A General Theory (and Some Evidence) of Expectation Traps in Monetary Policy

Roc Armenter*
Federal Reserve Bank of New York

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Abstract

I show that multiple equilibria are a general property of economies under full monetary policy discretion. Three simple conditions are sufficient to rule out, generically, a unique equilibrium in a static economy. The key departure from Barro and Gordon (1983) is to consider bounded welfare costs of inflation. I also show that in a two Markov equilibrium economy the inflation response to certain perturbations is, generically, qualitatively different in each equilibrium. Finally, I discuss some evidence on inflation dynamics which supports the hypothesis that U.S. monetary policy was caught in an expectation trap during the high inflation episode of the 70s.

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1 Introduction

Does monetary policy discretion lead to equilibrium indeterminacy? Multiple equilibria are certainly worrisome if the policymaker cannot coordinate private sector expectations. The economy may end up in a welfare inferior equilibrium path and unnecessary macroeconomic volatility may arise from shifts in expectations. Following Chari, Christiano and Eichenbaum (1998), I label the multiple equilibria as "expectation traps" to capture the idea that the monetary authority is trapped into validating the private sector expectations.

In this paper I study the conditions for equilibrium multiplicity when monetary policy is the choice of a benevolent policymaker with full discretion. In particular, monetary policy can be reviewed at any point in time so the policymaker lacks the ability to commit to a policy plan. As it well known since the contribution of Kydland and Prescott (1977) and Calvo (1978), the optimal monetary policy may not be time consistent in this context and hence it may be incompatible with rational expectations.

I argue that, under full policy discretion, expectation traps should be considered the rule rather than the exception. To rule out, generically, a unique equilibrium it is sufficient that

- the welfare costs of inflation are bounded,
- first order welfare effects are sufficient to characterize policy, and
- unexpected inflation provides a stimulus to the real economy.

All my analysis is conducted in a one-period economy. This underscores that the equilibrium multiplicity can arise in static economies and thus my result does not hinge on reputation issues or any other mechanism arising solely in dynamic economies.

The main appeal of the theory is its generality. The third condition introduces the time inconsistency problem. The second is basically a technical condition, well-known to the users of optimal control in policy problems, which ensures that monetary policy is a continuous function of private sector expectations.

The first condition has the most economic content and is the main culprit behind equilibrium multiplicity. The economy presented in Barro and Gordon (1983) satisfies all conditions but bounded welfare costs of inflation, a seemingly inoffensive by-product of their choice of a quadratic loss function. To summarize, all of the three conditions are more plausible than their converse—and the conditions are just sufficient to rule out a unique equilibrium.

The theory points out that the well known inflation bias of Barro and Gordon (1983) should not be the main concern arising from the lack of commitment. Instead, it is the possibility that a simple shift in the private sector expectations can drive the economy into persistent high inflation. The solutions are different, too. An inflation cap can be set to coordinate expectations in the low inflation equilibrium without constraining the monetary policy response to plausible shocks. On the other hand, a conservative central banker—as in Rogoff (1985)—can be a double-edged sword. While reducing inflation in some equilibria, a conservative central banker *increases* inflation in other equilibria.

The downside of the theory is that its applied value may be limited because the sufficient conditions are not easily evaluated. Whether they hold may depend on certain parameter values or specifications. This actually explains some of the confusion around the economics of expectation traps. Hopefully the theory will establish that expectation traps are not artifacts associated with particular models or parameter values.

Several recent papers corroborate the generality of the theory by finding expectation traps in a wide variety of monetary economies where policy is endogenously determined in equilibrium. These papers focus on Markov perfect equilibria: the equilibrium multiplicity results do not hinge on trigger strategies. Albanesi, Chari and Christiano (2003) explores a cash/credit good model and show that monetary policy discretion may lead to multiple equilibria. King and Wolman (2004) also finds multiple equilibria in a simple new Keynesian model with two-period staggered pricing. Allowing for endogenous price rigidity Siu (2004) shows that, once again, equilibrium multiplicity arises. Siu (2004) also argues that equilibrium uniqueness can be restored by adding a small cost term, linear in inflation—effectively violating the condition on bounded welfare costs of inflation. Armenter and Bodenstein (forthcoming) studies an economy with a nominal working capital requirement and find that multiple equilibria are a robust feature of the model. The theory presented here rationalizes the remarkable ubiquity of expectation traps.

There is also a large literature which analyzes whether a particular monetary policy rule induces equilibrium multiplicity. For inflation targeting, see Bernanke and Woodford (1997). There are many papers dealing with indeterminacy and nominal interest rate rules: see Benhabib, Schmitt-Grohe and Uribe (2000) and Carlstrom and Fuerst (2001), as well as Clarida, Gali and Gertler (2000) for a more applied approach. An important difference with the research on full policy discretion is that exogenous policy rules often leads to equilibrium indeterminacy, in the sense that there are a continuum of equilibria. In contrast, only isolated Markov equilibria arise under full policy discretion.

