# Of Nutters and Doves*

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**Abstract**

We argue that, under a large degree of extrinsic inflation persistence, there is a strong yet simple case for inflation targeting even if we are uncertain about many other dimensions of the economy. If inflation persistence is large and driven by extrinsic sources, even an excessively strict inflation-targeting regime is preferable to full policy discretion. Our result is entirely built on stabilization policy: long-run inflation rates are optimal under full policy discretion. It is instead the medium-term dynamics of inflation expectations that render the policy response under discretion worse than inaction.

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# 1 Introduction

Is inflation targeting appropriate for the U.S.? A large body of theory holds that some form or another of inflation targeting approximates optimal monetary policy well. On the other hand, there is little empirical evidence that inflation targeting would improve U.S. monetary policy.[1] Lots of theory but few facts will leave some policymakers skeptical about inflation targeting. Indeed, many have urged caution with the normative implications of misspecified models: optimal monetary policy, and by extension the proper design of an inflation target, is shaped by every feature of a model. Uncertainty about parameters, about the state of the economy or even about how to communicate policy leave the policymakers with no guarantee that inflation targeting will improve upon the status quo.

We argue that a strong case for inflation targeting can be made if there is a large degree of extrinsic inflation persistence. In this case, any inflation targeting regime, even if it is overly strict on inflation, is preferable to full policy discretion. Our result has nothing to do with long-term inflation rates. We show that zero flexibility actually outperforms policy discretion *in stabilizing the economy*. We can be then confident that inflation targeting will improve upon full policy discretion even if we are uncertain about the exact design of the best inflation targeting regime. Most of the existing research has focused on finding simple rules that perform reasonably well across alternative models. Our result is different, in that allows to rank a whole class of rules, as given by different inflation targeting regimes, above full policy discretion.

The distinction between extrinsic and intrinsic inflation persistence is important for our result. Extrinsic inflation persistence arises from the underlying persistence of the exogenous shocks to the economy. Crucially, extrinsic inflation persistence induces dispersion in nominal prices, which is at the core of both the real effects and the welfare costs of inflation. Intrinsic persistence stems from the price-setting practices, like indexation or rule-of-thumb price-setters, and it has no first-order effect on price dispersion. The degree of intrinsic persistence does not affect the ranking of inflation targeting and policy discretion.

We evaluate monetary policy using a simple, standard New Keynesian model. In particular, we consider the monetary policy response to a cost-push shock under two scenarios: full policy discretion and strict inflation targeting. In both scenarios the central bank lacks the ability to commit to future policy decisions and takes inflation expectations as given. What distinguishes the two scenarios are the policymaker's objectives. Under full policy discretion, the policymaker weighs output and inflation volatility exactly as society does—we call this policymaker the "dove." In the second scenario the policymaker is totally oblivious to output

[1] Bernanke and Woodford (2005) contains an excellent collection of papers on inflation targeting, both from the normative and positive perspective.

variation—an "inflation nutter," in the wording of King (1997).

Say there is a persistent cost-push shock, putting downward pressure on the output gap for the current and future periods. Under full policy discretion, the private sector adjusts its medium-term inflation expectations upwards as it correctly anticipates that the dove will allow inflation to rise in order to counter the expected output decline. The independent dynamics of inflation expectations amplify the initial shock. The dove's policy response is larger but it still barely offsets the negative impact of inflation expectations. The more persistent the shocks are, the lesser output stabilization is achieved and the more and longer inflation rises. Eventually it would have been better to keep inflation flat and let output bear all the adjustment—the inflation nutter's policy. We refer to the resulting policy response as "perverse."

We make no case in this paper for the inflation nutter as the optimal inflation targeting design. We know, at least since Svensson (1997), that the optimal design of inflation targeting involves some flexibility. Ours is instead a robustness result. If strict inflation targeting outperforms full policy discretion, then any inflation targeting regime will be preferable to full policy discretion. We venture that strong, yet intuitive robustness results like this are what it will take to claim definitive support for inflation targeting.

We emphasize that the perverse policy phenomenon essentially comes down to the degree of extrinsic inflation persistence, as given by the persistence of the underlying shocks. Key determinants of optimal monetary policy play little or no role when shocks are persistent enough; for example, the weight on output variation in the social welfare function or the slope of the Phillips curve. How much persistence is needed is, by itself, not very sensitive to different structural parametrizations. There is a simple economic argument behind this claim: whenever inflation is effective at stabilizing output, which helps the case for policy discretion, it also generates large price dispersion—which hurts the case for policy discretion.

Determining the sources of inflation persistence should be a priority: a large degree of extrinsic persistence would make a strong case for inflation targeting even if we are uncertain about many other dimensions of the economy. Unfortunately, there is no clear evidence in this direction. The overall degree of inflation persistence has been low in the U.S. for the last twenty years. This could be due to better policy or due to the absence of shocks. Moreover, aggregate inflation data cannot be used to distinguish between extrinsic and inflation persistence.[2] Data on disaggregated prices has not been too kind to most sticky price models, but evidence seems to indicate that there is little price indexation.

Our findings here are related to the literature on the stabilization bias.[3] It is known that

[2] See Coenen and Levin (2004) and de Walque, Smets and Wouters (2006).

[3] Among the early work on the stabilization bias are Jonsson (1997), Svensson (1997), and Clarida, Gali and Gertler (1999).

it is necessary to commit to a history dependent rule in order to implement the optimal policy response, even for i.i.d. shocks. However, to the best of our knowledge, this paper is the first to point out that flexibility can lead to welfare-reducing stabilization policy, strengthening the case for inflation targeting.[4]

Researchers have long sought simple policy rules that perform reasonably well across alternative monetary models. Schmitt-Grohe and Uribe (forthcoming) and Schmitt-Grohe and Uribe (2006) argue that robust policy should not be very sensitive to output fluctuations. Levin and Williams (2003) argue that robust rules exist only when output deviations are important for monetary objectives. Rudebusch (2001) shows how model and parameter uncertainty can rationalize Taylor rules estimated with U.S. data. There is also a large and growing literature on robust inflation targeting design. See Giannoni and Woodford (2005), Svensson and Williams (2005), Orphanides and Williams (2006) and Giannoni (2006).

The paper is organized as follows. In the next section we discuss the perverse policy phenomenon. In Section 3 we put our results in the context of robust inflation targeting design. Section 4 briefly discusses evidence on inflation persistence and its decomposition between intrinsic and extrinsic sources. Section 5 offers a short look into the structural model. And then we conclude.

# 2 The Perverse Policy Phenomenon

We illustrate our argument with a very simple model consisting of a loss function in output and inflation deviations, a New Keynesian Phillips curve and an exogenous process for cost-push shocks.

There are two possible sources of inflation persistence. The first source stems from the persistence of cost-push shocks themselves. As their name asserts, these shocks produce pressure on firms' prices and can generate inflation dynamics. We assume the cost-push shock $u_t$ follows an autoregressive process

$$u_t = \rho u_{t-1} + \varepsilon_t \tag{1}$$

where $|\rho| < 1$ and $\varepsilon_t$ is *iid* with zero mean.[5] Following the literature, we refer to this source as extrinsic inflation persistence.

The second source of inflation persistence is the result of the price-setting mechanism. The best-known case is price-indexation: a fraction of the nominal prices is updated accord-

[4] Armenter and Bodenstein (2005) points out that terms-of-trade shocks lead to the perverse policy responses, so a commitment to a fixed exchange rate can be desirable even if inflation rates are low.

[5] Because of our very simple environment, one should take $u_t$ to be any shock that induces a inflation-output volatility trade-off.

ing to a backward-looking rule which corrects for past inflation realizations. We specify a partial indexation rule of the form

$$\log p_t(i) = \log p_{t-1}(i) + \gamma \pi_{t-1}$$

for prices $p_t(i)$ which are optimally reset at date $t$. This constitutes the intrinsic source of inflation persistence. We find it useful to use the inflation differential

$$\tilde{\pi}_t = \pi_t - \gamma \pi_{t-1}$$

where $\gamma \in [0, 1]$ and $\pi_t$ is the deviations of inflation from its long run level.

Angeloni, Aucremanne, Ehrmann, Gali, Levin and Smets (2006) provide an excellent discussion of the sources of inflation persistence in the context of the standard New Keynesian model. Angeloni et al. (2006) list two additional sources of inflation persistence: persistent deviations from rational expectations and persistent measurement errors. We do not downplay the role of expectations-based persistence but regard them as a matter of transparency.

