A Note on Incomplete Factor Taxation*

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Abstract

Optimal capital taxes may be positive in the steady state in Ramsey models with an incomplete set of factor taxes. I show this possibility crucially depends on how fiscal policy is constrained at date $t = 0$. If the government is barred from manipulating the value of initial assets, the Chamley-Judd result reappears: the optimal capital tax is always zero in the steady state.

Keywords: Ramsey equilibrium, Incomplete Factor Taxation.

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1 Introduction

Optimal capital taxes may be positive in the steady state if the government cannot tax *every* factor of production at the rate of choice. This result, first established in Correia (1996) and Jones, Manuelli, and Rossi (1997), counters the famous finding of Judd (1985) and Chamley (1986) that capital income should not be taxed in the long run. Given that the Chamley-Judd result has proven to be robust along many other dimensions, it is important to understand the rationale for permanent capital taxes under incomplete factor taxation.[1]

In this note I show that the optimal capital tax under incomplete factor taxation depends on how policy is constrained at date $t = 0$. The standard assumption is to set the capital tax at date $t = 0$ to some arbitrary value. In this case, optimal capital taxes may be non-zero in steady state — exactly as in Correia (1996). I argue that this result stems from the possibility of taxing initial asset wealth by distorting capital accumulation in the long run. If the government is barred from any manipulation of the value of initial assets, the optimal capital tax is always zero in the steady state. Crucially, claims to the ownership of the untaxed factor are included in the calculation of total assets.

All distortions in a Ramsey framework arise from the restrictions on the set of fiscal instruments. The optimal policy thus seeks to replicate lump sum taxes whenever possible. Date $t = 0$ offers an excellent opportunity to do so, as the initial asset holdings are pre-determined. For example, the government can impose a capital levy at date $t = 0$ generating large revenues and not an iota of distortions. Even if the date $t = 0$ capital tax is restricted, the government can affect the value of assets through labor taxation. These options are distortionary, but less so at date $t = 0$ than at later dates.

I consider a simple model with an untaxed factor in fixed supply. With a complete set of Arrow-Debreu securities I can always define and price an asset which entitles the owner to the stream of revenues generated by the untaxed factor. In order to emphasize its contribution to initial wealth, I model the endowment of the untaxed factor as the "fruit" of a "Lucas tree."[2]

First I solve for the optimal policy subject to an arbitrary value for the capital tax rate at date $t = 0$. As in Correia (1996), I find that the optimal capital tax is zero in the steady state if and only if there is strong separability between capital and the untaxed factor. If capital is a complement (substitute) of the untaxed factor, then the steady state capital tax is positive (negative). Note that, independently of the sign, the optimal capital tax depresses the value of the Lucas tree by reducing (increasing) the supply of a complementary (substitutive) input.

Things are very different if I constrain policy to preserve the shadow value of total initial asset holdings at some arbitrary value. In this case, I find that the optimal capital tax is always zero in the steady state. In other words, once capital taxes cannot be used to depress the value of the Lucas tree at date $t = 0$, the Chamley-Judd result reappears.

[1] Atkeson, Chari and Kehoe (1999) generalize this result for a broad class of deterministic Ramsey models. Zhu (1992) and Farhi (2006) show that it also holds with aggregate uncertainty with complete and incomplete markets, respectively. See Albanesi and Armenter (2007) for a general treatment of intertemporal distortions in the second best.

[2] Just to be clear, my argument does not rely on the existence of a physical asset: the Lucas-tree model is used for expositional convenience.

My result also illustrates how policy restrictions at date $t = 0$ can determine *qualitative* properties of the optimal capital tax rate *in the long run.* Thus looking at the steady state does not insulate the results from our modelling choices at date $t = 0$. It is not clear how to make an informed decision about which are the appropriate restrictions to impose at date $t = 0$. In particular, a constraint on the initial asset value is not necessarily more restrictive than a constraint on the initial capital tax. A noteworthy property of the constraint on the initial asset value is that the resulting Ramsey allocations are "timeless" from date $t = 0$ onwards, that is, they are an invariant function of the state of the economy for all periods, including date $t = 0$.