The theoretical discussion is complemented with a brief discussion of empirical evidence. In an influential article, Clarida et al. (2000) estimate a forward-looking version of the Taylor rule.[1] They find evidence that policy was accommodative for the Pre-Volcker period (60:1-79:2) but not for the Volcker-Greenspan period (79:3-96:4).[2] Clarida et al. (2000) interpret

[1] Taylor (1993) show that a nominal interest rate based policy rule—the Taylor rule as it has come to be known—provides a good forecast of monetary policy in the U.S. See Woodford (2001) for an overview of the link between optimal monetary policy and Taylor rules.

[2] Judd and Rudebusch (1998) estimate a slightly different version of the Taylor rule yet find similar results.

these results in terms of an historical policy change. In a context of full policy discretion, the instability of the Taylor result suggests that U.S. monetary policy was in an expectation trap during the high inflation episode of the 70s. Two additional results from the theory are instrumental for an empirical evaluation of the hypothesis. First, every Markov equilibrium is locally unique. Second, expectation traps naturally induce inflation responses which are qualitatively different in each equilibrium—the sign switch phenomenon. The empirical exercise is very much in the spirit of Bisin and Topa (2005) and makes clear models with multiple equilibria have enough restrictions to be properly tested.

The first piece of evidence concerns the well known large persistence of inflation. Most statistical tests do not reject the unit root hypothesis for several measures of U.S. inflation in the post-war period. A model with multiple Markov equilibria suggests that the persistent component of inflation is driven by expectation shifts, which do not need to follow a stationary process. However, the local uniqueness of Markov equilibria is not compatible with the inflation rate following a random walk. Instead, the theory points that changes in expectations map into inflation shifts: the residual persistence of inflation should be much lower. And this is exactly the main finding in the recent literature on inflation mean breaks. I provide a short survey of the field in Section 6.

I also test the sign switch phenomenon. Using the estimated breaks in the inflation mean from Corvoisier and Mojon (2004), I split the sample between low and high inflation regimes. I then estimate an inflation equation using the Generalized Method of Moments, very much in the spirit of Clarida et al. (2000). I find that the inflation response to the output gap is qualitatively different during the low and high inflation subsamples. The sign-switch is significant across several sets of instruments.

In short, expectation traps naturally generate the observed persistence and dynamics of inflation. In contrast, these empirical phenomena pose a surmounting challenge to unique equilibrium models.

The remainder of the paper is organized as follows. Section 2 describes the framework used. Section 3 provides the sufficient conditions for equilibrium multiplicity. The sign switch phenomenon is studied in Section 4. In Section 5, I illustrate the results with two simple version of well-known examples of equilibrium multiplicity. Evidence in favor of expectation traps is discussed in Section 6. Finally, Section 7 concludes.

Gali, Lopez-Salido and Valles (2003) proceed by evaluating the monetary policy response to identified shocks across subsamples. In the reported impulse-response functions, the sign switch phenomenon is quite obvious, specially in terms of the nominal interest rate.

2 The Economy

The economy is populated by a set I of private sector agents and a monetary authority. The timing is as follows. First, each private sector agent chooses an action $z_i \in Z$. The monetary authority observes the agents' decisions $\mathbf{z} = \{z_i\}_{i \in I}$ and then sets the inflation rate $\pi \geq \beta$, where the lower bound β is imposed by feasibility.[3] Finally, given $\mathbf{z}$ and π, agents make remaining decisions, markets clear and equilibrium allocations $\mathbf{x} = \{x_i\}_{i \in I}$ are determined according to private sector equilibrium function $\mathbf{x} = \Psi(\mathbf{z}, \pi)$. Private sector agent i welfare is given by utility function $u_i(x_i)$.

For example, a fraction of firms may set their nominal prices in advance. The monetary authority decides the inflation rate taking the "sticky" prices as given. With knowledge of the monetary authority's decision and sticky prices, the remaining fraction of firms set their nominal prices, households make their consumption and leisure decisions and markets clear.

Private sector agents have to form expectations π_i^e about the inflation rate in order to act optimally. Under the assumptions of rational expectations and common knowledge, private sector expectations do not differ across agents, $\pi_i^e = \pi^e$. Agent i chooses then the action z_i that maximizes $u_i(\Psi_i(\{z_i, z_{-i}\}, \pi^e))$, taking as given the actions of the rest of the private sector z_{-i}. Let $\mathbf{z}^*$ be the solution to

$$z_i^* \in \arg\max_{z \in Z} u_i\left(\Psi_i\left(\left\{z, z_{-i}^*\right\}, \pi^e\right)\right) \tag{1}$$

for all $i \in I$. I assume there exists a solution $\mathbf{z}^*$ for all $\pi^e \geq \beta$. Let function $\zeta(\pi^e)$ map a private sector expectation $\pi^e \geq \beta$ into a vector of actions $\mathbf{z}^*$ solving the system (1).