The distinction between extrinsic and intrinsic inflation persistence is crucial. Extrinsic inflation persistence creates price dispersion and therefore it is a determinant of both the real output effect and welfare costs of inflation. In contrast, intrinsic inflation persistence is innocuous as it does not induce price dispersion by itself.

The relationship between inflation and the output gap is given by a New Keynesian Phillips curve,

$$\tilde{\pi}_t = \kappa x_t + \beta E_t \{\tilde{\pi}_{t+1}\} + u_t, \tag{2}$$

where $\kappa > 0$. The cost-push shock $u_t$ introduces a trade-off between inflation and output volatility. Again only the inflation differential $\tilde{\pi}_t$ has real effects.

We specify the following period social welfare loss function

$$l(\pi_t, x_t) = (\pi_t - \gamma \pi_{t-1})^2 + \lambda x_t^2$$

or

$$l(\pi_t, x_t) = \tilde{\pi}_t^2 + \lambda x_t^2 \tag{3}$$

where $x_t$ is the output gap deviations from its long-run level, and $\lambda > 0$ is the society's weight on output versus inflation volatility. We note two aspects of our loss function (3). First, we consider only economies where the long run output and inflation rates are optimal. In the parlance of Barro and Gordon (1983), there is no inflationary bias and therefore all welfare differences arise from the policy response to shocks. Second, only the inflation differential $\tilde{\pi}_t = \pi_t - \gamma \pi_{t-1}$ has direct welfare costs. As discussed above, indexation does not induce price dispersion per se and therefore has no direct welfare costs.

A careful construction of the Phillips curve and the social welfare loss function based on first principles can be found in Woodford (2003). In Section 5.2 we briefly discuss the structural foundations in order to understand better our results.

Total welfare loss at date $t$ is given by

$$L_t = E_t \left\{ \sum_{j=0}^{\infty} \beta^j \left( \tilde{\pi}_{t+j}^2 + \lambda x_{t+j}^2 \right) \right\}$$

where $0 < \beta < 1$ is the intertemporal discount rate and $E_t$ is the expectation operator conditional on information available at date $t$.

All that remains is a description of monetary policy. For simplicity the policy instrument is assumed to be the inflation rate. More importantly, the central bank cannot commit to any plan of future policy decisions. In other words, at date $t$ the central bank sets the inflation rate $\pi_t$ but has no direct control over inflation rates on future dates $t+1, t+2...$ The key implication of operating without commitment is that private sector inflation expectations are beyond the control of the central bank.

Within this framework we analyze two different scenarios for monetary policy: full policy discretion and strict inflation targeting. The two scenarios differ on how the policymaker weighs the output gap in his objectives. Let

$$z(\pi_t, x_t; \phi) = (\pi_t - \gamma \pi_{t-1})^2 + \phi x_t^2,$$

denote the period objective function of the policymaker where $\phi \geq 0$. The full policy discretion scenario is characterized by the dove, who weighs the output gap exactly as society does, $\phi = \lambda$. The strict inflation targeting scenario has an inflation nutter, $\phi = 0$, in command of the central bank.

We emphasize that each scenario features a different central banker operating the same central bank. In particular, both policymakers operate with the same policy instrument (inflation) under lack of commitment.

## 2.1 Full Policy Discretion: The Dove

In our first scenario monetary policy is set by a central bank that weighs output and inflation variation exactly as society does. The policymaker in place, whom we refer to as the dove, has an objective function given by (3). We index this scenario with a superscript *pd*.

Private sector expectations on future inflation $E_t \{\pi_{t+1}, \pi_{t+2}, ...\}$ are taken as given. We use a linear Markov equilibrium concept detailed in the Appendix. In a Markov equilibrium, private sector expectations are a linear function of the state of the economy. This rules out reputation-based mechanisms which, as it is well known, can sustain better and worse policy outcomes.

In the Appendix we show that in the unique linear Markov equilibrium, inflation at date $t$ is given by the solution to

$$\min_{\tilde{\pi}_t, x_t} \tilde{\pi}_t^2 + \lambda x_t^2$$

subject to the Phillips curve (2) and taking $E_t\{\tilde{\pi}_{t+1}\}$ as given. This is isomorphic to the standard problem under policy discretion, with the inflation differential $\tilde{\pi}_t$ in place of actual inflation $\pi_t$. While it is easy to see why $\pi_t$ and $\tilde{\pi}_t$ are equivalent instruments, we have to prove that the Markov equilibrium is equivalent and, in particular, the formation of private sector expectations can be expressed in terms of the inflation differential, $E_t\{\tilde{\pi}_{t+1}\}$. This is shown in the Appendix.

The first order condition characterizing the solution to the central bank's problem is

$$\tilde{\pi}_t = \frac{\lambda}{\kappa^2 + \lambda}\left(\beta E_t\{\tilde{\pi}_{t+1}\} + u_t\right). \tag{4}$$

We can view (4) as the policy decision that describes how the central bank reacts to the shock and inflation expectations.

Rational expectations dictate that the central bank's future decision is the determinant of private sector inflation expectations. The policy decision (4) at date $t+1$, conditional on the information at date $t$, is

$$E_t\{\tilde{\pi}_{t+1}\} = \frac{\lambda}{\kappa^2 + \lambda}\left(\beta E_t\{\tilde{\pi}_{t+2}\} + E_t\{u_{t+1}\}\right).$$

The private sector correctly anticipates that output will be off its optimal level at date $t+1$, leading the central bank to let inflation deviate from its long run level as well. Hence the private sector adjusts its inflation expectations to the shock forecast, $E_t\{u_{t+1}\}$. Using the policy decision (4) at dates $t+2, t+3...$ we determine $E_t\{\tilde{\pi}_{t+2}\}$, $E_t\{\tilde{\pi}_{t+3}\}$ ... and then solve for $E_t\{\tilde{\pi}_{t+1}\}$

$$\begin{aligned} E_t\{\tilde{\pi}_{t+1}\} &= \frac{\lambda}{\kappa^2 + \lambda} E_t\{u_{t+1}\} + \left(\frac{\lambda}{\kappa^2 + \lambda}\right)^2 \beta E_t\{u_{t+2}\} + ... \\ &= \frac{\lambda}{\kappa^2 + \lambda} \sum_{j=0}^{\infty} \left(\frac{\beta\lambda}{\kappa^2 + \lambda}\right)^j E_t\{u_{t+1+j}\}. \end{aligned}$$

It is then the whole expected path $\{E_t u_{t+j}\}_{j=1}^{\infty}$ which determines inflation expectations at date $t$. The shock follows the autoregressive process (1), so $E_t\{u_{t+j}\} = \rho^j u_t$ and the private sector inflation expectations at date $t$ are

$$E_t\left\{\tilde{\pi}_{t+1}\right\} = \frac{\lambda}{\kappa^2 + \lambda\left(1-\beta\rho\right)}\rho u_t. \tag{5}$$

Note that the expectation is upon the inflation differential to track the cost-push shock.

It is not hard to see that the $j$-steps ahead inflation expectation is

$$E_t\left\{\tilde{\pi}_{t+j}\right\} = \frac{\lambda}{\kappa^2 + \lambda\left(1-\beta\rho\right)}\rho^j u_t.$$

Hence a persistent shock $\rho > 0$ induces inflation expectations to deviate for the medium term. Long-term expectations remain well anchored as the shock eventually fades. The solution (5) also makes clear inflation expectations *commove* with the shock as long as it is persistent, $\rho > 0$. We shall return to this: the response of the inflation expectations is at the core of the failure of the dove to conduct proper stabilization policy.

The policy decision (4) determines the inflation response once we substitute for the inflation expectations $E_t\left\{\tilde{\pi}_{t+1}\right\}$,

$$\tilde{\pi}_t^{pd} = \frac{\lambda}{\kappa^2 + \lambda\left(1-\beta\rho\right)}u_t. \tag{6}$$

The inflation differential is proportional to the shock and therefore inherits its statistical properties. In particular, the inflation differential will be as persistent as the shock.[6] This also confirms that the unconditional mean of inflation is zero, $E\pi_t = 0$.