The remainder of this note proceeds as follows. The next section describes a simple economy and defines a competitive equilibrium. I then introduce the first of the two Ramsey equilibrium concepts and show that the optimal capital tax may be non-zero in steady state. Section 4 offers an alternative Ramsey equilibrium concept where the government is completely barred from manipulating the value of initial assets: this time the optimal capital tax is always zero. Section 5 concludes.

2 The Economy

The economy is populated by a representative household, a representative firm, and the government. Time is infinite, denoted by subscript $t = 0, 1, ...$ and there is no uncertainty.

There are three factors of production in this economy: labor n_t, capital k_t, and an unnamed factor z_t. The representative household owns all the factors of production. The supply of labor is non-negative and capped by an unit time endowment, $0 \leq n_t \leq 1$. Capital evolves according to the law of motion

$$k_{t+1} \leq (1 - \delta) k_t + i_t$$

where i_t is investment in units of foregone consumption.

Finally, the representative household owns a Lucas tree which each period produces one unit of factor z_t. The factor z_t is not storable, so it is in fixed supply:

$$z_t \leq 1. \tag{1}$$

I would like to emphasize that the Lucas-tree model is for expositional purposes. In a Lucas-tree economy it is clear that the ownership of the income stream generated by the factor is an asset for the household. But such an asset can always be defined and priced — even if there is no "tree" to back it up. The assumption of fixed supply is for simplicity and it can be relaxed.

The government has to finance an exogenously given level of expenditures $g > 0$. To this end the government can set linear tax rates on capital and labor income, θ_t and τ_t respectively. It can also borrow and save using one period bonds b_t^g. However, it cannot tax any income originating from factor z_t. A policy is a sequence of taxes and government bonds, $\{\theta_t, \tau_t, b_{t+1}^g\}$.[3] I do not assume any bounds on the tax rates, and only the natural borrowing limit for b_t^g.

[3] I also assume the government can make non-negative transfers back to the households. This assumption is convenient for the primal approach to Ramsey equilibria. It is, though, irrelevant for any economy of interest in this framework, so I exclude the additional fiscal instrument from the definitions.

The representative household values consumption c_t and leisure $1 - n_t$ according to a standard utility function $u(c_t, 1 - n_t)$. Date t welfare is given by

$$U_t = \sum_{j=t}^{\infty} \beta^{j-t} u(c_j, 1 - n_j)$$

with $0 < \beta < 1$.

The flow budget constraint at date t for the household is

$$c_t + q_t b_{t+1} + i_t + p_t (s_{t+1} - s_t) \leq (1 - \tau_t) w_t n_t + (1 - \theta_t) r_t k_t + d_t s_t + b_t + T_t \tag{2}$$

where $\{w_t, r_t, d_t\}$ are the factor rates on labor, capital, and factor z_t; b_t are one period bonds sold at discount rate q_t; s_t is a claim to the Lucas tree traded at price p_t; and $T_t \geq 0$ are non-negative transfers from the government.

The household problem consists of choosing allocations $\{c_t, n_t, k_{t+1}\}$ and assets $\{b_{t+1}, s_{t+1}\}$ to maximize U_0 subject to (2) for all dates $t \geq 0$, taking as given prices and policy as well as the initial asset holdings $\{k_0, b_0, s_0\}$.

The representative firm is perfectly competitive and produces final output according to technology

$$y_t \leq F(k_t, n_t, z_t)$$

where F is a constant returns to scale production function with the standard properties. Formally, there are no profits. However, it is possible to interpret z_t as some entrepreneurial blueprint and think of the income from factor z_t as positive profits. The claims to the Lucas tree would be then claims to the ownership of the firm.

The government budget constraint at date t is given by

$$g + b_t^g \leq \tau_t w_t n_t + \theta_t r_t k_t + q_t b_t^g. \tag{3}$$

Note that $b_t^g > 0$ is an outstanding obligation for the government.

Market clearing conditions for bonds and claims to the Lucas tree are simple

$$\begin{aligned} b_t &= b_t^g, \\ s_t &= 1. \end{aligned}$$

The aggregate resource constraints are

$$c_t + k_{t+1} + g_t \leq F(k_t, n_t, z_t) + (1 - \delta) k_t \tag{4}$$

and (1) for all $t \geq 0$.