The monetary policy decision is to maximize private sector welfare

$$\max_{\pi \geq \beta} \int_I u\left(\Psi_i\left(\mathbf{z}^*, \pi\right)\right) di$$

taking as given private sector actions $\mathbf{z}^*$. The monetary authority is benevolent since it chooses policy to maximize private sector welfare; it is informed as it has full knowledge of the economy and actions; and it has full policy discretion because its policy choice is only constrained by feasibility.

For the purposes of the paper, the economy is effectively summarized by the indirect welfare function

$$v(\pi, \pi - \pi^e) = \int_I u\left(\Psi_i\left(\zeta\left(\pi^e\right), \pi\right)\right) di. \tag{2}$$

[3] The notation is inspired by the zero nominal interest rate bound but the non-negativity of prices would also bound the inflation rate from below.

I will use the indirect welfare function v to characterize both policy equilibria and the structure imposed in the economy. Throughout the paper, I assume the necessary conditions upon u, Ψ and ζ such that $v(\pi, \pi - \pi^e) : \Re^2 \to \Re$ is twice continuously differentiable and it is bounded above for $\{\pi, \pi^e\} \in [\beta, \infty)$.

Note the indirect welfare function is written in terms of actual inflation π and its unexpected component $\pi - \pi^e$. This turns out to be quite convenient and it is not void of intuition either. Most economists agree that the monetary authority trades off the welfare costs of inflation against the real stimulus that unexpected inflation can provide. Both actual and expected inflation may have an impact upon private sector welfare. Picking up the previous example, both sticky nominal prices—function of expectations only—and flexible nominal prices impact market allocations and hence welfare.

For the sake of simplicity, the inflation rate π is taken to be the monetary policy instrument. Implicitly, I have assumed there exists a monotone relationship between whatever the actual policy instrument is and inflation. However the proposed framework can encompass many other possibilities. For example, assume that the monetary authority chooses an inflation rate *target* π. Actual inflation is given by $\hat{\pi}(s) = \pi + \varepsilon(s)$ for a random variable $s \in S$ distributed according to G. Let x_i be a state contingent allocation plan $x_i = \{x_i(s)\}_{s \in S}$, given by a state s private sector equilibrium function $\psi_i(\mathbf{z}, \hat{\pi}(s), s)$. One can then write the private sector equilibrium function $\Psi_i(\mathbf{z}, \pi) = \{\psi(\mathbf{z}, \pi + \varepsilon(s), s)\}_{s \in S}$. Assume v is a von Neumann-Morgenstern utility function $u(x_i) = \int_s v(x_i(s)) G(ds)$ and the indirect welfare function v is well defined in terms of the inflation target π and expectation π^e. A similar argument can be done for economies with multiple private sector equilibrium allocations associated with same pair π and $\mathbf{z}$, indexing each equilibrium by a sunspot variable.[4]

I will analyze policy equilibria which feature the monetary authority decision as an equilibrium object. The policy equilibrium is built upon the rational expectations hypothesis and the optimality of the monetary authority decision. The former implies that the private sector agents correctly forecast the monetary authority decision, $\pi = \pi^e$. Optimality requires that, given private sector expectation π^e, the monetary authority chooses the inflation rate π that maximizes private sector welfare.

For simplicity, I will dispense of actions $\mathbf{z}$ and allocations $\mathbf{x}$ in the definition of policy equilibrium. While the distinction is void in the context of a static economy, I will denote the policy equilibrium as Markov equilibrium. The nomenclature aims to emphasize that the results are not related to the well-known multiplicity of Nash equilibria in infinite horizon economies.

[4] Indirect welfare function v will not be well defined for economies where system (1) has multiple solutions $\mathbf{z}^*$ for the same private sector expectation π^e. However, being the point of the paper the generality of equilibrium multiplicity, this omission is irrelevant.

Definition 1 *A Markov Equilibrium is* π^m *such that*

$$\pi^m \in \arg\max_{\pi \geq \beta} v\left(\pi, \pi - \pi^m\right).$$

Note that the monetary authority treats the private sector expectations, $\pi^e = \pi^m$, as given. In equilibrium, out of all possible choices, it is optimal for the monetary authority to validate the private sector expectation. A policy π is denoted time inconsistent if π does not constitute a Markov equilibrium.

The following notation is on place:

$$v^j\left(x_1, x_2, ..., x_n\right) = \frac{\partial v\left(x_1, x_2, ..., x_n\right)}{\partial x_j}.$$

Consider the best policy response $\pi^*\left(\pi^e\right)$ given private sector expectations π^e,

$$\pi^*\left(\pi^e\right) \in \arg\max_{\pi \geq \beta} v\left(\pi, \pi - \pi^e\right).$$

The necessary first order condition associated with π^* is

$$v^1\left(\pi^*, \pi^* - \pi^e\right) + v^2\left(\pi^*, \pi^* - \pi^e\right) \leq 0 \tag{3}$$

with strict equality if $\pi^* > \beta$.