Finally, we use the Phillips curve to solve for the output gap

$$x_t^{pd} = -\frac{\kappa}{\kappa^2 + \lambda\left(1-\beta\rho\right)}u_t \tag{7}$$

and the period welfare loss is given by

$$l_t^{pd} = \left[\left(\frac{\lambda}{\kappa^2 + \lambda\left(1-\beta\rho\right)}\right)^2 + \lambda\left(\frac{\kappa}{\kappa^2 + \lambda\left(1-\beta\rho\right)}\right)^2\right]u_t^2. \tag{8}$$

## 2.2 Strict Inflation Targeting: The Inflation Nutter

The second scenario features an inflation nutter in charge of the central bank. The inflation nutter seeks to minimize inflation variation without any regard for output variation. This

[6] As a result, overall inflation is an auto-regressive of second order. See Section 4.2. for a discussion of the aggregate process.

has also been called strict or pure inflation targeting in the literature. We index this scenario with the superscript $it$.

The central bank's problem is trivial, as it chooses to implement zero inflation at all periods, $\pi_t^{it} = 0$ for all $t \geq 0$. Private sector inflation expectations follow, $E_t \{\pi_{t+1}\} = 0$. The Phillips curve (2) implies that the output gap is

$$x_t^{it} = -\frac{1}{\kappa} u_t. \tag{9}$$

The welfare period loss is trivially given by

$$l_t^{it} = \lambda \frac{1}{\kappa^2} u_t^2. \tag{10}$$

## 2.3 Welfare Comparison

The dynamics of inflation and output across the two scenarios are very much as one would expect. Consider a cost-push shock $u_t > 0$ which puts downward pressure on output. Inflation under policy discretion rises on impact, while it stays flat under strict inflation targeting

$$\tilde{\pi}_t^{pd} = \frac{\lambda}{\kappa^2 + \lambda (1 - \beta \rho)} u_t > 0 = \tilde{\pi}_t^{it}.$$

We have already discussed that the inflation differential under policy discretion is as persistent as the shock. The rise in inflation has a stabilizing effect on output that is absent under the inflation nutter. Combining (7) and (9), we have that

$$\frac{x_t^{pd}}{x_t^{it}} = \frac{\kappa^2}{\kappa^2 + \lambda (1 - \beta \rho)} < 1$$

so output gap deviations are smaller under full policy discretion. The inflation nutter, by being oblivious to everything but inflation, induces excessive output variation. The dove instead trades off some price dispersion for a smoother output response. It would seem like the dove's decision to do so indicates that stabilization policy under full policy discretion will be unambiguously better. Not so fast.

Note how the dynamics change as the persistence of the shock increases. For the same realization of the shock, inflation under policy discretion rises by more and for longer.[7] Yet less output stabilization is achieved. This certainly does not help the case for full policy discretion! As we take the degree of extrinsic persistence to its upper bound, $\rho \to 1$, and

[7] Recall the inflation differential inherits the persistence of the shock.

take the intertemporal discount rate to be approximately 1, we have that $\beta\rho \to 1$ and the resulting dynamics are

$$\begin{aligned} \tilde{\pi}_t^{pd} &= \frac{\lambda}{\kappa^2} u_t > \tilde{\pi}_t^{it} = 0 \\ x_t^{pd} &= x_t^{it}. \end{aligned}$$

The policy response is, in absolute terms, welfare reducing: inflation rises yet output displays no moderation. This is what we call the perverse policy response phenomenon. Clearly, the period social welfare is strictly lower under full policy discretion than under strict inflation targeting,

$$l_t^{pd} = \left(\frac{\lambda}{\kappa^2} + 1\right) \frac{\lambda}{\kappa^2} u_t^2 > \frac{\lambda}{\kappa^2} u_t^2 = l_t^{it},$$

as seen by evaluating (8) and (10).[8] By continuity it is clear that there exists $\rho < 1$ such that the inflation nutter outperforms the dove.

In short, for sufficiently persistent shocks, zero flexibility is preferred to discretion. Note that this holds for any magnitude or sign of the cost-push shock $u_t$. Moreover, we made no assumption of the values of $\lambda$ and $\kappa$: for any parametrization there is a sufficiently high degree of extrinsic persistence such that strict inflation targeting dominates.[9] We have shown this in what seemed to be the ideal scenario for policy discretion: without differences in long run inflation and in the aftermath of a shock. Instead of a sound stabilization policy, we find the dove engineering a large, persistent, costly, and futile rise in inflation.

The independent response of inflation expectations under policy discretion is behind this result. As discussed above, under full policy discretion inflation expectations commove with the shock over the medium term,

$$E_t \{\tilde{\pi}_{t+1}\} = \frac{\lambda\rho}{\kappa^2 + \lambda(1 - \beta\rho)} u_t.$$

The comovement of inflation expectations amplifies the initial shock. To see how, note the New Keynesian Phillips curve (2) is perceived by the central bank as an "Old" Keynesian Phillips curve since inflation expectations are taken as given,

$$\begin{aligned} \tilde{\pi}_t &= \kappa x_t + \beta E_t \{\tilde{\pi}_{t+1}\} + u_t, \\ &= \kappa x_t + \left(\frac{\beta\lambda\rho}{\kappa^2 + \lambda(1 - \beta\rho)} + 1\right) u_t. \end{aligned}$$

[8]Note the period welfare loss function is well defined for the limiting case $\beta\rho \to 1$.

[9]The exact threshold does depend on the parametrization. If one takes low values of the intertemporal rate $\beta$, it is also possible that the threshold would imply a random walk for the cost-push shock. We briefly discuss the structural determinants of the precise level of persistence needed in Section 5.

The more persistent the shock, the larger the amplifying role of inflation expectations. The policy response barely offsets the negative output impact of inflation expectations. This explains why inflation rises but no effective output stabilization is achieved. It would have been better to keep inflation flat and let output bear all the adjustment—the inflation nutter's policy.

At this point it is useful to compare both scenarios with the optimal policy response. We refer the interested reader to Woodford (2003) for a complete analysis of the optimal monetary policy. The optimal policy requires a commitment technology that is not available to the central bank. Hence neither the dove nor the inflation nutter can implement it.

Figure 1 displays the dynamics of the inflation differential $\tilde{\pi}_t$, actual inflation $\pi_t$ and the output gap for the two scenarios as well as for the optimal policy response. The cost-push shock has an auto-correlation of .9 — sufficient to trigger the perverse policy response phenomenon.[10] We plot the response for up to twelve quarters.

The upper panel in Figure 1 displays the response of the inflation differential — what is directly under control of the monetary authority. The responses under full policy discretion and strict inflation targeting are the dotted and the solid lines respectively. The optimal policy response is given by the dashed line. Relative to the optimal policy response, inflation under full policy discretion overreacts on impact. Note that under the optimal policy response inflation dies out quite fast—after one year it is very close to its long run level. The rationale behind the optimal policy response is to avoid the feedback from high medium term inflation expectations. In contrast, inflation is quite persistent under full policy discretion. The dynamics of the actual inflation level are quite revealing.

The output gap is displayed in the lower panel of Figure 1. First, we observe that the dove effectively achieves some output smoothing compared to the inflation nutter. The comparison with the optimal monetary policy, though, is illustrative of the perverse effect of inflation expectations. On impact output falls much less under the optimal monetary policy than under full policy discretion—despite the inflation response being much larger in the latter. The optimal policy calls for the output gap to deteriorate in the medium term. Overall, the dove achieves some mild output stabilization in the medium term in exchange for large, persistent inflation and subpar output stabilization in the short term. The inflation nutter, on the other hand, forgoes all the output stabilization in the short term for a flat response of inflation.

[10] Remaining parameters are $\kappa = .06$, $\beta = .99$, $\gamma = .5$, and $\lambda = .01$.