Definition 1 *A **Competitive Equilibrium** is an allocation $\{c_t, n_t, k_{t+1}, b_{t+1}, s_{t+1}\}$, a set of prices $\{w_t, r_t, d_t, q_t, p_t\}$ and a policy $\{\theta_t, \tau_t, b_{t+1}^g\}$ such that allocations solve the household problem, firms maximize profits, the government budget constraint holds at all dates, and all markets clear.*

I next characterize the conditions for a competitive equilibrium. The household problem produces a set of necessary and sufficient first order conditions (f.o.c.). For bonds, capital, and labor supply, the f.o.c. are

$$\begin{aligned} u_t^c q_t &= \beta u_{t+1}^c, && (5) \\ u_t^c &= \beta u_{t+1}^c \left((1-\theta_{t+1}) r_{t+1} + 1 - \delta \right), && (6) \\ (1-\tau_t) w_t u_t^c &= -u_t^n, && (7) \end{aligned}$$

at every date $t \geq 0$. I have used superscripts to indicate partial derivatives. The necessary first order condition associated with the household's decision to hold claims s_t is

$$p_t = \frac{\beta u_{t+1}^c}{u_t^c} \left(p_{t+1} + d_{t+1} \right). \tag{8}$$

Clearly, one can price claims at date $t = 0$,

$$p_0 = \sum_{t=1}^{\infty} q^t d_t \tag{9}$$

where

$$q^t = q_1 ... q_t.$$

There are also transversality conditions associated with each of the household's assets.

The representative firm is perfectly competitive and equates the marginal product of each factor to its rental rate,

$$\begin{aligned} r_t &= F_t^k, && (10) \\ w_t &= F_t^n, && (11) \\ d_t &= F_t^z. && (12) \end{aligned}$$

3 Ramsey Equilibrium with a Preset Capital Tax at Date $t = 0$

A Ramsey equilibrium is usually defined as the competitive equilibrium that maximizes household welfare U_0. However, without any restriction on taxes at date $t = 0$, the Ramsey equilibrium becomes trivial. My definition of Ramsey equilibrium for this section follows the standard procedure in the literature of setting the capital tax at date $t = 0$ to some arbitrary value, usually zero.

Definition 2 *A Ramsey equilibrium given initial capital tax $\hat{\theta}_0$ is the competitive equilibrium with highest household welfare U_0 such that $\theta_0 = \hat{\theta}_0$.*

I use the primal approach to solve for Ramsey equilibria. The first step is to characterize the set of allocations that can be decentralized as a competitive equilibrium. It turns out that a single constraint — known as the implementability constraint — is sufficient to characterize this set. A Ramsey equilibrium can then be solved for as the outcome of a constrained efficiency problem on the allocation space.

The next Proposition formally states the equivalence result. Chari and Kehoe (1999) contains a detailed account of the steps involved in deriving the implementability constraint and proving the equivalence.

Proposition 3 *An allocation $\{c_t, n_t, k_{t+1}\}$ belongs to a Ramsey equilibrium given initial capital tax $\hat{\theta}_0$ if and only if it solves*

$$\max U_0$$

subject to

$$\sum_{t=0}^{\infty} \beta^t \left\{u_t^c \left(c_t - F_t^z\right) + u_t^n n_t\right\} \geq u_0^c \left\{b_0 + \left(\left(1 - \hat{\theta}_0\right) F_0^k + 1 - \delta\right) k_0\right\} \tag{13}$$

and the resource constraint (4) at all dates $t \geq 0$, taking $\left\{\hat{\theta}_0, k_0, b_0\right\}$ as given.

Proof. In the Appendix ■

Next I derive the necessary first order conditions associated with the Ramsey problem in primal approach. It is convenient to simplify some notation. Let

$$\begin{aligned} V\left(c_t, n_t\right) &\equiv u_t^c c_t + u_t^n n_t, \\ H\left(c_t, n_t, k_t\right) &\equiv -u_t^c F_t^z. \end{aligned}$$

At date $t > 0$ the necessary f.o.c. for c_t and k_{t+1} are

$$\begin{aligned} u_t^c &= \lambda_t - \phi\left(V_t^c + H_t^c\right), \\ \lambda_t &= \beta \lambda_{t+1}\left(F_{t+1}^k + 1 - \delta\right) + \phi H_t^k, \end{aligned}$$

where λ_t is the Lagrangian multiplier for the resource constraint (4) at date t and ϕ is the Lagrangian multiplier associated with the implementability constraint (13). These conditions are usually not sufficient.

