\sum_{s_{t+1}}\pi\left(s_{t+1}|s_t\right) u_c^1\left(s^{t+1}\right) b\left(s^{t+1}\right) = 0$$

where $u_c^1\left(s^t\right) = \frac{\partial u^1\left(c_1\left(s^t\right)\right)}{\partial c_1}$ and so on.

The first-order conditions imply the following arbitrage condition between bond prices and capital rental rate,

$$1 = \sum_{s_{t+1}} q\left(s^t, s^{t+1}\right)\left[\left(1-\tau^k\left(s^{t+1}\right)\right) r^k\left(s^{t+1}\right)+1-\delta\right]. \tag{1}$$

Using (1), the sequence of budget constraints can be collapsed in an intertemporal budget constraint,

$$\sum_{t=0}^{\infty}\sum_{s^t} q_0\left(s^t\right) c\left(s^t\right) \leq a\left(s_0\right) \tag{2}$$

with bond prices rewritten

$$q_0\left(s^t\right) q\left(s^t, s^{t+1}\right) = q_0\left(s^{t+1}\right).$$

It is straightforward to solve for date 0 prices, using the previously derived first-order conditions

$$q_0\left(s^t\right) = \beta^t\pi\left(s^t|s_0\right)\frac{u_c^1\left(s^t\right)}{u_c^1\left(s_0\right)}. \tag{3}$$

Note that since $u_c^1 > 0$, then $q_0\left(s^t\right)$ is finite and strictly positive for all s^t, and the budget constraint can be evaluated with strict equality.

The solution to the worker household problem (*Wor.*-HH) is given by

$$-\frac{u_n^2\left(s^t\right)}{u_c^2\left(s^t\right)} = \left(1-\tau^n\left(s^t\right)\right) w\left(s^t\right) \tag{4}$$

$$c_2\left(s^t\right) \leq \left(1-\tau^n\left(s^t\right)\right) w\left(s^t\right) n\left(s^t\right) \tag{5}$$

for all $s^t \in S^t$, $t \geq 0$. Since the worker household does not have access to savings, the sequence of budget constraints cannot be collapsed into a date 0 intertemporal budget constraint.

The representative firm problem implies that at every s^t, factor prices equal the respective marginal product:

$$\begin{aligned} F_k\left(s^t\right) &= r^k\left(s^t\right), \\ F_n(s^t) &= w\left(s^t\right). \end{aligned} \tag{6}$$

Finally, the sequence of government budget constraints $(G.B.C.)$ can be collapsed in a date 0 intertemporal budget constraint,

$$\sum_{t=0}^{\infty}\sum_{s^t} q_0\left(s^t\right)\left(\tau^n\left(s^t\right) w\left(s^t\right) n\left(s^t\right) + \tau^k\left(s^t\right) r^k\left(s^t\right) k\left(s^t\right) - g\left(s_t\right)\right) \geq b_0^g \tag{7}$$

It is possible to show that the resource constraint $(R.C.)$ must be binding since u_1^c and u_2^c are strictly positive. Then it is straightforward to show that (7) must be binding with strict equality as well.

Remark 1 *Private sector competitive equilibrium allocations x and prices p given a feasible fiscal policy τ and initial conditions (k_0, b_0, s_0) are completely characterized by equation (2), and equations (1), (3), (4), (5), (6) and resource constraint (R.C.) for all $s^t \in S^t, t \geq 0$.*

Note that there is no need to include the government intertemporal budget constraint (7), as it follows from (2) combined with $(R.C.)$ and (5) at every node $s^t \in S^t$, $t \geq 0$.

The private-sector competitive equilibrium is defined for any triplet (k_t, b_t, s^t). The following lemma states a standard result linking competitive equilibria at nodes that are a continuation of each other.

Lemma 1 *If triplet $\{x, \tau, p\}$ constitutes a private-sector competitive equilibrium at date $t = 0$, then it constitutes a private-sector competitive equilibrium at any continuation node $\left(k\left(s^{t-1}\right), b\left(s^t\right), s^t\right)$ for all $s^t \in S^t$, $t \geq 0$.*

Proof. Conditions 2 to 4 of the definition of a competitive equilibrium will obviously be satisfied from any date $t > 0$ onward. Assume there exists some node s^t such that x does not solve one (or both) of the two representative household problems. This will imply that there exists x' such that at least one household achieves strictly more utility than under x. Set plan x'' such that $x''_j\left(s^j\right) = x_j\left(s^j\right)$ for $j < t$, and $x''_j\left(s^j\right) = x'_j\left(s^j\right)$ for $j \geq t$. It is straightforward to show that x'' satisfies the correspondent budget constraints at all $j \geq t$ given p and τ. Plan x'' also delivers strictly more utility; therefore it would contradict the optimality of x at date 0 ■

3 Set of Pareto Efficient Policies

In this section I characterize the set of Pareto efficient (or second best) allocations and corresponding policies. The allocations are second best because they must constitute a private-sector equilibrium and, by assumption, there are limited fiscal instruments available to the policymaker. In particular, it is the absence of lump sum taxation that will generally set first and second best allocations apart.

The set of allocations has to be further restricted or the policy problem would easily become trivial. Let $X_0(\alpha)$ denote the set of allocations x such that there is a private-sector competitive equilibria $\{x, p, \tau\}$ with

$$u_c^1(s_0)\left[\left(\left(1-\tau^k(s_0)\right)F_k(k_0, n(s_0)) + 1 - \delta\right)k_0 + b_0\right] = \alpha. \tag{8}$$

For the rest of the paper, α is fixed to an arbitrary positive number.

Without any restriction, it would be possible to implement a zero-distortion fiscal policy by taxing initial assets and financing government expenditure entirely out of the date 0 tax refunds. Several papers in the literature proceed by restricting instead the initial capital tax $\tau^k(s_0)$—see Chari, Christiano and Kehoe (1994) and Chari and Kehoe (1998) among others. Conditions of the form (8) rule out taxing initial value of assets through any fiscal scheme. They have the advantage of being able to easily accommodate the study of optimal fiscal policy from a timeless perspective, introduced in Woodford (2003).

The definition of Pareto efficiency is given over private-sector competitive equilibria.

Definition 2 *A private-sector competitive equilibrium $\{x, p, \tau\}$ is* ***Pareto efficient*** *if there is no $x' \in X_0(\alpha)$ such that*

$$\begin{aligned} U^1(x', s_0) &\geq U^1(x, s_0) \\ U^2(x', s_0) &\geq U^2(x, s_0) \end{aligned}$$

and at least one condition holds with strict inequality.

A policy τ is said to be Pareto efficient if there exist allocations x and prices p such that $\{x, p, \tau\}$ is Pareto efficient. The same for a Pareto efficient allocation.

It is well known that each element in the Pareto efficient set corresponds to the maximum of a utilitarian social welfare function (SWF) with a precise Pareto weight distribution.[7] I use this property to introduce a different characterization of the set of Pareto efficient policies. Below I define a class of policy equilibria, each indexed by the *policymaker type* λ, which spans the set of Pareto efficient policies.

[7]See Mas-Colell, Whinston and Green (1995), page 558, for a discussion and proof. The assumption of a convex utility possibility set is used in the proof.