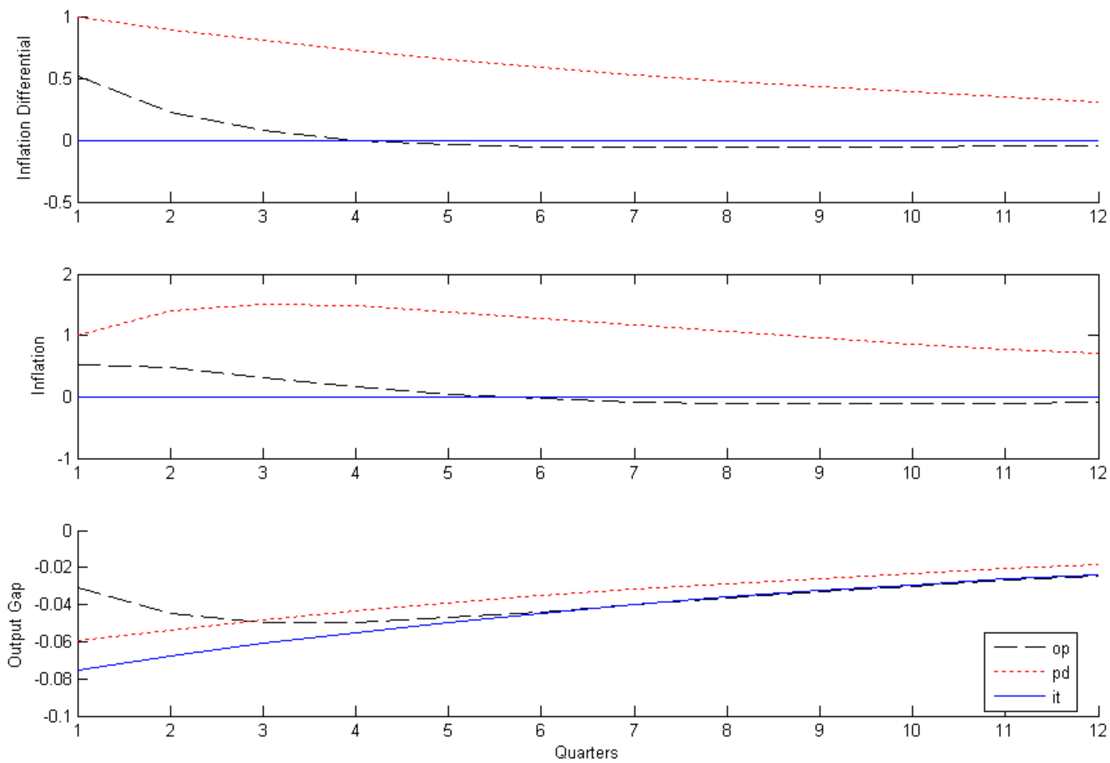


Figure 1: **Inflation and Output Dynamics** The solid line is strict inflation targeting, the dotted line is full policy discretion, and the dashed line is the optimal policy response. See text for details.

# 3 Implications for Inflation Targeting Design

Full policy discretion and strict inflation targeting are just two out of many possible designs of inflation targeting. In this section we study a larger class inflation targeting regimes. As pointed out by Svensson (1997), the optimal level of flexibility in an inflation targeting regime does not correspond to either the inflation nutter or the dove. However, ranking the inflation nutter above the dove ensures a move to inflation targeting is welfare-improving even if the precise optimal degree of flexibility is unknown.

Formally, we return to the general formulation of the policymaker's objective function,

$$z\left(\tilde{\pi}_t, x_t; \phi\right) = \tilde{\pi}_t^2 + \phi x_t^2. \tag{11}$$

We now consider any policymaker with $\phi \in [0, \lambda]$—this is the class of inflation targeting regimes we focus on. The inflation nutter, $\phi = 0$, corresponds to zero flexibility; the dove, $\phi = \lambda$, features no inflation targeting; and any intermediate values $\phi \in (0, \lambda)$ corresponds to a different degree of flexibility.[11]

We can solve for inflation and output dynamics for an arbitrary $\phi$ using (2) and (1),

$$\begin{aligned} \tilde{\pi}_t &= \frac{\phi}{\kappa^2 + \phi\left(1 - \beta\rho\right)} u_t, \\ x_t &= -\frac{\kappa}{\kappa^2 + \phi\left(1 - \beta\rho\right)} u_t. \end{aligned}$$

The period loss function is then given by

$$l_t\left(\phi\right) = \left[\left(\frac{\phi}{\kappa^2 + \phi\left(1 - \beta\rho\right)}\right)^2 + \lambda\left(\frac{\kappa}{\kappa^2 + \phi\left(1 - \beta\rho\right)}\right)^2\right] u_t^2 \tag{12}$$

defined for $\phi \geq 0$.

With the "generalized" loss function (12) we can easily compute the optimal degree of flexibility $\lambda^*$, that is, the one that minimizes the resulting equilibrium period loss. Taking first order conditions we find that

$$\lambda^* = \lambda\left(1 - \beta\rho\right). \tag{13}$$

[11] What does link the policymaker's weight on output $\tilde{\lambda}$ to policy flexibility? Most inflation targeting regimes feature a tolerance range. If inflation steps out of the range the central bank is held accountable. Mishkin and Westelius (2005) show how to think of the tolerance range as a penalty function on inflation deviations. The tighter the range, the harsher the central bank's penalty for inflation deviations and the lower the relative weight on output deviations.

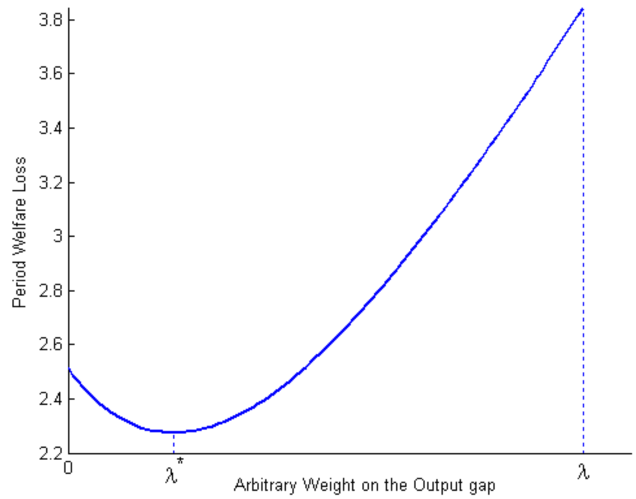


Figure 2: **The Optimal Degree of Flexibility in Inflation Targeting** Baseline parameters.

The first thing to note is that the optimal degree of flexibility lies somewhere between strict inflation targeting and full policy discretion, $0 < \lambda^* \leq \lambda$. The inflation nutter is the optimal policymaker only for the limiting case $\beta\rho \to 1$; the dove only when cost-push shocks have no persistence $\rho = 0$ or society does not value the future $\beta = 0$.

Figure 2 plots the period welfare loss as a function of the policymaker's weight on the output gap, $\phi$, as in (11). We indicate the location of the optimal degree of flexibility $\lambda^*$ as well as the output weight in the social welfare function (3), $\lambda$. Note that $l(0) < l(\lambda)$, i.e., the inflation nutter ranks above the dove.

The precise level of flexibility as given in (13) is a function of all of the parameters in the economy and there is considerable uncertainty with respect to many of them. Even small deviations of the better "known" parameters can lead to very different optimal levels of flexibility. Moreover, there may be difficulties in computing the output gap and/or communicating the degree of flexibility to the public. Hence there is the concern that a country will be worse off under an incorrectly chosen/communicated inflation targeting framework than under full policy discretion.

However, in the event cost-push shocks are sufficiently persistent, our analysis shows that any inflation targeting regime is better than full policy discretion—even if the targeting regime is more strict than optimal. Coming back to Figure 2, if we establish that $l(0) < l(\lambda)$, then $l(\phi) \leq l(\lambda)$ for all $\phi \in [0, \lambda]$ and uncertainty about the optimal degree of flexibility should not deter policymakers from adopting inflation targeting.

# 4 Inflation Persistence in the U.S.

In this section we ask whether a strong case for inflation targeting in the U.S. can be made on the basis of our results. The short answer is no, as there is not conclusive evidence that extrinsic inflation persistence is large in the U.S. for the last twenty years. We do, however, call for further research on the sources of inflation persistence. In particular, the present state of very low inflation volatility could be the outcome of a lack of shocks. We should then press to understand the dynamics of less tranquil times for when stabilization policy is actually called into action.

We highlight the following three facts about U.S. inflation dynamics:

- There is a growing consensus that inflation persistence has been quite low in the last twenty years. However, the fall in persistence has been accompanied with a fall in volatility. Thus, it is hard to assess whether the fall in inflation persistence results from the disappearance of a propagation mechanism or the lack of shocks.

- It is not possible to decompose the sources of inflation persistence by looking at aggregate data. It is thus necessary to have a micro-foundation of the Phillips curve and test the model on the basis of disaggregated data.

- Aggregate data, though, points to at most one significant source of inflation persistence. In particular, the smooth fall in the auto-correlation function of inflation is at odds with having both high extrinsic and intrinsic persistence.

In the remaining of the section we discuss further these points based on data and the existing literature.