Evaluating both conditions at the steady state, I obtain

$$\beta^{-1} - \left(F^k + 1 - \delta\right) = \frac{\phi H^k}{\beta \lambda}. \tag{14}$$

As long as $H^k \neq 0$, condition (14) implies that there is a permanent distortion in the intertemporal margin.[4] Using competitive equilibrium conditions (6) and (10) evaluated at the steady state

$$\beta^{-1} = \left(1 - \theta_\infty\right) F^k + 1 - \delta$$

[4] I am assuming that the implementability constraint (13) is binding $\phi > 0$ — otherwise the economy is at the first best. This is the only sense in which the precise value of $\hat{\theta}_0$ matters for the result: if the initial capital levy can be very large, there may be no need to raise any further distortionary taxation.

I can recover the steady state capital tax θ_∞. It is clear that, as in Correia (1996), the optimal capital tax is zero in the steady state if and only if $H^k = 0$, that is, when the cross-derivative F^{kz} is zero. Indeed, the sign of the capital tax is the sign of F^{kz}:

$$\begin{aligned} sign(\theta_\infty) &= sign\left(F^k + 1 - \delta - \beta^{-1}\right) \\ &= sign(-H^k) \\ &= sign(F^{zk}). \end{aligned}$$

If capital and factor z are complements, then the optimal capital tax is positive. If they are substitutes, though, the optimal policy calls for a capital subsidy in the steady state. Note that the optimal policy is effectively depressing the demand for factor z_t by either taxing a complement or subsidizing a substitute. This has the unambiguous effect of lowering the value of the Lucas tree all the way back to date $t = 0$. I will return to this observation in the next Section.

4 Ramsey Equilibrium with a Preset Value for Initial Assets

The government can manipulate the value of initial assets at date $t = 0$ even if capital taxes are constrained. For example, labor taxes affect the shadow value of debt and the return to capital at $t = 0$. This can be readily seen from the necessary f.o.c. in a Ramsey equilibrium given an initial capital tax $\hat{\theta}_0$. At any date $t > 0$, the f.o.c. for labor is

$$\lambda_t F_t^n + \phi\left(V_t^n + H_t^n\right) = -u_t^n.$$

At date $t = 0$, the f.o.c. is

$$\lambda_0 F_0^n + \phi\left(V_0^n + H_0^n\right) = -u_0^n + \phi\left\{u_0^{cn}\left\{b_0 + \left(\left(1 - \hat{\theta}_0\right) F_0^k + 1 - \delta\right) k_0\right\} + u_0^c\left(1 - \hat{\theta}_0\right) F_0^{kn} k_0\right\}.$$

The additional term reflects that taxation at date $t = 0$ affects the right hand side of the implementability constraint (13). In particular, *reducing the value of assets at date* $t = 0$ *relaxes the constraint.*

I consider a different Ramsey equilibrium concept where the government is prevented from manipulating the total value of initial assets. I define the shadow value of total initial assets

$$A_0 = u_0^c\left\{b_0 + (p_0 + d_0)\, s_0 + \left((1 - \theta_0)\, r_0 + 1 - \delta\right) k_0\right\}$$

for allocations and prices belonging to some competitive equilibrium. The main result goes through if I use the face value of assets (instead of the shadow value) or I preset the value of each of the assets rather than their total.

Definition 4 *A Ramsey equilibrium given initial asset value* $\hat{A}_0$ *is the competitive equilibrium with highest household welfare* U_0 *such that* $A_0 = \hat{A}_0$*.*

What is crucial about the above definition is that the claims to the Lucas tree are included as an asset. Of course, this is quite natural in the framework presented here. In more general terms, whatever entitles the household to the stream of factor income can be considered a contribution to initial asset wealth. To be clear, I am not arguing the above is the "correct" definition for a Ramsey equilibrium: my aim here is to clarify the rationale for taxing capital in incomplete factor models. It is important to note that a restriction on the initial asset value is neither stronger nor weaker than a restriction on the initial capital tax. That is, for any two given values for initial assets $\hat{A}_0$ and initial capital tax $\hat{\theta}_0$, there exists (typically) many competitive equilibria with $A_0 = \hat{A}_0$ but $\theta_0 \neq \hat{\theta}_0$, and many competitive equilibria with $\theta_0 = \hat{\theta}_0$ but $A_0 \neq \hat{A}_0$ as well. So it is not possible to establish for any pair $\left(\hat{\theta}_0, \hat{A}_0\right)$ which Ramsey equilibrium will achieve higher welfare.