A Markov equilibrium π^m must be the best response to expectations $\pi^e = \pi^m$. Hence, a necessary condition associated with a Markov equilibrium is

$$v^1\left(\pi^m, 0\right) + v^2\left(\pi^m, 0\right) \leq 0 \tag{4}$$

with strict equality if $\pi^m > \beta$.

I characterize optimal monetary policy as a baseline case. One can visualize the optimal monetary policy problem as an alternative equilibrium concept—the Ramsey equilibrium—where the monetary authority decision precedes all private sector decisions. The choice of monetary policy is restricted by rational expectations, i.e. for any choice of π, $\pi^e = \pi$. But now private sector expectations are not taken as given by the monetary authority.

Definition 2 *A Ramsey equilibrium is* π^r *such that*

$$\pi^r \in \arg\max_{\pi \geq \beta} v\left(\pi, 0\right).$$

The necessary first order condition associated with a Ramsey equilibrium is

$$v^1(\pi^r, 0) \leq 0 \tag{5}$$

with strict equality if $\pi^r > \beta$.

Do Ramsey equilibria constitute a Markov equilibrium? In other words, is the optimal monetary policy time-consistent? Not surprisingly, the economy here points to the welfare impact of the unexpected component of inflation $\pi - \pi^e$ for an answer. If the monetary authority gains nothing from deviating from the private sector expectations, then any Ramsey equilibrium is a Markov equilibrium. If unexpected inflation brings sufficiently large welfare gains, then rational expectations rule out optimal monetary policies. In terms of first order welfare effects, if $v^2(\pi^r, 0)$ is large enough then π^r does not satisfy the necessary condition for a Markov equilibrium (4).

However, commitment may still be valuable even if optimal monetary policy is time consistent. The reason is that even if all Ramsey equilibria are Markov equilibria, there is no guarantee that every Markov equilibrium is a Ramsey equilibrium. Absent commitment or an alternative equilibrium selection mechanism, the monetary authority can find itself trapped into validating inflation expectations far from the optimal monetary policy. In first order terms, the fact that π^r is a solution to (4) does not rule out other Markov equilibria π^m which does not satisfy (5).

3 Equilibrium Multiplicity

In this section I provide sufficient conditions to rule out, generically, an unique Markov equilibrium. The conditions are formalized in terms of the indirect welfare function $v(\pi, \pi - \pi^e)$. A small discussion accompanies each condition.

The results hold 'generically,' i.e., counterexamples are characterized by a restriction upon the economy's parameters which would not be robust to any arbitrarily small perturbation. See Mas-Colell, Whinston and Green (1995), page 595, for a brief introduction on genericity analysis.

The first condition is that the welfare costs of inflation are bounded. It has a straight mathematical formulation.

Condition 1 *For all $\pi \geq \beta$, $v(\pi, 0) \geq B > -\infty$.*

Note that v must be bounded only along the locus of rational expectations $\pi = \pi^e$.

It is hard to argue that inflation has unbounded costs. Hyperinflation would eventually render fiduciary money valueless, so a barter economy provides a natural lower bound. In other words, the economy may demonetize. Yet there is no need for such an extreme. Wage and bond indexation bound the welfare costs of inflation. Another example is Siu (2004), featuring a monetary economy where firms can pay a fix cost up-front in order to access a full flexible pricing technology. Once all firms have incurred the cost, inflation has no further real effects.

The second condition states that, in a neighborhood of π^e, first order conditions are sufficient to characterize the monetary authority's best response. This is often not the case in the application of optimal control techniques to policy design. The reason is that the indirect welfare function, $v(\pi, \pi - \pi^e)$, is likely not be concave in π for all (π, π^e). This has not stopped researchers from using first order conditions to characterize candidate policies and then cautiously check the global properties with numerical methods. It is often reported that first order conditions seem to be indeed sufficient.[5]

I formalize this heuristic approach by assuming that first order welfare changes in a neighborhood of the expected inflation rate are sufficient to characterize the policy choice. Loosely speaking, the condition says that the expected inflation rate π^e is the correct initial guess for any maximization algorithm based on first order conditions.

Condition 2 *For all $\pi^e \geq \beta$, there exists a neighborhood of π^e, $A(\pi^e)$, such that if $\pi^* \in A(\pi^e)$ is a critical point of $v(\pi, \pi - \pi^e)$ then $\pi^* \in \arg\max_{\pi \geq \beta} v(\pi, \pi - \pi^e)$.*

Note that Condition 2 only requires the existence of any neighborhood $A(\pi^e)$, understood as an open, connected subset over the real line. There is no need for that neighborhood to be arbitrarily small or to hold any other property. The condition is thus significantly weaker than just assuming that first order conditions (evaluated anywhere) are sufficient.

To illustrate Condition 2, Figure 1 depicts two examples of $v(\pi, \pi - \pi^e)$ for a given π^e. *Example A* satisfies Condition 2. Note $v(\pi, \pi - \pi^e)$ features convex regions as well as local minima away from π^e. Hence, Condition 2 is a weaker restriction than global concavity of v. *Example B* illustrate a case where v does not hold Condition 2. The global maximum is in a different concave region of v than π^e, which is a local maxima. Hence, it is not possible to construct any connected neighborhood of π^e which does not include an additional critical point, π^e itself.