## 4.1 U.S. Inflation Persistence

Virtually all common inflation measures in the U.S. display a high auto-correlation for the post-war period. At a quarterly frequency over the period 1964-2006, the GDP deflator has an auto-correlation of .87 — the same as the PCE and only slightly higher than the auto-correlation of CPI.

However, there is now a growing consensus that inflation persistence has been lower for the last twenty years.[12] A simple observation is that all auto-correlation estimates are

[12] See Pivetta and Reis (2004) for a dissent.

significantly lower for the period 1986 − 2006, and even lower for 1996 − 2006. The fall in persistence, though, has been accompanied with a fall in inflation and output volatility.

Researchers have tried to encompass the sample instability in a single framework. It has been pointed out that ignoring breaks in the inflation mean induces a upward bias in the persistence estimates of inflation. Corvoisier and Mojon (2004) uses the Altissimo-Corradi test of multiple breaks for several OECD countries and Benati (2004) does a similar exercise using the Bai-Perron test instead. See also Levin and Piger (2004). Evidence of a unit root becomes tenuous at best if one allows for mean breaks. Corvoisier and Mojon (2004) states that "the roots of autoregressive models of inflation are actually far below unity." Benati (2004) concludes that "conditional on estimated breaks, inflation shows little persistence" and "the null of the unit root can be strongly rejected for a vast majority of series."

Another well-known line of research has explored "regime switches." Sims and Zha (2004) argues that it is changes in the volatility of innovations, rather than their persistence, that are key to explain US monetary policy. See also Owyang and Ramey (2004).

It has also been argued that inflation persistence is significantly lower than estimated due to aggregation bias. An aggregate time series inherits its persistence from its most persistent component. For example, the average between a random walk process and white noise is a random walk. However, the quantitative importance of the aggregation bias is still open to debate. Altissimo, Mojon and Zaffaroni (2004) analyze inflation persistence across over 400 sectorial series for the Euro area. They find a large heterogeneity of estimates across sectors and show how fast adjustment at the micro level can be reconciled with the sluggish aggregate dynamics.

## 4.2 The Limitations of Aggregate Data

Our simple model can be used to understand why aggregate data is of no use to identify the sources of inflation. Under any monetary policy regime, the equilibrium inflation dynamics are given by

$$\tilde{\pi}_t = \rho \tilde{\pi}_{t-1} + \eta \varepsilon_t$$

where $\varepsilon_t$ is iid, $\rho$ is the persistence of cost-push shocks (extrinsic persistence) and $\eta$ would depend on the exact monetary framework.[13] Substituting back the formulation of the inflation differential $\tilde{\pi}_t = \pi_t - \gamma \pi_{t-1}$, where $\gamma$ is the degree of indexation (intrinsic persistence) we end up with an auto-regressive process of order two,

$$\pi_t = (\rho + \gamma)\, \pi_{t-1} - \gamma \rho \pi_{t-2} + \eta \varepsilon_t. \tag{14}$$

[13] For strict inflation targeting, $\eta = 0$, and inflation is a constant.

Note that both parameters $\rho$ and $\gamma$ enter symmetrically — preventing identification.[14] If we estimate an autoregressive process of order 2, we can equate

$$\psi_1 = \rho + \gamma,$$
$$\psi_2 = -\rho\gamma.$$

If a pair $(\rho, \gamma)$ solves the above system, so it does $\tilde{\rho} = \gamma$ and $\tilde{\gamma} = \rho$.

In a recent paper, de Walque et al. (2006) explicitly acknowledges this problem in work with a large DSGE model and Bayesian estimation methods. While more complete models of price indexation or rule-of-thumb price-setters allow some identification, estimates remain very sensitive to the precise assumptions. Coenen and Levin (2004) also discuss how lagged inflation can be interpreted as backward-looking price indexation or learning about the central bank's objectives.

The alternative is to look at disaggregate price data for price indexation mechanisms. In a well-known paper, Bils and Klenow (2004) look at the behavior of retail prices and conclude that price durations in the data are quite at odds with the parametrizations used in macroeconomic models. The distribution of price changes is not clustered around average inflation, as a price indexation model would suggest. Most of the recent research has focused on the performance of standard sticky price models. For example, Klenow and Kryvtsov (2005) argue that time-dependent pricing models match well the observed price behavior and Nakamura and Steinsson (2007) reevaluate menu cost models.

The research undertaken by the Inflation Persistence Network at the European Central Bank has directly addressed the issue (see Alvarez, Dhyne, Hoeberichts, Kwapil, Le Bihan, Lunneman, Martins, Sabbatini, Stahl, Vermeulen and Vilmunen (2006) for an overview of the work on disaggregated price data). Price indexation does not seem to be very common in European countries at least during the recent period of stable inflation. Rule-of-thumb behavior seems to be more common but not a key determinant.

## 4.3 At Most One Source of Inflation Persistence

While aggregate data cannot identify the source of inflation persistence, estimates strongly indicate that, at most, there is one source of inflation persistence. We can conclude this from the simple observation that, if both $\rho > 0$ and $\gamma > 0$, (14) implies that the second lag must have a negative coefficient.[15]

[14] Note that in our very simple model, only $\rho$ and $\gamma$ affect the persistence of inflation — this is not true in more complete specifications. See Wolman (1999) for a discussion of inflation dynamics under different price-setting assumptions.

[15] For more elaborated indexation rules of order $m$, one always find that the last lag in the inflation process is the product of $\rho$ and the coefficient $\gamma_m$ on the last lag.

We explored various inflation measures (GDP deflator, CPI, and PCE) and different lag specifications, but we could not find an instance of the last significant lag being negative. This is because inflation dynamics display a very smooth decay. While having an autoregressive process of order two allows to reproduce the hump-shaped dynamics of inflation, the negative coefficient seems to be counterfactual.

# 5 A Short Look at the Structural Model

We have argued that, if extrinsic inflation persistence is high enough, any inflation targeting regime is preferred to full policy discretion. We now look at a simple structural model for the determinants of the precise level of extrinsic inflation persistence needed. Of notice it is that the volatility of shocks is completely irrelevant. Moreover, the persistence threshold is not very sensitive to the degree of nominal frictions.

## 5.1 The Persistence Threshold

The welfare period losses under full policy discretion and strict inflation targeting, (8) and (10), can be written as functions of the shock and reduced form parameters, $l_t^{pd}(u_t; \lambda, \kappa, \rho)$ and $l_t^{it}(u_t; \lambda, \kappa)$, respectively. We obviate the dependence with respect to the intertemporal discount rate $\beta$. As discussed above, the period welfare loss under full policy discretion is strictly increasing in the degree of extrinsic persistence, $\rho$. We can characterize the welfare ranking across scenarios in terms of a persistence threshold. Let $\bar{\rho}(\lambda, \kappa)$ be the solution to

$$l_t^{it}(u_t; \lambda, \kappa) = l_t^{pd}(u_t; \lambda, \kappa, \bar{\rho}(\lambda, \kappa))$$

such that $\beta\rho < 1$. Strict inflation targeting outperforms full policy discretion for all parametrizations $\{\lambda, \kappa, \rho\}$ such that $\rho \geq \bar{\rho}(\lambda, \kappa)$. This leads to

$$2\lambda(1 - \beta\bar{\rho}(\lambda, \kappa)) + \left(\frac{\lambda}{\kappa}\right)^2 (1 - \beta\bar{\rho}(\lambda, \kappa))^2 = \lambda \tag{15}$$

which makes clear that the realization of the shock $u_t$ is irrelevant for the threshold. Hence the only relevant statistical property of the shock is its persistence—in particular, volatility does not affect the welfare ranking of the two scenarios.