This alternative Ramsey equilibrium concept can also be solved through the primal approach. The only difference is the implementability constraint, which is modified to accommodate the date $t = 0$ constraint on asset value.

Proposition 5 *An allocation $\{c_t, n_t, k_{t+1}\}$ belongs to a Ramsey equilibrium given an initial asset value $\hat{A}_0$ if and only if it solves*

$$\max U_0$$

subject to

$$\sum_{t=0}^{\infty} \beta^t \{u_t^c c_t + u_t^n n_t\} \geq \hat{A}_0. \tag{15}$$

and the resource constraint (4) at all dates $t \geq 0$, taking $\left\{\hat{A}_0, k_0, b_0\right\}$ as given.

I now show that the steady state capital tax is always zero in a Ramsey equilibrium given an initial asset value.[5] As before, I derive the necessary first order conditions for c_t and k_{t+1} associated with the primal approach

$$\begin{aligned} u_t^c &= \lambda_t + \phi V_t^c \\ \lambda_t &= \beta \lambda_{t+1} \left(F_{t+1}^k + 1 - \delta\right) \end{aligned}$$

for any $t \geq 0$, where I have used the same notation as before. In steady state these two conditions simplify to

$$u^c + \phi V^c = (u^c + \phi V^c)\, \beta \left(F^k + 1 - \delta\right)$$

or

$$1 = \beta \left(F^k + 1 - \delta\right).$$

There are, thus, no capital taxes in the steady state.

The contrasting result makes it clear that the rationale for taxing capital in models of incomplete factor models is to depress the value of assets at date $t = 0$. Recall that, when othe capital tax rate is pre-set at date $t = 0$, the optimal policy had the unambiguous effect

[5] The optimal capital tax in steady state does not depend on what value for the initial assets $\hat{A}_0$ is used.

of depressing the value of the Lucas tree. In other words, it achieved a lower asset value A_0. On the margin, to lower the asset value at date $t = 0$ is akin to a lump sum tax and allows to reduce distortionary taxation. Indeed, a lump sum tax, a higher capital tax, or a default on government debt have the same effect of relaxing the implementability constraint (15) in the definition of a Ramsey equilibrium given an initial asset value; so they are, on the margin, equivalent to a reduction in the initial asset value.

A noteworthy property of allocations under a Ramsey equilibrium given an initial asset value is that they are timeless, that is, they can be characterized by a time-invariant function of the state of the economy k_t in all periods, date $t = 0$ included. This can be easily seen from the fact that all allocations enter the implementability constraint (15) symmetrically.[6]

Proposition 6 *Let allocation* $\{c_t, n_t, k_{t+1}\}$ *belong to a Ramsey equilibrium given an initial asset value* $\hat{A}_0$*. Then there exist functions* φ_c*,* φ_n*, and* φ_k *such that*

$$
\begin{aligned}
c_t &= \varphi_c(k_t), \\
n_t &= \varphi_n(k_t), \\
k_{t+1} &= \varphi_k(k_t),
\end{aligned}
$$

for all $t \geq 0$*.*

Proof. It can be shown that the necessary first order conditions associated with the primal approach to a Ramsey equilibrium given initial asset value $\hat{A}_0$ constitute a time-invariant system. It follows that there exists a solution given by a time-invariant function ■

5 Conclusions

My result emphasizes the dangers associated with making arbitrary restrictions on the set of available tax instruments. In particular, the properties of the steady-state optimal taxes are not independent of whatever policy restrictions the researcher imposes at date $t = 0$.

That said, it is now clear what the logic of taxing capital in the long run in models of incomplete factor taxation is. Shall this rationale be brought forward in policy discussions? I believe most economists will feel uncomfortable with an argument for long-run capital taxation based on the exceptional inelasticity of assets at date $t = 0$. To constrain the government's ability to manipulate the value of initial assets has the clear advantage of dictating policies solely as a function of the state of the economy. In this case, the Chamley-Judd result stands: the optimal capital tax is always zero in the steady state.