Condition 2 is also closely related to the properties of the best response function of the monetary authority. Condition 2 implies that, in a neighborhood of a Markov equilibrium π^m, the first order condition (3) is sufficient to characterize the monetary authority problem. Generically, the implicit function theorem can be applied then at π^m, and $\pi^*(\pi^e)$ is a differentiable function around π^m.

[5] There are counterexamples within the literature. See Albanesi et al. (2003) and Armenter and Bodenstein (forthcoming).

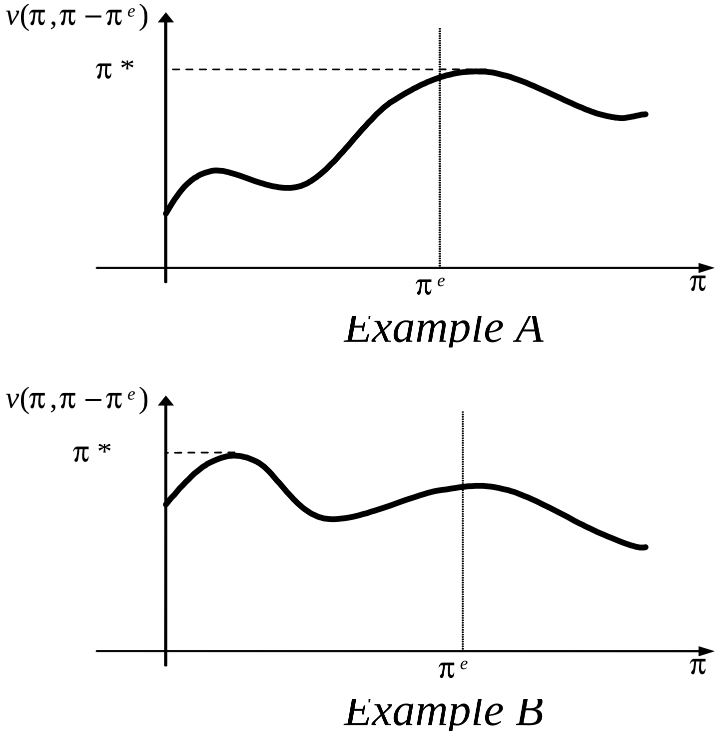

Figure 1: Illustrating Condition 2

This is summarized in the following remark.

Remark 3 *Let Condition 2 be satisfied and* π^m *be a Markov equilibrium. Then, generically, for* π^e *in a neighborhood of* π^m*,*

$$\pi^*(\pi^e) = \arg\max_{\pi \geq \beta} v(\pi, \pi - \pi^e)$$

is a differentiable function.

Now it is straightforward to show that any Markov equilibrium π^m is, generically, locally unique. Otherwise, the best policy response function $\pi^*(\pi^e)$ would need to have slope equal to 1 exactly at π^m—a non-generic condition.

Lemma 4 *Let Condition 2 hold. Then, generically, any Markov equilibrium is locally unique.*

It only remains to bring in the temptation of unexpected inflation for the monetary authority. If deviations from the private sector expectations had no impact whatsoever on welfare, then the set of Markov equilibria would trivially coincide with the Ramsey equilibrium set. Any equilibrium multiplicity would be then welfare irrelevant.

In order to link unexpected inflation to a stimulus in the real economy, Condition 3 assumes that given any inflation rate π, the monetary authority prefers that the private sector expects slightly less inflation. Using the present notation, the condition states that, given any $\pi > \beta$, arbitrarily close expectations $\pi_0^e < \pi$ and $\pi_1^e > \pi$ are strictly welfare ordered, i.e., $v(\pi, \pi_0^e) > v(\pi, \pi_1^e)$.

The formal statement uses the differentiability of v, so the assumption can be extended to the corner $\pi = \beta$.

Condition 3 *For all* $\pi \geq \beta$*,* $v^2(\pi, 0) \geq C > 0$.

Note Condition 3 does not imply welfare is always increasing in unexpected inflation—that would rule out any Markov equilibrium! Condition 3 applies for all $\pi \geq \beta$, even asymptotically in the sense that $\lim_{\pi\to\infty} v^2(\pi, 0) \neq 0$. The latter point is important in order to rule out economies where unbounded inflation expectations are validated only on the limit. I will discuss this further when presenting some examples based on Barro and Gordon (1983). Later in this section I show that is also possible to relax Condition 3 under an alternative set of assumptions by being explicit about the possibility that the economy demonetizes.

Condition 3 is closely related to the robustness of the time inconsistency problem, yet it does not imply that optimal monetary policy is time inconsistent. Note that if $\pi^r = \beta$, then (4) and (5) can be satisfied even if $v^2(\pi^r, 0) > 0$.