We find that $\bar{\rho}$ is the smallest solution to the quadratic equation (15),

$$\beta\bar{\rho}(\lambda, \kappa) = 1 + \frac{\kappa^2}{\lambda} - \sqrt{\frac{\kappa^2}{\lambda}\left(1 + \frac{\kappa^2}{\lambda}\right)}. \tag{16}$$

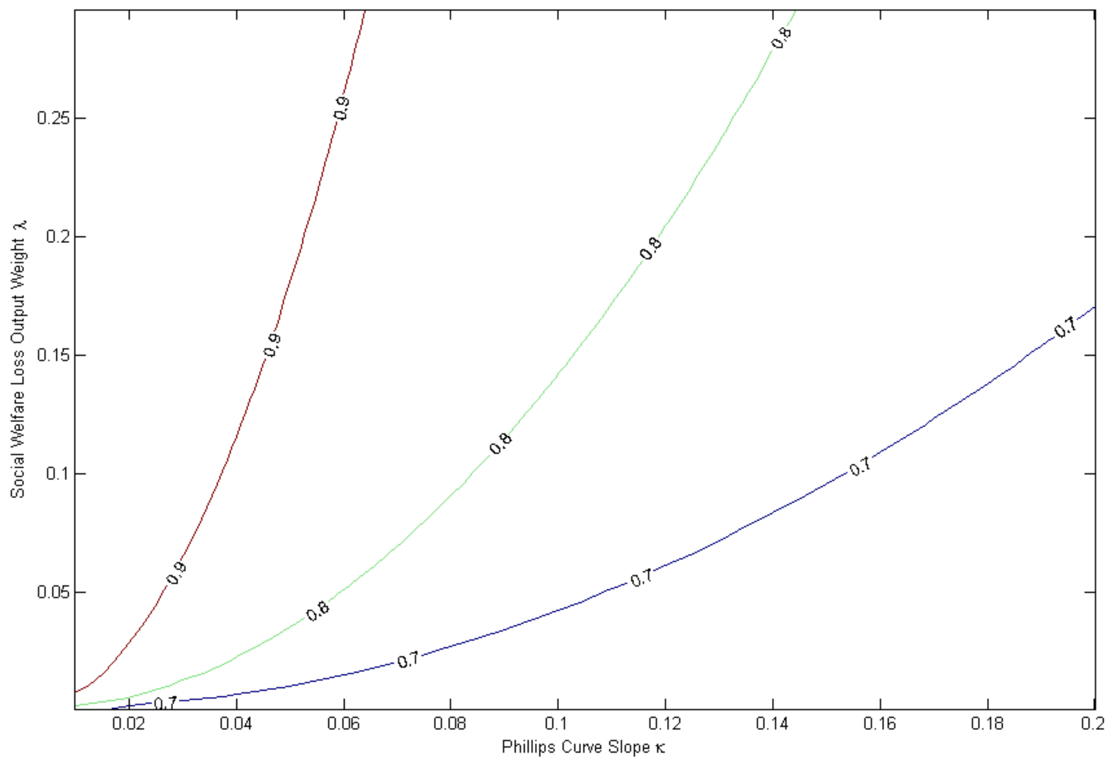


Figure 3: **Welfare Comparison on the full Parameter Space** Each contour line plots the pairs $\{\kappa, \lambda\}$ with a constant persistence threshold.

For all $\kappa, \lambda > 0$ there exists $\bar{\rho}$ such that $\bar{\rho}\beta < 1$.

The persistence threshold $\bar{\rho}(\lambda, \kappa)$ is increasing in $\lambda$ and decreasing in $\kappa$.[16] Both comparative statics are as one would expect them. For a given level of persistence, the case for full policy discretion is stronger the more society cares about output variation relative to inflation variation, i.e., the higher $\lambda$. Hence the persistence threshold increases with $\lambda$. Similarly, full discretion performs better if the Phillips curve is flatter, i.e., $\kappa$ is lower, as inflation is more effective in stabilizing output.

The welfare comparison for the full parameter space $\{\lambda, \kappa, \rho\}$ is displayed in Figure 3.

[16]For comparative statics, it is convenient to rewrite (16) as $(\beta\rho - 1)^2 / (2\beta\bar{\rho} - 1) = \kappa^2/\lambda$.

The value for the Phillips curve coefficient, $\kappa$, is in the horizontal axis; the output gap weight in the social welfare loss function, $\lambda$, is on the vertical axis. The contour lines indicate pairs $\{\lambda, \kappa\}$ which map into a constant persistence threshold $\bar{\rho}$. For parametrizations $\{\lambda, \kappa\}$ to the southeast of a contour line, strict inflation targeting outperforms full policy discretion. Our claim that the perverse policy response arises as long as the shocks are sufficiently persistent is clearly illustrated: the higher the persistence, the larger the parameter subspace where strict inflation targeting dominates.

Note that the contour lines become more steeper for higher values of persistence. This implies that the perverse policy phenomenon becomes quite robust to parametrizations of $\lambda$ and $\kappa$ for shocks with a half-life past three quarters.

## 5.2 A Structural Approach

Equations (3) and (2) can be derived from first principles and parameters can be given a structural interpretation. The reader is referred to Woodford (2003) for details. The basic structure of the economy features a representative household who consumes a variety of goods and a continuum of firms that behave as monopolistic competitors. A nominal friction is introduced a la Calvo: each period a randomly draw fraction $\alpha$ of firms cannot optimally reset their nominal price — instead it is updated according to the indexation rule.

The output weight in the period social welfare loss (3) is given by

$$\lambda = \frac{\kappa}{\theta},$$

where $\theta$ is the elasticity of substitution across varieties. The slope of the Phillips curve is

$$\kappa = \frac{(1-\alpha)(1-\alpha\beta)}{\alpha} \frac{(\sigma^{-1}+\omega)}{1+\omega\theta},$$

where $\alpha$ is the fraction of firms that do not change the price; $\sigma$ is the intertemporal elasticity of substitution, and $\omega$ is the elasticity of the real marginal cost with respect to own output. Note that the degree of price indexation does not arise anywhere.

As discussed above, we are interested in the persistence threshold $\bar{\rho}(\lambda, \kappa)$ such that for $\rho \geq \bar{\rho}(\lambda, \kappa)$ the welfare under strict inflation targeting is higher than under full policy discretion. With the exception of the elasticity of substitution $\theta$, all structural parameters will affect $\lambda$ and $\kappa$ equally as the structural model dictates them to be proportional. This is important because, as discussed in the previous subsection, the output weight and the slope of the Phillips curve have opposite effects on the persistence threshold. These countervailing effects imply that the persistence threshold is not very sensitive to different structural parametrizations.

We explore this point further by computing the persistent threshold for different degrees of price stickiness. Figure 4 displays our results. The fraction of sticky prices is on the horizontal axis and we set the elasticity of the real marginal cost with respect to own output on the vertical axis.[17] Each contour line plots parameter pairs such that the persistence threshold $\bar{\rho}$ is constant. For a given persistence $\rho$, strict inflation targeting outperforms full policy discretion for parametrizations to the left of the respective contour line. Standard estimates for price stickiness range somewhere between 3 and 5 quarters, implying $\alpha$ between .75 and .85. For the elasticity of the real marginal cost with respect to own output Rotemberg and Woodford (1997) take a value close to $\omega = .5$.[18]

Nominal frictions strengthen, only slightly, the case for full policy discretion. For a high degree of price stickiness, the Phillips curve is basically flat and inflation is very effective at stabilizing output. However, the effect of price stickiness is small. This is perhaps surprising as the slope of the Phillips curve is very sensitive to the level of price stickiness. An increase in the average price duration from two to four quarters cuts the slope to one third. The structural relationship between the slope of the Phillips curve and the output weight in the welfare loss is behind this. When only a limited number of firms can change prices, even small amounts of inflation require large price dispersion. High price stickiness enables the dove to perform more output stabilization but it also implies that the shortcomings of policy discretion (excess inflation) are more costly.

## 5.3 Fixing the Stabilization Bias

It has been pointed out that commitment is needed to implement the optimal policy response even when policy discretion features no inflationary bias. The difference between the discretionary and the optimal policy responses is known as the stabilization bias. Our analysis shows that the stabilization bias can be strong enough to make flexibility welfare reducing.

Most of the research on the stabilization bias has focused on the conditions needed to implement the optimal monetary policy when the central bank lacks commitment. As Woodford (2003) shows, the optimal policy response is (generically) history-dependent, i.e., it is a function of present *and past* realizations of shocks. Therefore no parametrization $\phi$ in our characterization of the policymaker's objectives (11) can implement the optimal policy response.

[17] The remaining parameters are set to the following baseline values. The model is evaluated at a quarterly frequency. The intertemporal discount rate $\beta$ is set to .995 in order to replicate a 2% annual real interest rate. The elasticity of substitution matches a 20% markup, $\theta = 6$. Finally the intertemporal elasticity of substitution corresponds to log-preferences, $\sigma = 1$.

[18] In our simple model, the elasticity is a key determinant of the degree of complementarity, as given by $\frac{1+\omega\theta}{\sigma^{-1}+\omega}$.