A policymaker of type λ will value allocations x according to a utilitarian SWF, indexed by a constant Pareto weight $\lambda \geq 0$ assigned to the representative capitalist household. Hence, policymaker λ preferences over x are given by the **λ-welfare function**,

$$W^{\lambda}\left(x, s^{t}\right) = \lambda U^{1}\left(c_{1}, s^{t}\right) + U^{2}\left(c_{2}, n, s^{t}\right).$$

Note that for all policymaker types $\lambda > 0$, this is a Paretian SWF, i.e., for any x, x', if x is Pareto superior to x', then $W^{\lambda}\left(x, s^{t}\right) > W^{\lambda}\left(x', s^{t}\right)$.[8] Shortcut $W_{0}^{\lambda}\left(x\right)$ will be used for $W^{\lambda}\left(x, s_{0}\right)$.

The literature often assumes that the policymaker preferences over welfare distributions are given by a symmetric utilitarian SWF. The policymaker ψ corresponds to a symmetric SWF, as ψ is the measure of capitalist households. Note, though, that I treat the ψ-welfare function, W^{ψ}, as *a* social welfare function and not *the* social welfare function.

The policy equilibrium concept is named Ramsey equilibrium. As with all the objects involving a policy decision in this paper, my definition of policy equilibrium is required to be indexed by the policymaker type λ. A λ-Ramsey equilibrium is the best private-sector equilibrium according to the λ-welfare function.[9] The resulting policy will be referred to as λ-Ramsey policy. As discussed above, the whole class of λ-Ramsey equilibria for all $\lambda \geq 0$ spans the set of Pareto efficient private-sector competitive equilibria.

Definition 3 *A* ***λ-Ramsey Equilibrium*** *is a private-sector competitive equilibrium* $\{x, p, \tau\} \in X_{0}\left(\alpha\right)$ *such that there is no* $x' \in X_{0}\left(\alpha\right)$ *with*

$$W_{0}^{\lambda}\left(x'\right) > W_{0}^{\lambda}\left(x\right).$$

I will use the so called primal approach in order to solve for the λ-Ramsey policy. It is possible to solve for all prices and taxes given allocations using the necessary and sufficient conditions for the private-sector competitive equilibrium. Using this relationship, the policy problem can be thought of as choosing a feasible allocation plan subject to some carefully specified implementability constraints. These constraints must ensure that the choice set of the primal approach policy problem is identical to the set of private-sector competitive equilibrium allocations. There is excellent work detailing and illustrating the applications of the primal approach: see Chari et al. (1994), Chari and Kehoe (1998), and further references. The following proposition derives the implementability constraints for the two-class economy presented here.

[8] The set of policymaker types needs to be enlarged to include the policymaker $\lambda = \infty$, with

$$W^{\infty}\left(x, s^{t}\right) = U^{1}\left(c_{1}, s^{t}\right).$$

This proves that value sets are closed. For the same reason, policymaker type 0 cannot be excluded. Note that $\lambda = \infty$ and $\lambda = 0$ are only weakly Paretian.

[9] If for any fiscal plan τ there are multiple private-sector equilibria with distinct welfare properties, only the competitive equilibrium with higher λ-welfare is a λ-Ramsey equilibrium. This is implied by the second point of the definition below.

Proposition 2 *Let D be the set of allocations x that satisfy:*

1. *For some $\tau^k(s_0) \leq 1$,*

$$\sum_{t=0}^{\infty} \sum_{s^t} \beta^t \pi\left(s^t|s_0\right) u_c^1\left(s^t\right) c_1\left(s^t\right) = A(s_0) \qquad (\textit{Cap.-IC})$$

where

$$A(s_0) = u_c^1(s_0)\left[\left(\left(1-\tau^k(s_0)\right) F_k(k_0, n(s_0)) + 1 - \delta\right) k_0 + b_0\right].$$

2. *For all $s^t \in S^t, t \geq 0$,*

$$u_c^2\left(s^t\right) c_2\left(s^t\right) + u_n^2\left(s^t\right) n\left(s^t\right) = 0. \qquad (\textit{Wor.-IC})$$

3. *For all $s^t \in S^t, t \geq 0$, the resource constraint (R.C.) holds.*

Then the set D coincides with the set of competitive equilibrium allocations at date $t = 0$,

$$X_0 = D$$

Proof. First, I will show that $x \in X_0$ implies $x \in D$. Feasibility is trivial. Substitute the first-order condition with respect to bonds (3) into (2) and the implementability constraint (*Cap.-IC*) follows. In a similar fashion, for each s^t use (4) into (5) to show that (*Wor.-IC*) holds. Details about the strict sign are given in the description of the private-sector competitive equilibrium.

Now I prove that $x \in D$ implies $x \in X_0$, i.e., for any allocation plan satisfying (*Cap.-IC*) and (*Wor.-IC*) and (*R.C.*) for all s^t, there exists prices p and a fiscal plan τ such that they constitute a private-sector competitive equilibrium. For this, let $x \in D$ satisfy the implementability constraints. I propose the following candidates for prices:

$$q_0\left(s^t\right) = \beta^t \pi\left(s^t|s_0\right) \frac{u_c^1\left(s^t\right)}{u_c^1(s_0)}$$

and

$$\begin{aligned} r^k\left(s^t\right) &= F_k\left(s^t\right), \\ w\left(s^t\right) &= F_n(s^t). \end{aligned}$$

These prices are well defined, i.e., they are positive and finite. Note that $c\left(s^t\right) = 0$ cannot satisfy (*Cap.-IC*) since $u_c^1(s_0) > 0$. By construction, (3) and (6) are satisfied. The capitalist household budget constraint (2) only needs some algebra. Following the identical procedure, let $\tau^n\left(s^t\right)$ solve

$$-\frac{u_n^2\left(s^t\right)}{u_c^2\left(s^t\right)} = \left(1-\tau^n\left(s^t\right)\right) w\left(s^t\right).$$

This $\tau^n(s^t)$ will exist given the interiority conditions on c_2 and n. Again, by construction, (4) is satisfied and so is (5). To retrieve the capital tax, just solve the arbitrage condition (1) ■

Note that the implementability constraints restrict allocations to X_0 and not $X_0(\alpha)$.

Now all is set to state the λ-Ramsey problem in primal approach, which consists of finding the allocations x that are compatible with a private-sector competitive equilibrium and maximize the λ-welfare function.[10]

Definition 4 *The λ-**Ramsey problem** (in primal approach) is to set x and $\tau^k(s_0)$ to maximize*

$$W_0^\lambda(x)$$

subject to (Cap.-IC) and (8) as well as (R.C.) and (Wor.-IC) for all $s^t \in S^t$, $t \geq 0$, where b_0, k_0 are given.

Judd (1985) documents several features of the set of Pareto efficient policies in a similar economy. Without any doubt, the most striking result is that the capital tax will converge to zero for all Pareto efficient policies. Hence, even the policymaker type 0, who assigns zero weight to the capitalist household welfare, will not tax capital in the long run.

The result applies and can be extended in the economy presented here. The next proposition states that any λ-Ramsey equilibrium allocation can be implemented by a zero capital tax from date $t = 1$ onward. It is well known that there is an indeterminacy on the capital tax schedule: there may be alternative policies, not featuring a zero capital everywhere, that are Pareto efficient. However, all of them share a zero ex-ante capital tax. The extension of the result to policy from date $t = 1$ onward is a particular property of isoelastic preferences. Proposition 3 is, indeed, an extension of Proposition 7 in Chari and Kehoe (1998) to the two-class economy presented here.