All is set to rule out economies with an unique Markov equilibrium. Note that $v \in C^2$ or conditions 1-3 do not imply equilibrium existence. It is not too difficult to come up with sufficient conditions for the existence of a Markov equilibrium, but they are not relevant to the multiplicity result.

Theorem 5 *Let conditions 1-3 be satisfied. Then, generically, if a Markov equilibrium exists, it is not unique.*

Proof. The proof proceeds as follows. First I provide a sufficient condition, evaluated at a given inflation π, for the existence of a Markov equilibrium with higher inflation than π. Then I show that the condition is satisfied in the neighborhood of a Markov equilibrium.

Lemma 6 *Let conditions 1-3 be satisfied. If $v^1(\pi, 0) + v^2(\pi, 0) < 0$ for any $\pi \geq \beta$, then there exists a Markov equilibrium $\tilde{\pi} > \pi$.*

Proof. Assume first $v^1(\pi', 0) + v_2(\pi', 0) < 0$ for all $\pi' > \pi$. Then

$$v^1(\pi', 0) \leq -C < 0 \tag{6}$$

by condition 3 for all $\pi' \geq \pi$. Use the Taylor theorem and condition 1 to show

$$v(\pi, 0) + v^1(\hat{\pi}, 0)(\pi' - \pi) \geq B$$

for some $\hat{\pi} \in (\pi, \pi')$, for all $\pi' > \pi$. Combined with (6),

$$v(\pi, 0) - (\pi' - \pi) C \geq v(\pi, 0) + v^1(\hat{\pi}, 0)(\pi' - \pi) \geq B$$

which is a contradiction as $C > 0$.

Hence, for some $\pi' > \pi$, $v^1(\pi', 0) + v^2(\pi', 0) \geq 0$. Then by $v \in C^2$, there exists $\tilde{\pi} \in [\pi, \pi']$ such that $v^1(\tilde{\pi}, 0) + v^2(\tilde{\pi}, 0) = 0$. Condition 2 implies that $\tilde{\pi}$ is a Markov equilibrium, as π^e always belong to a neighborhood of π^e and $\tilde{\pi}$ is a critical point of $v(\pi, \pi - \tilde{\pi})$ ■

Let π^m be the Markov equilibrium whose existence is a premise of the theorem.

Consider first the case $\pi^m = \beta$. The necessary first condition (3) associated with the Markov definition implies that

$$v^1(\beta, 0) + v^2(\beta, 0) \leq 0.$$

Note that the strict equality is non-generic: it imposes a point restriction upon economy primitives. Hence, generically, Lemma 6 says there exists an additional Markov equilibrium $\tilde{\pi} \in (\beta, \pi]$.

Now consider the case $\pi^m > \beta$. For clarity, let $h(\pi) \equiv v^1(\pi, 0) + v^2(\pi, 0)$. By $v \in C^2$, $h : [\beta, \infty) \to \Re$ is a differentiable function. The necessary first order condition (3) implies

$$h(\pi^m) = 0.$$

If

$$\frac{d}{d\pi}[h(\pi^m)] < 0,$$

then for some $\pi > \pi^m$, $h(\pi) < 0$ or $v^1(\pi, 0) + v^2(\pi, 0) < 0$. Then Lemma 6 applies as and there exists an additional Markov equilibria $\tilde{\pi} > \pi^m$.

If

$$\frac{d}{d\pi}h(\pi^m) > 0,$$

then for some $\pi < \pi^m$, $h(\pi) < 0$. Consider first $h(\beta) \leq 0$. Then $\pi = \beta$ is a Markov equilibrium since $v^1(\beta, 0)+v^2(\beta, 0) \leq 0$, so the lower bound β is a critical point of $v(\pi, \pi - \beta)$ and Condition 2 applies.

If $h(\beta) > 0$, then $h(\pi) < 0$ with $\pi < \pi^m$ implies that for some $\tilde{\pi} \in [\beta, \pi^m)$, $h(\tilde{\pi}) = 0$. Hence $v^1(\tilde{\pi}, 0) + v^2(\tilde{\pi}, 0) = 0$, so $\pi = \tilde{\pi}$ is a Markov equilibrium using again condition 2.

Finally, case $\frac{d}{d\pi}(v^1(\pi^m, 0) + v^2(\pi^m, 0)) = 0$ is non-generic ■

The intuition for linking bounded welfare costs of inflation and expectation traps can be seen with some visual aid. Figure 2 contains the standard displays to characterize Markov equilibria. Private sector expectations π^e are on the horizontal axis, and the solid line depicts the policy best response, $\pi^*(\pi^e) \in \arg\max_\pi v(\pi, \pi - \pi^e)$. Any crossing with the 45-degree line is a Markov equilibrium as $\pi^*(\pi^m) = \pi^m$. Figure 2 displays the policy response in Barro and Gordon (1983), featuring unique equilibrium.