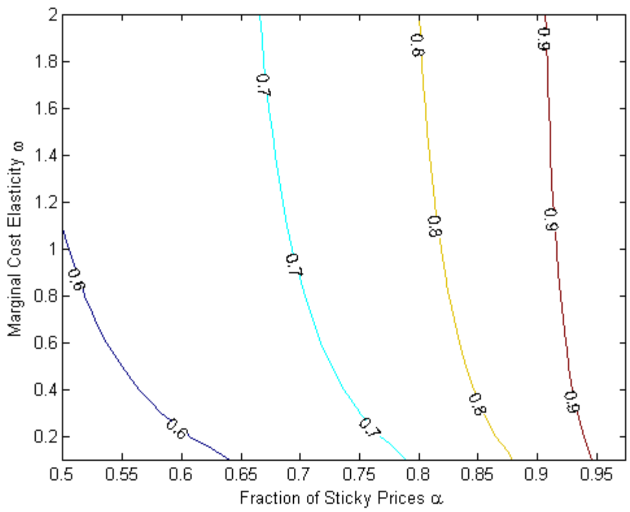


Figure 4: **Robustness Exercises: Marginal Cost Elasticity** Each contour line plots the pairs $\{\alpha, \omega\}$ with a constant persistence threshold. Strict inflation targeting outperforms full policy for parametrizations to theleft of the contour line. See text for details on unspecified parameters.

A possibility, put forward by Svensson (2003) and Giannoni and Woodford (2005), is a *targeting rule*. The central bank targets an inflation target which is a function of present and past realizations of the output gap, $\pi^*(x_t, x_{t-1}, ...)$. The targeting rule can always be designed such that the optimal policy response is implemented.

Indeed, some researchers have departed from policymaker's objectives of the functional form (11) to incorporate some history-dependence. A well known proposal is price-level targeting—see Vestin (2003). The central bank objectives are given by deviations of the price level, $p_t^2 + \lambda x_t^2$. The price level is history-dependent in the sense that it reflects the whole history of past inflation rates. Vestin (2003) shows that in a special case the optimal policy is implemented and more generally it can be well approximated. Cecchetti and Kim (2005) go further and consider "hybrid targeting," a combination inflation and price level targeting.

# 6 Conclusions

Optimal monetary policy sits at the core of the debate on inflation targeting. Researchers have been analyzing the extent to which the optimal monetary policy—ideally across a variety of acceptable models—is approximated by some form of inflation targeting. Policymakers, however, often feel uncomfortable about optimal policy analysis given the large uncertainty about the economy's workings.

We argue that there are conditions such that any inflation targeting regime, even if it is overly strict on inflation, is preferable to full policy discretion. A sufficient condition is that the underlying shocks are persistent, i.e., there is a large degree of extrinsic inflation persistence. Other determinants of optimal monetary policy play little or no role. It is thus possible to be simultaneously uncertain about the optimal inflation targeting regime and confident that a move towards inflation targeting will improve monetary policy. This is the kind of reassurance policymakers are after.

Our argument is built upon the fact that zero flexibility outperforms policy discretion in stabilizing the economy in response to a persistent shock. This theoretical point is important by itself. Rules are commonly thought to provide some gains in long-term inflation at the cost of forgone stabilization policy. Our result shows that the loss of stabilization policy can be an advantage of rules.

Serious quantitative analysis is needed before we draw any firm conclusions. Such analysis will face important challenges. First and foremost, there is the question of whether full policy discretion is an appropriate description of current U.S. monetary policy. Second, large-scale monetary models have many sources of shocks: it would be important to evaluate them jointly. Finally, one would need to confidently evaluate the sources of inflation persistence in the data as only extrinsic persistence induces the perverse policy phenomenon. As demanding as it is, such a research agenda has a better outlook than chasing a reassuring characterization of the optimal monetary policy.

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# A Linear Markov Equilibrium for the Full Policy Discretion Scenario

As mentioned in the in main text, we restrict attention to linear Markov equilibria. Thus, inflation expectations are tied to the state of variables of the model.

In our model with price indexation, past inflation is an endogenous state variable of the economy. Thus, finding the equilibrium policy rules under full policy discretion can be quite complicated. However, we show here, that if we reformulate the monetary authority's problem in terms of $\tilde{\pi}_t = \pi_t - \gamma\pi_{t-1}$, and allow the policymaker to choose $\tilde{\pi}_t$ as opposed to $\pi_t$, the reformulated problem has no endogenous state variables (and is isomorph with the model without indexation, i.e. $\gamma = 0$).

As already stated in the main text, the Phillips curve of the economy with indexation is given by

$$\pi_t - \gamma\pi_{t-1} = \kappa x_t + \beta E_t\left(\pi_{t+1} - \gamma\pi_t\right) + u_t. \tag{17}$$

The monetary authority minimizes the expected discounted loss, $L_t = E_t\left\{\sum_{j=0}^{\infty}\beta^j l_{t+j}\right\}$ with the period loss function being given by

$$l_t = \left(\pi_t - \gamma\pi_{t-1}\right)^2 + \lambda x_t^2. \tag{18}$$

We follow Svensson (1999) and Vestin (2003) by stating the problem of the monetary authority in recursive form. The monetary authority solves

$$\begin{aligned} V\left(\pi_{t-1}, u_t\right) &= \min\left[\left(\pi_t - \gamma\pi_{t-1}\right)^2 + \lambda x_t^2 + \beta E_t V\left(\pi_t, u_{t+1}\right)\right] \\ &\quad +\theta_t\left[\pi_t - \gamma\pi_{t-1} - \kappa x_t - \beta E_t\left(\pi_{t+1} - \gamma\pi_t\right) - u_t\right]. \end{aligned} \tag{19}$$

Since we focus on linear Markov equilibria, we conjecture that inflation follows:

$$\pi_t = b_1 u_t + b_2\pi_{t-1}. \tag{20}$$

Two immediate consequences are, that the value function is quadratic

$$V\left(\pi_{t-1}, u_t\right) = a_0 + a_1 u_t + \frac{1}{2}a_2 u_t^2 + a_3 u_t\pi_{t-1} + a_4\pi_{t-1} + \frac{1}{2}a_5\pi_{t-1}^2$$

and that under the additional assumption of rational expectations formation inflation expectations are given by

$$E_t\pi_{t+1} = b_1 E_t u_{t+1} + b_2\pi_t. \tag{21}$$

Using the (21) in (19) and taking first order conditions, leaves us with the following set of equations:

$$\begin{aligned}
\left(1-\beta\left(b_2-\gamma\right)\right)\pi_t-\gamma\pi_{t-1}-\left(1+\beta b_1\rho\right)u_t &= \kappa x_t && (22)\\
2\lambda x_t-\theta_t\kappa &= 0 && (23)\\
2\left(\pi_t-\gamma\pi_{t-1}\right)+\beta\frac{\partial E_tV\left(\pi_t,u_{t+1}\right)}{\partial\pi_t}+\theta_t\left(1+\beta\gamma\right)-\theta_t\beta\frac{\partial E_t\pi_{t+1}}{\partial\pi_t} &= 0 && (24)
\end{aligned}$$

where $E_t\frac{\partial\pi_{t+1}}{\partial\pi_t}$ and $E_tV_\pi\left(\pi_t,u_{t+1}\right)$ are given by

$$\begin{aligned}
\frac{\partial E_t\pi_{t+1}}{\partial\pi_t} &= b_2\\
\frac{\partial E_tV\left(\pi_t,u_{t+1}\right)}{\partial\pi_t} &= a_3E_tu_{t+1}+a_4+a_5\pi_t.
\end{aligned}$$

Substituting out for $x_t$ and $\theta_t$, delivers a relationship between $\pi_t$, $\pi_{t-1}$ and $u_t$:

$$\begin{aligned}
\pi_t &= \frac{2\gamma\left\{1+\frac{\lambda}{\kappa^2}\left[1-\beta\left(b_2-\gamma\right)\right]\right\}}{\left\{\left(2+\beta a_5\right)+2\frac{\lambda}{\kappa^2}\left[1-\beta\left(b_2-\gamma\right)\right]^2\right\}}\pi_{t-1} && (25)\\
&\quad-\frac{\left\{\beta a_3\rho-2\frac{\lambda}{\kappa^2}\left[1-\beta\left(b_2-\gamma\right)\right]\left(1+\beta b_1\rho\right)\right\}}{\left\{\left(2+\beta a_5\right)+2\frac{\lambda}{\kappa^2}\left[1-\beta\left(b_2-\gamma\right)\right]^2\right\}}u_t\\
&\quad-\frac{\beta a_4}{\left\{\left(2+\beta a_5\right)+2\frac{\lambda}{\kappa^2}\left[1-\beta\left(b_2-\gamma\right)\right]^2\right\}}
\end{aligned}$$