Proposition 3 *For all $\lambda \geq 0$, there exists a λ-Ramsey policy τ with $\tau^k(s^{t+1}) = 0$ for all $s^{t+1} \in S^{t+1}$, $t \geq 1$.*

Proof. In the Appendix A ■

4 Time-consistency Properties of Pareto Efficient Policies

It is well known that optimal fiscal policy is usually time inconsistent in the representative agent framework. At any date $t \geq 1$, the continuation of the optimal fiscal plan is generally

[10] Appendix A contains a brief discussion of the necessary first-order conditions associated with the λ-Ramsey problem.

suboptimal. The eventual inelasticity of the capital supply implies that ex-post capital taxes indeed mimic lump sum taxation. It becomes then ex-post welfare superior to tax existing capital stock and to decrease distortionary labor taxation. Hence, policy discretion implies that rational expectations rule out the optimal fiscal policy. I analyze here the time-consistency properties of Pareto efficient policies for the two-class economy presented above.

The following theorem is the key result of the paper. It establishes that, at any date, the continuation of a Pareto efficient policy is Pareto efficient. In other words, it is not possible to revise any ex-ante Pareto efficient fiscal policy at any node and improve the welfare of a household type without leaving the other type strictly worse off.

Theorem 4 *Let $\{x, p, \tau\}$ be Pareto efficient. Then for all $s^t \in S^t$, $t \geq 0$, there is no $x' \in X\left(k\left(s^{t-1}\right), b\left(s^t\right), s_t\right)$ such that*

$$\begin{aligned} U^1\left(x', s^t\right) &\geq U^1\left(x, s^t\right) \\ U^2\left(x', s^t\right) &\geq U^2\left(x, s^t\right) \end{aligned}$$

and at least one condition holds with strict inequality.

Proof. Assume $\{x, p, \tau\}$ is Pareto efficient and there exists $x' \in X\left(k\left(s^{t-1}\right), b\left(s^t\right), s_t\right)$ such that $U^1\left(x', s^t\right) \geq U^1\left(x, s^t\right)$, $U^2\left(x', s^t\right) \geq U^2\left(x, s^t\right)$ and at least one with strict inequality. Because $x' \in X\left(k\left(s^{t-1}\right), b\left(s^t\right), s_t\right)$, x' must satisfy (*Wor.-IC*) and (*R.C.*) for all $s^j \in S^j$, $j \geq t$. Let x'' be equal to x' for all $s^j \in S^j\left(s^t\right)$, $j \geq t$ and equal to x everywhere else. Using Proposition 2, it is easy to see that $x'' \in X_0$. If $x'' \in X_0(\alpha)$, then it contradicts x being Pareto efficient. Therefore $x'' \notin X_0(\alpha)$.

Under isoelastic preferences, (*Cap.-IC*) and (8) combine to show that $x \in X_0(\alpha)$ implies $U^1\left(x, s_0\right) = \frac{\alpha}{1-\sigma}$. Since $x'' \in X_0$ but $x'' \notin X_0(\alpha)$, $U^1\left(x'', s_0\right) \neq U^1\left(x, s_0\right)$. The definition of x'' rules out $U^1\left(x'', s_0\right) < U^1\left(x, s_0\right)$, therefore $U^1\left(x'', s_0\right) > U^1\left(x, s_0\right)$ and $U^1\left(x', s_t\right) > U^1\left(x, s_t\right)$.

Consider now an alternative allocation plan $\tilde{x}$ with $\tilde{c}_1\left(s_0\right) = c_1'\left(s_0\right) - \varepsilon$ and $\tilde{x}_1 = x_1''$ everywhere else. By continuity of U^1, there exists $\varepsilon > 0$ small enough such that $U^1\left(\tilde{x}, s_0\right) = U^1\left(x, s_0\right)$. This implies that $\tilde{x}$ satisfies (*Cap.-IC*) and (8), both the worker and capitalist household are at least as well as under x, (*Wor.-IC*) holds everywhere, and the resource constraint (*R.C.*) holds everywhere and at s_0 with a strict inequality sign. It is straightforward to show that any allocation for which the resource constraint (*R.C.*) does not bind everywhere is in the interior of the utility possibility set $\left\{v \in \Re^2 : v_1 \leq U^1\left(x, s_0\right), v_2 \leq U^2\left(x, s_0\right), x \in X_0\right\}$. Therefore there exists $\hat{x} \in X_0$ with $U^1\left(\hat{x}, s_0\right) = U^1\left(\tilde{x}, s_0\right)$ and $U^2\left(\hat{x}, s_0\right) > U^2\left(\tilde{x}, s_0\right)$. But this contradicts x being Pareto efficient as $U^1\left(\tilde{x}, s_0\right) = U^1\left(x, s_0\right)$ implies that $\tilde{x} \in X_0(\alpha)$ ■

The zero capital tax result is key to understand Theorem 4. For any Pareto efficient policy, worker households are financing all government expenditures through a distortionary labor tax. In contrast, capitalist households are not being taxed at all. Any ex-post taxation of existing capital reduces overall distortion but also increases the fiscal burden of capitalist households. Capitalist households cannot be compensated without bringing back the very same distortionary labor taxation. Any gains from intertemporally shifting tax distortions could have been achieved at date 0 as well.

If future capital taxes were positive, there would have been an obvious way to bring the efficiency gains to both households: the tax revenues could be used to reduce both labor and future capital taxation. It is safe to say that most optimal fiscal theory features a zero or close to zero capital tax.[11]

A straightforward implication is that no policy revision will receive unanimous approval. Constitutions that grant veto rights to every political group will ensure the credibility of efficient fiscal policy. It is important to emphasize that Theorem 4 applies for any Pareto efficient policy. Minority veto rights do not restrict the redistributive possibilities available.

There is a technical interpretation of Theorem 4 that leads to interesting implications. Theorem 4 states that the continuation of a Pareto efficient policy at any given node $\left(k\left(s^{t-1}\right), b\left(s^{t}\right), s^{t}\right)$ will always lay in the frontier of the utility possibility set spanned by $X\left(k\left(s^{t-1}\right), b\left(s^{t}\right), s^{t}\right)$. Given the convexity of the utility possibility set, the continuation will achieve the maximum of a utilitarian SWF indexed by a precise Pareto weight distribution. In the language of the previous section, the continuation of a Pareto efficient policy will be the choice of a precise policymaker type λ^{*}. If at every date the same Pareto weight distribution indexed both the continuation and the initial choice from the set of Pareto efficient policies at date 0, then the policy would naturally become time-consistent for policymaker λ^{*}.

I introduce first a definition to formally develop these ideas. A policy will be λ-time-consistent if at any node the policymaker λ does not find any strictly λ-welfare superior fiscal policy. Hence, λ-time-consistency is a property of a pair of a policy τ and a policymaker λ. Note that the policymaker type is constant.

Definition 5 *Let $\{x, p, \tau\}$ constitute a private-sector competitive equilibrium at date $t=0$. Fiscal plan τ is λ-**time-consistent** if for all $s^{t} \in S^{t}, t \geq 0$, there is no $\{x', p', \tau'\}$ such that*

$$W^{\lambda}\left(x', s^{t}\right) > W^{\lambda}\left(x, s^{t}\right)$$

and $x' \in X\left(k\left(s^{t-1}\right), b\left(s^{t}\right), s^{t}\right)$.

Note this is a strong time-consistency requirement. At node s^{t}, policymaker λ can set any fiscal plan τ' *as if full commitment were available.* No fiscal policy given by Markovian or history-dependent policies would satisfy this requirement in the representative agent framework as long as there is positive taxation in equilibrium.

[11] While the capital tax is zero in the long run for a very general set of preferences, Proposition 4 is specific of the isoelastic preferences assumed. Other preferences may feature a positive capital tax along the transition path and Theorem 5 would only apply to long run policy.