[6] Thus, it is necessary to use the *shadow* value of assets in the definition of Ramsey equilibrium for allocations to be timeless.

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Proof of Proposition 3. I prove that an allocation $\{c_t, n_t, k_{t+1}\}$ belongs to a competitive equilibrium with an initial capital tax $\hat{\theta}_0$ if and only if the resource constraint (4) holds at all dates $t \geq 0$ and the implementability constraint (3). Once the equivalence between the two sets is established, the Proposition follows.

I first prove $\Longrightarrow$. Say $\{c_t, n_t, k_{t+1}\}$ conforms a competitive equilibrium with an initial capital tax $\hat{\theta}_0$ with prices $\{w_t, r_t, d_t, q_t\}$ and a policy $\left\{\theta_t, \tau_t, b^g_{t+1}\right\}$. Using (5) I rewrite equilibrium conditions (6) and (8) as non-arbitrage conditions

$$\begin{aligned} 1 &= q_t\left(\left(1-\theta_{t+1}\right) r_{t+1} + 1 - \delta\right), \\ p_t &= q_t\left(p_{t+1} + d_{t+1}\right), \end{aligned}$$

for any date $t \geq 0$. The arbitrage conditions can be then used to collapse the sequence of household budget constraints (2) into a date $t = 0$ intertemporal budget constraint

$$\sum_{t=0}^{\infty} q^t \left\{c_t - \left(1-\tau_t\right) w_t n_t - d_t - T_t\right\} = b_0 + \left(\left(1-\theta_0\right) r_0 + 1 - \delta\right) k_0$$

where I have used the market clearing condition for claims, $s_t = 1$, and the transversality conditions for each asset,

$$\begin{aligned} \lim_{t\to\infty} q^t s_{t+1} &= 0, \\ \lim_{t\to\infty} q^t k_{t+1} &= 0, \\ \lim_{t\to\infty} q^t b_{t+1} &= 0. \end{aligned}$$

Since $T_t \geq 0$ for all $t \geq 0$, I can rewrite the budget constraint as a weak inequality,

$$\sum_{t=0}^{\infty} q^t \left\{c_t - \left(1-\tau_t\right) w_t n_t - d_t\right\} \geq b_0 + \left(\left(1-\theta_0\right) r_0 + 1 - \delta\right) k_0. \tag{16}$$

The implementability constraint (13) can be then obtained by substituting for prices and after-tax factor returns using (5), (7), and (12) at all dates. To obtain the right hand side, multiply both side by $u_{c,0} > 0$ and use (10) at $t = 0$. The resource constraint holds trivially by the definition of a competitive equilibrium.

Now I prove $\Longleftarrow$. Say $\{c_t, n_t, k_{t+1}\}$ is a feasible allocation such that (13) holds. I construct a candidate competitive equilibrium as follows. Candidate prices $\{w_t, r_t, d_t, q_t, p_t\}$ and tax rates $\{\theta_{t+1}, \tau_t\}$ are given by (5)-(8), with $\theta_0 = \hat{\theta}_0$. It is immediate that the intertemporal household budget constraint (16) follows. I can then obtain a sequence of bond holdings $\{b_t\}$ using the flow budget constraint for the households (2) which is guaranteed to hold the transversality condition. By construction, the allocations solve the household problem and firms maximize profits.

Finally, the resource constraint is satisfied at every period by assumption. The government budget constraint is then implied by the Walras' law. This completes all the equilibrium conditions ■

Proof of Proposition 5. I obviate most of the proof as it proceeds through exactly the same steps as in Proposition 3. The only difference is in the derivation of the date $t = 0$ household budget constraint. Using the arbitrage condition (9) I can rewrite (16) as

$$\sum_{t=0}^{\infty} q^t \left\{c_t - (1 - \tau_t)\, w_t n_t\right\} \geq b_0 + p_0 + d_0 + \left((1 - \theta_0)\, r_0 + 1 - \delta\right) k_0$$

The equilibrium conditions then imply the new implementability constraint (15). Once both sides are multiplied by u_0^c, the right hand side is equal to the shadow value of initial assets $\hat{A}_0$

■