As evident in Figure 2, the key for uniqueness is that the monetary authority undershoots inflation expectations as they grow large, $\pi^*(\pi^e) < \pi^e$ as $\pi^e \to \infty$.[6] While this may seem intuitive, it is at odds with bounded welfare costs of inflation.

Along the rational expectation locus—the 45-degree line featuring $\pi^e = \pi$—the unexpected component of inflation is kept at 0. Hence, the monetary authority can reap the marginal welfare gains from a small $\varepsilon > 0$ unexpected component for any inflation level. On the other hand, the marginal cost of inflation must fall with the level of inflation if the welfare costs of inflation are to be bounded.

[6] Note that if the best response function cuts only once the 45-degree line with a slope greater than one, then the lower bound β would become a Markov equilibrium as well.

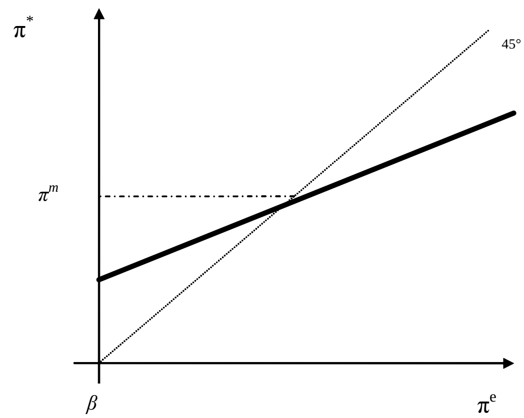

Figure 2: Best Response Policy Function: Unique Equilibrium Case

Hence, as the inflation expectation grow large $\pi^e \to \infty$, the best response must eventually be overshooting expectations $\pi^* (\pi^e) > \pi^e$ as the marginal cost of inflation falls below the gains from the unexpected component. This is the precise sense in which the best policy response function in Figure 2 is not compatible with bounded welfare costs of inflation.

Just to be clear, Conditions $1 - 3$ are sufficient, but not necessary. It is not clear that there any meaningful necessary conditions for multiplicity. Obviously the model must have some degree of non-linearity. As discussed early, it is not even necessary that the optimal monetary policy is time inconsistent for expectation traps to arise.

3.1 A Short Discussion on Stability

A legitimate concern is the robustness of expectations traps under learning. Maybe there is a unique stable Markov equilibrium. Indeed, Figure 3 seems to suggest that this is the case. It plots the best response function of the monetary authority, $\pi^* (\pi^e)$—Markov equilibria π^a and π^b are the fixed points of this function.

It is clear that in Figure 3 the slope of the best response function will be greater than unity in the high inflation equilibrium π^b. This suggests that only the low inflation π^a is stable under learning. The situation is reminiscent of the stability analysis of the fiscal

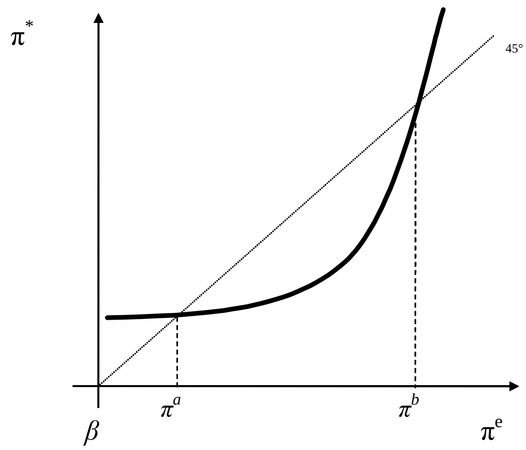

Figure 3: An Economy with Two Markov Equilibria

theory of inflation.[7]

However, the best policy response function depicted $\pi^*(\pi^e)$ is not the relevant object to evaluate the stability of equilibria. Recall that the monetary authority reacts to private sector actions $\mathbf{z}$ which arise from expectations through (1). Agent i learning rule would be naturally in terms of the rest of actions of the private sector z_{-i} and the actual inflation rate π. It is certainly not standard to state a learning rule in terms of the expectations of other agents. The best policy response as a function of $\mathbf{z}$ can have a slope less than unity in every Markov equilibrium. Hence, the high inflation would also be stable under learning.

In short, expectation traps can be stable under learning. Indeed, learning holds great promise for the theory. It can sharpen the predictions of the theory for inflation dynamics, explaining both the high turbulence period of the 70's and the very successful conduct of monetary policy in the 90's.

[7] See Marcet and Sargent (1989), who concluded that high inflation equilibria were not stable under least-squares learning. However, a recent paper by Marcet and Nicolini (2003) shows that boundedly rational learning does not rule out recurrent deviations from a stable stationary equilibrium.

3.2 An Alternative Set of Conditions: Demonetization

It should be emphasized that Conditions 1-3 are by no means necessary. Here I will discuss an alternative set of conditions to allow for "demonetization.". This captures the possibility that, in face of high inflation, the economy completely insulates itself from any monetary phenomena. Demonetization provides a lower bound for the welfare costs of inflation but it also may leave the unexpected component of inflation without any real impact. The multiplicity results stands, as demonetization naturally becomes a Markov equilibrium.