Before proceeding to the next result, I introduce a slightly modified version of the λ-Ramsey problem in primal approach. To be precise, the implementability constraint associated with the capitalist household problem, (*Cap.-IC*), is dropped.[12] Hence, it is an unconstrained version of the λ-Ramsey problem for which it is not possible to conclude that the resulting allocations will be part of any private-sector competitive equilibrium. Yet it proves very useful for the study of the time-consistent properties of Pareto efficient policies.

Definition 6 *The* ***unconstrained λ-Ramsey problem*** *given (k_t, b_t, s_t) is to set x to maximize*

$$W^{\lambda}\left(x, s^t\right)$$

subject to (R.C.) and (Wor.-IC) for all $s^j \in S^j$, $j \geq t$, (k_t, b_t) given.

The reader may guess that the unconstrained λ-Ramsey policy problem at node s_0 corresponds to the relevant fiscal policy problem when lump sum taxes (and transfers) upon the capitalist agent are available. To see this, note that in the unconstrained λ-Ramsey problem the policymaker can dictate the full consumption path by the capitalist household with no other constraint than feasibility. Importantly, once lump sum tax are in, the time inconsistency problem is out: the continuation of a unconstrained plan is the solution of the unconstrained problem. Appendix A contains further details about the solution and properties of the unconstrained λ-Ramsey problem.

I show that not every Pareto efficient policy is time inconsistent in the following sense: there exists a policymaker type λ^* such that she does not find any ex-post revision of her optimal policy to be welfare superior. In the terminology introduced above, there is a policymaker type λ^* such that the λ^*-Ramsey policy (the policy choice among the Pareto efficient set) is λ^*-time-consistent (so no deviation at any time is λ^*-welfare superior).

I need to assume that the utility possibility set generated by X_0 is convex. Admittedly, this is a condition about equilibrium objects rather than model primitives. However, I am not concerned because the assumption is fully motivated by simplicity: one can introduce lotteries (and correspondent cumbersome notation) to convexify the utility possibility set.

Proposition 5 *Assume the utility possibility set*

$$\left\{v \in \Re^2 : v_1 \leq U^1\left(x, s_0\right), v_2 \leq U^2\left(x, s_0\right), x \in X_0\right\}$$

is convex. Then there exists $\lambda^* > 0$ such that the λ^*-Ramsey policy is λ^*-time-consistent.

Proof. In the Appendix B ■

Loosely speaking, I am stating the existence of a policymaker type λ^* such that lump sum taxes (or transfers) upon the capitalist agent are a redundant instrument. Given a chance to

[12] Constraint (8) also loses any significance and it can be dropped as well. Note that capital tax $\tau^k\left(s_0\right)$ is then unrestricted.

use lump sum taxes on the capitalist household, policymaker type λ^* decline as she is willing to tolerate the distortion associated with labor taxation in order to satisfy her redistributive objectives. Hence, when capital supply is eventually inelastic, nothing has changed, and policymaker type λ^* does not find any welfare gain in raising capital taxation. The market completeness allows the policymaker to trade distortions across states, so policymaker λ^* does not find it optimal to deviate from her Pareto efficient policy after any history.

Proposition 5 stands in stark contrast to the robustness of the time-inconsistency problem in representative agent models. It is clear that the policymaker's equity considerations play a key role in the result. Note that what distinguishes one policymaker type from another is how welfare distributions are ranked—no exogenous dislike for certain policies is introduced. The result builds upon the classic efficiency-redistribution trade-off, stacked against the temptation to raise ex-post capital taxation.

The reader is referred to the Appendix B for the rigorous proof of Proposition 5. Here I provide only a sketch of the argument. It can be shown that for a certain λ-welfare function, the solutions to the λ-Ramsey problem and the unconstrained λ-Ramsey problem coincide. This is the precise sense in which lump sum taxation of capitalists is redundant: implementation constraint (*Cap.-IC*) holds with equality but is not binding. Then the proof builds upon the fact that the solution to the unconstrained λ-Ramsey problem at date 0 solves the unconstrained problem at any node s^t. In other words, which involve a conscious abuse of terminology, the unconstrained λ-Ramsey allocation plan is time-consistent. Hence, the λ-Ramsey policy will be time-consistent too.[13]

5 Delegation

The previous section established that the continuation of a Pareto efficient policy is always Pareto efficient. Hence, a possible solution to the time-inconsistency problem is to give veto rights to every household. However, Pareto efficiency is an incomplete relationship, so minority veto rights may be problematic for decision making on several other dimensions.[14] In this section, I show the possibilities of fiscal policy delegation as a solution to the time-consistency problem.

I will focus on Pareto efficient policies from a timeless perspective. The concept of optimal fiscal policy from a timeless perspective is discussed in detail in Woodford (2003) and references therein. In short, the timeless perspective requires that allocations, prices, and policy are written as a time-invariant function of the state of the economy.

[13] An alternative, and maybe more technical explanation is that the frontiers of the utility possibility sets spanned by $X_0(\alpha)$ and X_0 have a non-empty intersection.

[14] Coalition governments—where parties with smaller constituencies may have veto power—have been linked, theoretically and empirically, with delayed stabilization. The seminal contributions are Alesina and Drazen (1991) and Roubini and Sachs (1989), respectively. See Franzese (2005) for a recent survey of the field of "veto actors" from political science.

In this economy it is possible to define a λ-Ramsey equilibrium from a timeless perspective as a continuation of a λ-Ramsey equilibrium.

Definition 7 *A λ-Ramsey equilibrium from a timeless perspective is a private-sector competitive equilibrium $\{x, p, \tau\} \in X(k_{t-1}, b_t, s^t)$ such that*

1. *$\{x, p, \tau\}$ is the continuation of a λ-Ramsey equilibrium at node s^t with $k(s^{t-1}) = k_{t-1}$ and $b(s^t) = b_t$.*

2. *There exists time-invariant functions γ_x, γ_p and γ_τ such that*

$$\begin{aligned} x\left(s^j\right) &= \gamma_x\left(k\left(s^{j-1}\right), b\left(s^j\right), s_j\right), \\ p\left(s^j\right) &= \gamma_p\left(k\left(s^{j-1}\right), b\left(s^j\right), s_j\right), \\ \tau\left(s^j\right) &= \gamma_\tau\left(k\left(s^{j-1}\right), b\left(s^j\right), s_j\right) \end{aligned}$$

for all $s^j \in S^j(s^t)$, $j \geq t$, with $k(s^{t-1}) = k_{t-1}$ and $b(s^t) = b_t$.

The next theorem is the second important result of the paper. It establishes that, given a Pareto efficient policy, its timeless component is time-consistent for a policymaker with a utilitarian SWF with precise Pareto weights. In other words, any λ-Ramsey equilibrium from a timeless perspective is λ'-time-consistent for some $\lambda' \geq 0$, where λ' will not be generally equal to λ.

Theorem 6 *Let τ be part of a λ-Ramsey equilibrium from a timeless perspective. Then there exists a policymaker type λ' such that τ is λ'-time-consistent.*

Proof. In the Appendix C ■

Theorem 6 is a strong result for delegation. A policymaker λ can implement her exact optimal fiscal policy from a timeless perspective by handing fiscal policy to a policymaker λ'. The delegate policymaker will have generally different preferences, but several desirable properties for a SWF are maintained. Note the delegation is between policymakers with a Paretian SWF and well-defined preferences for any public decision problem. There is no need to hand fiscal policy to a robot with exogenous tastes over policies.

It is easy to see how the result relies on Theorem 4. If there would be Pareto superior deviations, then no Paretian policymaker could implement any λ-Ramsey policy in a time-consistent way. But since there is no policy revision that is Pareto superior, Pareto weights can be chosen such that the policymaker's equity considerations are stacked against the efficiency gains associated with lump sum taxation—only available upon capitalist households. Isoelastic preferences and complete markets play a key role in establishing that the λ-Ramsey policy from a timeless perspective can be sustained everywhere by a single policymaker type λ'.