Furthermore, the envelope condition and our guess for the value function imply

$$\begin{aligned}
\frac{\partial E_tV\left(\pi_{t-1},u_t\right)}{\partial\pi_t} &= a_3u_t+a_4+a_5\pi_{t-1} && (26)\\
&= -2\gamma\left(\pi_t-\gamma\pi_{t-1}\right)-2\gamma\frac{\lambda}{\kappa}x_t\\
&= -2\gamma\left\{1+\frac{\lambda}{\kappa^2}\left(1-\beta\left(b_2-\gamma\right)\right)\right\}\pi_t\\
&\quad+2\gamma^2\left\{1+\frac{\lambda}{\kappa^2}\right\}\pi_{t-1}\\
&\quad+2\gamma\frac{\lambda}{\kappa^2}\left(1+\beta b_1\rho\right)u_t
\end{aligned}$$

or after using (25)

$$\begin{aligned}
\frac{\partial V(\pi_{t-1},u_t)}{\partial \pi_{t-1}} &= +2\gamma^2\left\{1+\frac{\lambda}{\kappa^2}\right\}\pi_{t-1} \\
&\quad -2\gamma\left\{1+\frac{\lambda}{\kappa^2}\left(1-\beta\left(b_2-\gamma\right)\right)\right\}\frac{2\gamma\left\{1+\frac{\lambda}{\kappa^2}\left[1-\beta\left(b_2-\gamma\right)\right]\right\}}{\left\{(2+\beta a_5)+2\frac{\lambda}{\kappa^2}\left[1-\beta\left(b_2-\gamma\right)\right]^2\right\}}\pi_{t-1} \\
&\quad +2\gamma\left\{1+\frac{\lambda}{\kappa^2}\left(1-\beta\left(b_2-\gamma\right)\right)\right\}\frac{\left\{\beta a_3\rho-2\frac{\lambda}{\kappa^2}\left[1-\beta\left(b_2-\gamma\right)\right]\left(1+\beta b_1\rho\right)\right\}}{\left\{(2+\beta a_5)+2\frac{\lambda}{\kappa^2}\left[1-\beta\left(b_2-\gamma\right)\right]^2\right\}}u_t \\
&\quad +2\gamma\frac{\lambda}{\kappa^2}\left(1+\beta b_1\rho\right)u_t \\
&\quad +2\gamma\left\{1+\frac{\lambda}{\kappa^2}\left(1-\beta\left(b_2-\gamma\right)\right)\right\}\frac{\beta a_4}{\left\{(2+\beta a_5)+2\frac{\lambda}{\kappa^2}\left[1-\beta\left(b_2-\gamma\right)\right]^2\right\}}.
\end{aligned} \tag{27}$$

Following the method of undetermined coefficients, we compare the coefficients in our conjectured equilibrium rules to the coefficients in the implied policy rules. Comparing the terms across the first line in (26) and (27), the following restrictions are imposed on $a_3$, $a_4$ and $a_5$:

$$\begin{aligned}
a_5 &= -2\gamma\left\{1+\frac{\lambda}{\kappa^2}\left(1-\beta\left(b_2-\gamma\right)\right)\right\}\frac{2\gamma\left\{1+\frac{\lambda}{\kappa^2}\left[1-\beta\left(b_2-\gamma\right)\right]\right\}}{\left\{(2+\beta a_5)+2\frac{\lambda}{\kappa^2}\left[1-\beta\left(b_2-\gamma\right)\right]^2\right\}} \\
&\quad +2\gamma^2\left\{1+\frac{\lambda}{\kappa^2}\right\}
\end{aligned} \tag{28}$$

$$\begin{aligned}
a_3 &= 2\gamma\left\{1+\frac{\lambda}{\kappa^2}\left(1-\beta\left(b_2-\gamma\right)\right)\right\}\frac{\left\{\beta a_3\rho-2\frac{\lambda}{\kappa^2}\left[1-\beta\left(b_2-\gamma\right)\right]\left(1+\beta b_1\rho\right)\right\}}{\left\{(2+\beta a_5)+2\frac{\lambda}{\kappa^2}\left[1-\beta\left(b_2-\gamma\right)\right]^2\right\}} \\
&\quad +2\gamma\frac{\lambda}{\kappa^2}\left(1+\beta b_1\rho\right)
\end{aligned} \tag{29}$$

$$a_4 = 2\gamma\left\{1+\frac{\lambda}{\kappa^2}\left(1-\beta\left(b_2-\gamma\right)\right)\right\}\frac{\beta a_4}{\left\{(2+\beta a_5)+2\frac{\lambda}{\kappa^2}\left[1-\beta\left(b_2-\gamma\right)\right]^2\right\}} \tag{30}$$

In addition (25) implies

$$b_2 = \frac{2\gamma\left\{1+\frac{\lambda}{\kappa^2}\left[1-\beta\left(b_2-\gamma\right)\right]\right\}}{\left\{(2+\beta a_5)+2\frac{\lambda}{\kappa^2}\left[1-\beta\left(b_2-\gamma\right)\right]^2\right\}} \tag{31}$$

$$b_1 = -\frac{\left\{\beta a_3\rho-2\frac{\lambda}{\kappa^2}\left[1-\beta\left(b_2-\gamma\right)\right]\left(1+\beta b_1\rho\right)\right\}}{\left\{(2+\beta a_5)+2\frac{\lambda}{\kappa^2}\left[1-\beta\left(b_2-\gamma\right)\right]^2\right\}} \tag{32}$$

$$0 = -\frac{\beta a_4}{\left\{(2+\beta a_5)+2\frac{\lambda}{\kappa^2}\left[1-\beta\left(b_2-\gamma\right)\right]^2\right\}} \tag{33}$$

An immediate conclusion from (33), is $a_4 = 0$.

To show that the model with indexation ($\gamma \neq 0$) and without indexation ($\gamma = 0$) are isomorph, consider the solution guess $b_2 = \gamma$. Then equation (28) implies $a_5 = 0$ since for the guess $b_2 = \gamma$,

$$-2\gamma^2 \left\{1 + \frac{\lambda}{\kappa^2}\right\} \frac{\left\{2 + 2\frac{\lambda}{\kappa^2}\right\}}{\left\{\beta a_5 + 2 + 2\frac{\lambda}{\kappa^2}\right\}} + 2\gamma^2 \left\{1 + \frac{\lambda}{\kappa^2}\right\} = a_5.$$

$a_3$ and $b_1$ are determined from (29) and (32) under the assumptions $a_4 = a_5 = 0$ and $b_2 = \gamma$:

$$\begin{aligned} b_1 &= -\frac{\beta a_3 \rho - 2\frac{\lambda}{\kappa^2}(1 + \beta b_1 \rho)}{2 + 2\frac{\lambda}{\kappa^2}}, \\ a_3 &= 2\left\{1 + \frac{\lambda}{\kappa^2}\right\} \frac{\left\{\beta a_3 \rho - 2\frac{\lambda}{\kappa^2}(1 + \beta b_1 \rho)\right\}}{2\left\{1 + \frac{\lambda}{\kappa^2}\right\}}\gamma + 2\gamma\frac{\lambda}{\kappa^2}(1 + \beta b_1 \rho), \end{aligned}$$

leading to $b_1 = \frac{\lambda}{\kappa^2 + \lambda(1 - \beta\rho)}$.

$a_3 = a_4 = a_5 = 0$ is exactly what we should expect if the problem with $\gamma \neq 0$ is isomorph with the original case in which $\gamma = 0$. Hence inflation follows

$$\begin{aligned} \pi_t &= b_1 u_t + b_2 \pi_{t-1} \\ &= \frac{\lambda}{\kappa^2 + \lambda(1 - \beta\rho)} u_t + \gamma \pi_{t-1} \end{aligned}$$

The output gap $x_t$ is then simply given by

$$x_t = -\frac{\kappa}{\kappa^2 + \lambda(1 - \beta\rho)} u_t.$$

These are the same formulas as derived in the main text.