Why is Theorem 6 restricted to Pareto efficient policy from a timeless perspective? The answer is that the timeless component is completely driven by efficiency. All redistribution implemented is front loaded: two different policymakers will generally not agree on the same allocation plan from the point of view of date 0. Hence there is a redistribution-efficiency trade-off in the choice of the policymaker type λ at date 0.[15] This trade-off is heavily tilted in favor of efficiency: redistributive taxation is cheap to give up for time-consistency because, as Judd (1985) established, Pareto efficient policies do not feature too much redistribution anyway.

Now I further explore delegation. What can be said about the precise policymaker's preferences that achieve time-consistency? Which redistributive goals exacerbate or ease the time-inconsistency problem?[16] To answer this, I consider a non-stochastic version of the economy presented above. Let policymaker type $\hat{\lambda}$ be given by

$$\hat{\lambda} u_c^1 \left(c_1^\infty \right) = u_c^2 \left(c_2^\infty, n^\infty \right)$$

where x^∞ denotes the steady state allocations (if any). The policymaker type $\hat{\lambda}$ "conforms" to the market-induced welfare distribution, at least up to a first order approximation. If the policymaker $\hat{\lambda}$ finds it optimal to deviate from the continuation of the $\hat{\lambda}$-Ramsey policy, it must be because of the net efficiency gains. The policymaker $\hat{\lambda}$ gives us a baseline to compare redistributive goals.

Proposition 7 shows that, if the labor tax is strictly positive in steady state, then the $\hat{\lambda}$-Ramsey policy is not $\hat{\lambda}$-time-consistent and it is λ'-time-consistent, where $\lambda' > \hat{\lambda}$. In other words, to implement the optimal fiscal policy with neutral redistribution goals, it is necessary for a policymaker to favor the capitalist household more.

Proposition 7 *Let $\hat{\lambda}$ be a policymaker type such that*

$$\hat{\lambda} u_c^1 \left(c_1^\infty \right) = u_c^2 \left(c_2^\infty, n^\infty \right)$$

where x^∞ corresponds to the non-stochastic steady state associated with the $\hat{\lambda}$-Ramsey policy. If $\tau_n^\infty > 0$, then the $\hat{\lambda}$-Ramsey policy is not $\hat{\lambda}$-time-consistent and it is λ'-time-consistent where $\lambda' > \hat{\lambda}$.

[15] Note this trade-off exists despite the fact that nondistortionary taxation—date $t = 0$ capital taxation—is available.

[16] The model is explicit only about relative factor income heterogeneity, but I will argue that wealth heterogeneity can be thought of as the underlying process. Rodriguez, Diaz-Gimenez, Quadrini and Rios-Rull (2002) present evidence for the United States. They report labor earnings, defined to include 85.7 % of business and farm income, and wealth distribution. One finding is that the earnings-poor are surprisingly wealthy: "*...a household who owned the average wealth of the households in the bottom earnings quintal would be in the very top of the fourth quintal of the wealth distribution*" (Rodriguez et al. (2002), page 6). It is also reported that high income is a good proxy of high share of capital income. Also, see Garcia-Mila, Marcet and Ventura (2001) for additional facts used in their calibration. Thus, the different set of weights for capitalist and worker households can effectively be related to the more traditional view of redistributive goals across the wealth distribution.

Proof. In the Appendix C ■

An interesting possibility is that a policymaker λ is too "capitalist friendly" and, in order to implement the λ-Ramsey policy in a time-consistent way, a policymaker $\lambda' < \lambda$ is required.[17] This seems to be at odds with the overall intuition, but it is not. From the date 0 perspective, transferring resources from workers to capitalists involves two distortions. First, distortionary labor taxation is necessary to raise revenues. Second, lump sum transfers are not available, only subsidies to capital. These are distortionary starting from date $t = 1$ onwards. A policymaker who strongly favours the capitalist household may subsidize capital—albeit only in the transition path, never on the long run. However, the key is that once the investment decisions have been set, capital subsidies are no longer distortionary, and the ex-post policy problem calls for additional transfers to the capitalist household. Hence, the λ-Ramsey policy would end up not being λ-time consistent.

I complement the results with a couple of numerical illustrations on delegation.[18] I will focus on steady state Pareto efficient fiscal policies for different parameters. Then I compute the exact policymaker type λ^* that renders each policy time-consistent. Figure 1 documents delegation across a range of steady state Pareto efficient policies, each indexed by a different value of government consumption as a share of total output.[19] The plot indicates the precise policymaker type λ^* such that the policy is λ^*-time-consistent for a constant value of α. More government consumption requires more fiscal pressure on labor, increasing the efficiency gains associated with lump sum taxation. Not surprisingly, then, the policymaker must be tilted more toward capitalists in order to overcome the temptation of ex-post capital taxation.

The theory points to debt playing a special role. While government consumption is neutral, debt service is an income for the capitalist household. Hence, ceteris paribus, a larger amount of debt implies a higher net wealth for capitalist households and shifts equity considerations toward the worker household. Figure 2 plots the policymaker type λ^* for a range of Pareto efficient policies in steady state for a constant α. Each policy corresponds to a

[17] Policymaker $\lambda' < \lambda$ is less capitalist friendly yet not worker friendly either.

[18] Armenter (2004) explores time-consistent policy under different redistributive objectives.

[19] This is not meant to be a calibration exercise. I take as a baseline steady state policy a government-consumption-to-output ratio of .23 and a debt-to-output ratio of .4.

I used the following form for the worker's preferences:

$$u^2(c_2, n) = \frac{c_2^{1-\nu}}{1-\nu} + \psi_0 \frac{(1-n)^{1-\psi}}{1-\psi}.$$

Most of the preferences' parameters were chosen to match standard values: $\beta = \frac{1}{1.02}$, $\sigma = 2$, $\nu = 4$, and $\psi = 1$. Parameter ψ_0 was calibrated to have $n = .7$ in the steady state baseline policy.

Technology is given by a Cobb-Douglas production function

$$F(k, n) = Ak^{\eta} n^{1-\eta}$$

with $\eta = .35$ and A set to normalize output under the baseline policy equal to 1. The depreciation ratio δ is set equal to .05.

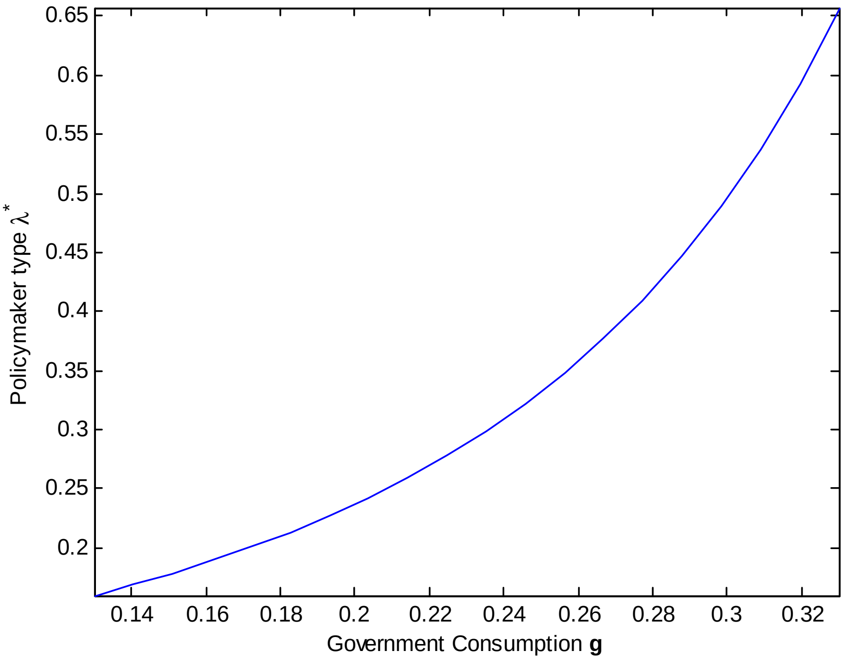

Figure 1: **Delegation of Pareto Efficient Fiscal Policies.** Different values of government consumption. See text for calibration details.

different level of debt service to total government spending, achieved by adjusting government spending g and the initial debt holdings b_0. Note total government spending is kept constant and, hence, unlikely the previous exercise, the fiscal pressure does not change. Yet a more capitalist-friendly policymaker is needed to implement the policy in a time-consistent way.

It is easy to see how comparative statics will work for government transfers to the worker household. As the transfers ease redistribution pressures to the worker households, ex-post capital taxation loses part of its appeal. The required capitalist-friendly policymaker may be then quite close to the symmetric policymaker.

6 Conclusions

The main conclusion of this paper is clear: equity considerations, while known not to shape Pareto efficient policies, are a key determinant of their time-consistency properties. The findings stand in contrast to the representative agent framework, where the time inconsistency is a robust property of the optimal fiscal policy.

The theory also introduces a couple of novel mechanisms that substitute for commit-

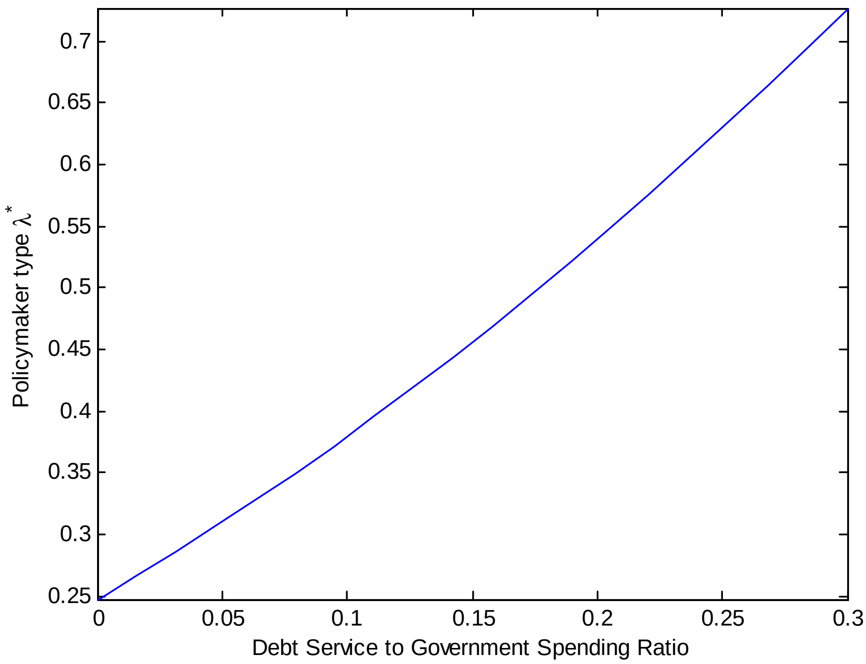

Figure 2: **Delegation of Pareto Efficient Fiscal Policies.** Different values of debt service to government spending ratio. See text for calibration details.

ment. First, to require that any policy revision is unanimously approved can safeguard the credibility of any Pareto efficient policy. Second, the time-consistency of fiscal policy can be ensured by delegation among Paretian policymakers with well-defined preferences over any policy problem. Both mechanisms are simple in concept and implementation compared to the alternatives present in the literature.

The results presented here improve the stance of time-consistent fiscal policy as a positive theory. First, there is no need to rule out contemporaneous capital taxation by assumption in order to achieve sensible equilibrium fiscal policy.[20] Second, differences in political institutions show great promise in explaining the variety of fiscal policy experiences across time and the globe. Society—or voters—may be willing to give up redistributive goals in order to preserve efficient fiscal policy. Whether they do or not depends crucially on political stability, among other things.[21] If political institutions are properly designed, there is no need to grant political independence to fiscal authorities.

[20] This assumption is present in Meltzer and Richard (1981), Krusell, Quadrini and Rios-Rull (1997), Klein and Rios-Rull (2002), and many others.

[21] See Armenter (2004).

References

Albanesi, S.: 2002, The time consistency of optimal monetary policy with heterogenous agents. Working Paper.

Alesina, A. and Drazen, A.: 1991, Why are stabilizations delayed?, *American Economic Review* **82**(4), 1170–88.

Armenter, R.: 2004, Redistribution, time consistent fiscal policy and representative democracy. Working Paper, Federal Reserve Bank of New York.

Bassetto, M.: 1999, Optimal fiscal policy with heterogeneous agents. Working Paper, Northwestern University.

Chamley, C.: 1986, Optimal taxation of capital income in general equilibrium with infinite lives, *Econometrica* **54**(3), 607–622.

Chari, V., Christiano, L. J. and Kehoe, P. J.: 1994, Optimal fiscal policy in a business cycle model, *Journal of Political Economy* **102**(4), 617–652.

Chari, V. and Kehoe, P. J.: 1990, Sustainable plans, *The Journal of Political Economy* **98**(4), 783–802.

Chari, V. and Kehoe, P. J.: 1998, Optimal fiscal and monetary policy. Research Department Staff Report, Federal Reserve of Minneapolis.

Dixit, A. and Londregan, J.: 2000, Political power and the credibility of government debt, *Journal of Economic Theory* **94**, 80–105.

Fischer, S.: 1980, Dynamic inconsistency, cooperation and the benevolent dissembling government, *Journal of Economic Dynamics and Control* **2**, 93–107.

Franzese, R. J.: 2005, Fiscal policy with multiple policymakers. Working Paper, The University of Michigan.

Garcia-Mila, T., Marcet, A. and Ventura, E.: 2001, Supply side interventions and redistribution. Working Paper, Universitat Pompeu Fabra.

Judd, K. L.: 1985, Redistributive taxation in a simple perfect foresight model, *Journal of Public Economics* **28**, 59–83.

Klein, P. and Rios-Rull, J.-V.: 2002, Time-consistent optimal fiscal policy. Working Paper, IER.

Krusell, P.: 2002, Time-consistent redistribution, *European Economic Review* **46**, 755–769.

Krusell, P., Quadrini, V. and Rios-Rull, J.-V.: 1997, Politico-economic equilibrium and economic growth, *journal of economic dynamics and control* **21**, 243–272.

Kydland, F. and Prescott, E. C.: 1977, Rules rather than discretion: The inconsistency of optimal plans, *Journal of Political Economy* **85**(3), 473–491.

Lucas, R. E. J. and Stokey, N. L.: 1983, Optimal fiscal and monetary policy in an economy without capital, *Journal of Monetary Economics* **12**, 55–93.

Mas-Colell, A., Whinston, M. D. and Green, J. G.: 1995, *Microeconomic Theory*, Oxford University Press, New York.

Meltzer, A. H. and Richard, S. F.: 1981, A rational theory of the size of the government, *The Journal of Political Economy* **89**(5), 914–927.

Persson, T. and Svensson, L. E.: 1986, International borrowing and time-consistent fiscal policy, *Scandinavian Journal of Economics* **88**(1), 273–295.

Persson, T. and Tabellini, G.: 1994, Representative democracy and capital taxation, *Journal of Public Economics* **55**, 53–70.

Rodriguez, S. B., Diaz-Gimenez, J., Quadrini, V. and Rios-Rull, J.-V.: 2002, Updated facts on the u.s. distributions of earnings, income and wealth, *Federal Reserve Bank of Minneapolis Quarterly Review* **26**(3), 2–35.

Rogoff, K.: 1985, The optimal degree of commitment to an intermediate monetary target, *Quarterly Journal of Economics* **100**, 1169–1189.

Roubini, N. and Sachs, J.: 1989, Political and economic determinants of budget deficits in the industrial democracies, *European Economic Review* **33**(2), 903–33.

Sleet, C. and Yeltekin, S.: 2002, Credibility and distribution: A simple model. Working Paper, Northwestern University.

Stokey, N.: 1989, Reputation and time consistency, *American Economic Review* **79**(2), 134–139.

Woodford, M.: 2003, *Interest and Prices*, Princeton University Press, Princeton.

A Solving The λ-Ramsey Problem (in Primal Approach)

In this Appendix, I discuss the solution to the λ-Ramsey policy problem, in its primal approach, and prove Proposition 3. As discussed in the main text, allocations x and initial capital tax $\tau^k(s_0)$ are restricted by (8). This is a different condition than assuming an arbitrary value for $\tau^k(s_0)$.

The necessity of the first order conditions of the λ-Ramsey fiscal policy imply that the allocations associated with the optimal fiscal policy must hold the following set of equations for all $s^t \in S^t, t \geq 1$,

$$\begin{aligned} \lambda u_c^1\left(s^t\right) + \phi_1 V_c^1\left(s^t\right) &= \theta\left(s^t\right) \\ u_c^2\left(s^t\right) + \phi_2\left(s^t\right) V_c^2\left(s^t\right) &= \theta\left(s^t\right) \\ u_n^2\left(s^t\right) + \phi_2\left(s^t\right) V_n^2\left(s^t\right) &= -\theta\left(s^t\right) F_n\left(s^t\right) \\ \sum_{s^{t+1}} \beta\pi\left(s^t|s_{t-1}\right)\theta\left(s^t\right)\left(F_k\left(s^t\right)+1-\delta\right) &= \theta\left(s^{t-1}\right) \end{aligned} \tag{9}$$

and at $t = 0$

$$\begin{aligned} \lambda u_c^1(s_0) + \phi_1 V_c^1(s_0) &= \theta(s_0) + (\phi_1 - \gamma) A_{c0} \\ u_c^2(s_0) + \phi_2(s_0) V_c^2(s_0) &= \theta(s_0) \\ u_n^2(s_0) + \phi_2(s_0) V_n^2(s_0) &= -\theta(s_0) F_n(s_0) + (\phi_1 - \gamma) A_{n0} \end{aligned} \tag{10}$$

where

$$\begin{aligned} V^1\left(s^t\right) &= u_c^1\left(s^t\right) c_1\left(s^t\right) \\ V^2\left(s^t\right) &= u_c^2\left(s^t\right) c_2\left(s^t\right) + u_n^2\left(s^t\right) n\left(s^t\right) \end{aligned}$$

Lagrangian multipliers are associated with each constraint as follows: ϕ_1 is associated with (*Cap.-IC*), $\phi_2(s^t)$ with (*Wor.-IC*) at s^t, $\theta(s^t)$ with (*R.C.*) at s^t, and finally γ is associated with the constraint (8). Allocations also satisfy the resource constraint (*R.C.*) with strict equality. Incidentally, note that if Lagrangian multipliers γ and ϕ_1 can be equated, then first order conditions (9) and (10) are identical. This enables constraints of the type (8) to characterize fiscal policy from a timeless perspective.

Now I proceed to the proof of Proposition 3. It uses the necessity of first order conditions (9).

Proof of Proposition 3. Under the assumption of isoelastic preferences for the capitalist household

$$V_c^1\left(s^t\right) = u_c^1\left(s^t\right)(1-\sigma).$$

The necessary first order conditions (9) imply that any λ-Ramsey equilibrium allocations x satisfy

$$u_c^1\left(s^t\right)\left(\lambda + \phi_1(1-\sigma)\right) = \theta\left(s^t\right).$$

Combining this with the intertemporal first order condition of the λ-Ramsey problem for $t \geq 1$

$$\sum_{s^{t+1}} \beta\pi\left(s^{t+1}|s_t\right) u_c^1\left(s^{t+1}\right)\left(F_k\left(s^{t+1}\right)+1-\delta\right) = u_c^1\left(s^t\right).$$

It is only left to show that $\tau^k(s^{t+1}) = 0$ for all $s^{t+1} \in S^{t+1}$, $t \geq 1$ constitutes a private-sector competitive equilibrium associated with the solution to the λ-Ramsey problem x. The only private-sector competitive equilibrium condition involving $\tau^k(s^{t+1})$ is the intertemporal Euler equation for the capitalist household:

$$u_c^1(s^t) = \beta \sum_{s_{t+1}} \pi(s_{t+1}|s_t)\, u_c^1(s^{t+1}) \left[\left(1 - \tau^k(s^{t+1})\right) r^k(s^{t+1}) + 1 - \delta\right].$$

Since $F_k(s^{t+1}) = r^k(s^{t+1})$, it is clear that $\tau^k(s^{t+1}) = 0$ for all $s^{t+1} \in S^{t+1}$, $t \geq 1$, is compatible with allocations x. Prices and other policy variables exist because of Proposition 2. In particular, the government budget constraint (7) is redundant, as it follows from the worker budget constraint (5), the resource constraint (*R.C.*), and the implementability condition (*Cap.-IC*) ■

For later reference, I include the necessary first order conditions of the unconstrained λ-Ramsey fiscal policy problem:

$$\begin{aligned} \lambda u_1^c(s^t) &= \theta(s^t) \\ u_c^2(s^t) + \phi_2(s^t) V_c^2(s^t) &= \theta(s^t) \\ u_n^2(s^t) + \phi_2(s^t) V_n^2(s^t) &= -\theta(s^t) F_n(s^t) \\ \sum_{s^{t+1}} \beta\pi(s^{t+1}|s_t)\, \theta(s^{t+1}) \left(F_k(s^{t+1}) + 1 - \delta\right) &= \theta(s^t) \end{aligned} \tag{11}$$

for all $s^t \in S^t$, $t \geq 0$, Lagrangian multipliers follow the previous notation but they do not necessarily have the same value. Note that (9) and (11) coincide if $\phi_1 = 0$, i.e., the implementability constraint for capitalists (*Cap.-IC*) is not binding.

B Proofs of the Time-consistency Properties Propositions

In this Appendix, I prove Proposition 5. I start, however, with a simple lemma with respect to the solution of the unconstrained λ-Ramsey policy problem and a proposition about its relationship to the λ-Ramsey policy.

Lemma 8 *Let x be the solution to the unconstrained λ-Ramsey policy problem at $t = 0$. Then x solves the unconstrained λ-Ramsey policy problem at any continuation state $(k(s^{t-1}), b(s^t), s^t)$ for all $s^t \in S^t$, $t \geq 0$.*

Proof. It follows trivially since the set of allocations satisfying (*R.C.*) and (*Wor.-IC*) from s^t onward is fully characterized by state $(k(s^{t-1}), b(s^t), s^t)$ ■

Proposition 9 *Let x^* be the solution to the unconstrained λ-Ramsey policy problem at date $t = 0$. If x^* satisfies (Cap.-IC), then x^* is the solution to the λ-Ramsey policy problem at date $t = 0$ and the λ-Ramsey policy τ^* is λ-time-consistent.*

Proof. It is obvious that if (*Cap.-IC*) is satisfied, then x^* must solve the λ-Ramsey policy problem as well.

Assume there exists allocation $x' \in X\left(k\left(s^{t-1}\right), b\left(s^t\right), s^t\right)$ with $W^\lambda\left(x', s_t\right) > W^\lambda\left(x^*, s_t\right)$. At any node s^t, x^* solves the unconstrained λ-Ramsey policy problem given $\left(k\left(s^{t-1}\right), b\left(s^t\right), s^t\right)$ as shown in Lemma 8. But because $x' \in X\left(k\left(s^{t-1}\right), b\left(s^t\right), s^t\right)$, x' must satisfy (*R.C.*) and (*Wor.-IC*) as well. Therefore, $W^\lambda\left(x', s_t\right) > W^\lambda\left(x^*, s_t\right)$ would contradict x^* being the solution to the unconstrained λ-Ramsey problem ■

Any allocation x that solves the unconstrained λ-Ramsey policy problem is then λ-time-consistent. Indeed, any allocation plan x that does not solve the unconstrained λ-Ramsey problem is not λ-time-consistent. The proof of Proposition 5 shows that there exists λ^* such that the unconstrained and constrained λ^*-Ramsey policies coincide everywhere.

Proof of Proposition 5. Assume α is such that constraint (8) implies a non-empty set of Pareto efficient policies. Otherwise, Proposition 2 is trivial.

Let

$$h\left(c_1\right) = \sum_{t=0}^{\infty} \sum_{s^t} \beta^t \pi\left(s^t | s_0\right) u_c^1\left(s^t\right) c_1\left(s^t\right).$$

If $\sigma = 1$, the set of Pareto efficient policies is non-empty if and only if $\alpha = (1-\beta)^{-1}$. Then constraint (*Cap.-IC*) is trivially satisfied and the solution to the constrained and unconstrained λ-Ramsey problem coincides for all λ.

Consider now $\sigma \neq 1$. Note that

$$h\left(c_1\right) = (1-\sigma) U^1\left(c_1, s_0\right).$$

Hence, the image of $h\left(c_1\right)$ is a linear projection of the image of $U^1\left(c_1, s_0\right)$. Consider the sets in $\Re$

$$\begin{aligned} LU &= \left\{U^1\left(c_1, s_0\right) : c_1 \text{ belongs to a } \lambda - \text{Ramsey equilibrium for some } \lambda \geq 0\right\}, \\ LU^* &= \left\{U^1\left(c_1, s_0\right) : c_1 \text{ solves a unconstrained } \lambda - \text{Ramsey problem for some } \lambda \geq 0\right\}. \end{aligned}$$

The assumption of a convex utility possibility set ensures that LU^* is a convex set as the choice set of any unconstrained λ-Ramsey problem is X_0. Because of the unconstrained nature of the second problem, it holds that $\sup\left\{LU^*\right\} \geq \sup\left\{LU\right\}$. But because LU must also be a subset of the utility possibility set spanned by X_0—possibly not the frontier—it is also true $\inf\left\{LU^*\right\} \leq \inf\left\{LU\right\}$. Finally, note that by including $\lambda = 0$ and $\lambda = \infty$, LU is closed. Hence $LU \subset LU^*$.

If there exists a λ'-Ramsey equilibrium allocation x' such that $h\left(c_1'\right) = \alpha$, it means that $\frac{\alpha}{1-\sigma} \in LU$ and therefore $\frac{\alpha}{1-\sigma} \in LU^*$. Hence there exists λ such that the solution x to the unconstrained λ-Ramsey policy problem satisfies $h\left(c_1\right) = \alpha$ and Proposition 9 ■

The proof of Proposition 5 uses the convexity of the utility possibility set and the isoelastic preferences for the capitalist household. As is discussed in the main text, I am not concerned about the former. However, how robust is the result to alternative sets of preferences? The precise condition needed is that the image of $h(c_1)$ is a monotone transform of the image of $U^1(c_1, s_0)$ on the domain of solutions to any unconstrained λ-Ramsey problem. Isoelastic preferences are a special case when the image of $h(c_1)$ is a linear projection of the image U^1. Note one can interpret $h(c_1)$ as $\frac{dU^1(\gamma c_1, s_0)}{d\gamma}$ evaluated at $\gamma = 1$ and rewrite the condition in terms of curvature of the utility possibility set.

C Proofs of Delegation Results

Here I include the proofs of Theorem 6 and Proposition 7.

Proof of Theorem 6. A λ-Ramsey equilibrium plan x must satisfy first order conditions (10) and (9). It is clear that the timeless component $\tilde{x}$ is given by (9) for all $t \geq 1$. Since

$$V_c^1\left(s^t\right) = u_c^1\left(s^t\right)(1-\sigma)$$

allocations $\tilde{x}$ satisfy the necessary first order conditions (11) for $t \geq 1$ of the unconstrained λ'-Ramsey problem where λ' is given by

$$\lambda' = \lambda + \phi_1(1-\sigma)$$

and ϕ_1 is the Lagrange multiplier associated with (*Cap.-IC*) in the λ-Ramsey problem.

The sufficiency of first order conditions cannot generally be established. However, plan $\tilde{x}$ is sitting at the utility possibility frontier associated with $X\left(k(s_0), b(s^1), s_1\right)$ for all $s_1 \in S$—see Theorem 4. By the convexity of the utility possibility set, there exists λ'' such that $\tilde{x}$ is the solution to the unconstrained λ''-Ramsey problem. Simple inspection of (11) shows that $\tilde{x}$ only satisfies the necessary first order conditions for λ'. Hence $\lambda' = \lambda''$ ■

Proof of Proposition 7. The existence of λ' follows from Theorem 6 and it is given by $\lambda' = \hat{\lambda} + \phi_1(1-\sigma)$—see proof of Theorem 6. Comparing necessary first order conditions (9) and (11), if $\phi_1 V_c^1 > 0$, then $\lambda' > \hat{\lambda}$, and $\lambda' = \hat{\lambda}$ if and only if $\phi_1 V_c^1 = 0$. Using $\hat{\lambda} u_c^1(c_1^\infty) = u_c^2(c_2^\infty, n)$, one can show that

$$\phi_1 V_c^1 = \phi_2 V_c^2$$

everything evaluated in the non-stochastic steady state allocations associated with the $\hat{\lambda}$-Ramsey policy. Assume that $\phi_1 V_c^1 < 0$. Then

$$\begin{aligned} \hat{\lambda} u_c^1 &> \theta \\ u_c^2 &> \theta \end{aligned}$$

where θ is the Lagrangian multiplier associated with the resource constraint (*R.C.*) for the $\hat{\lambda}$-Ramsey problem. But this would imply that raising an additional unit of resources and transferring it back to both agents would be $\hat{\lambda}$-welfare increasing. Therefore, $\phi_1 V_c^1 \geq 0$. With strict equality, then the economy is in the first best as

$$\begin{aligned} \lambda' u_c^1 &= \theta \\ u_c^2 &= \theta \end{aligned}$$

and therefore all taxes are 0. Hence, if $\phi_1 V_c^1 > 0$, $\lambda' > \hat{\lambda}$ and as the $\hat{\lambda}$-Ramsey policy plan does not satisfy the necessary first order conditions of the unconstrained $\hat{\lambda}$-Ramsey plan, it is not $\hat{\lambda}$-time-consistent ■