An excellent illustration is featured in Siu (2004). Firms can pay a up-front fix cost that allows them to post prices in real terms. For a high enough inflation expectation, all firms become flexible price setters, money becomes totally neutral and inflation is irrelevant, including the unexpected component. However, the high inflation expectation is trivially validated.

I formalize the demonetization possibility by modifying Conditions 1 and 3. In addition, it is required that there exists a Markov equilibrium strictly below the satiation point. Otherwise the model would be essentially real economy. In other words, at least one equilibrium should exist where monetary policy can have real implications.

Proposition 7 *Let Condition 2 hold. Assume that the following is true:*

1. *There exists $\bar{\pi}$ such that for all $\pi \geq \beta$, $v(\pi, 0) \geq v(\bar{\pi}, 0)$, with strict equality if $\pi \geq \bar{\pi}$.*

2. *For all $\pi \geq \beta$, $v^2(\pi, 0) \geq 0$.*

Then, generically, if there exists a Markov equilibrium $\pi^m < \bar{\pi}$, then it is not unique.

Proof. The proof repeats the steps in Theorem 5. Note that Lemma 6 also holds. For any $\pi \geq \beta$, there exists $\pi' \geq \bar{\pi}$ such that $v^1(\pi', 0) + v^2(\pi', 0) \geq 0$ since for all $\pi' \geq \bar{\pi}$, $v^1(\pi', 0) = 0$ as $v(\pi', 0) = v(\bar{\pi}, 0)$.

To complete the proof, use the fact that π^m belongs to $[\beta, \bar{\pi})$ ■

It may seem that the Proposition imposes some unnecessary structure by assuming demonetization. Can it be substituted by Condition 1? The answer is no. It is possible to find economies with a unique Markov equilibrium satisfying Conditions 1, 2 and $v^2(\pi, 0) \geq 0$ for all $\pi \geq \beta$. Yet I will argue that these economies share a very special property: unbounded inflation expectations get validated on the limit. One can make a case for hyperinflation as an equilibrium in this economies.

Formally, consider an economy with a unique equilibrium satisfying Conditions 1, 2 and $v^2(\pi, 0) \geq 0$ for all $\pi \geq \beta$. It can be shown that for any unbounded increasing sequence

$\{\pi_i^e\}_{i=1}^{\infty}$, the welfare gains of deviating from inflation expectations goes to 0, i.e.

$$\left\{\max_{\pi\geq\beta} v\left(\pi,\pi-\pi_i^e\right)-v\left(\pi_i^e,0\right)\right\}_{i=1}^{\infty}\rightarrow 0.$$

It is not straightforward how to define a hyperinflation as a Markov equilibrium. One can consider an alternative equilibrium definition as follows. Let $\hat{\pi}$ be a ε-Markov equilibrium if $\max_{\pi\geq\beta} v\left(\pi,\hat{\pi}-\pi\right)-v\left(\hat{\pi},0\right)<\varepsilon$. This economy has the property that the set of ε-Markov equilibria is unbounded for any $\varepsilon>0$. I.e., for any ε, there exists $\bar{\pi}$ such that any $\pi\geq\bar{\pi}$ is ε-Markov equilibrium.

4 The Sign Switch Phenomenon

I extend the previous notation to make explicit the economy's dependence on some of the fundamentals. These can be thought as parametric choices or random variables whose realization is common knowledge before agents take their actions $\mathbf{z}$.

Let $\xi\in\Xi$ be a fundamental of the economy, where Ξ is a convex set. It is not required that ξ is an exhaustive characterization of the economy and explicit dependence on other parameters/variables is omitted for notational convenience. Indeed, ξ may be—but it does not need to be—a combination of parameters, as long as Ξ is a convex subset of the overall parameter space.[8]

I assume that fundamental ξ enters the welfare function $v\left(\pi,\pi-\pi^e;\xi\right)$ nicely, it is C^2 with respect to $\xi\in\Xi$ as well. A Markov equilibrium $\pi^m\left(\xi\right)$ can be defined for each value of $\xi\in\Xi$. It is further assumed that the economy has only two Markov equilibria.

The following Proposition shows that the qualitative response of equilibrium inflation to perturbations of ξ are different in each equilibrium as long as the perturbations satisfy a simple condition. The possibility that simple comparative statics deliver opposite signs in each equilibrium is labelled the sign switch phenomenon.

Proposition 8 *Let conditions 1-3 be satisfied. Assume that for all $\xi\in\Xi$,*

$$\left[\frac{\partial^2}{\partial\xi\partial\pi}v\left(\pi,\pi-\hat{\pi};\xi\right)\right]_{\pi=\hat{\pi}}\neq 0$$

for all $\hat{\pi}\geq\beta$; and assume that for each $\xi\in\Xi$ there exist just two distinct equilibria $\pi_1^m\left(\xi\right)$ and $\pi_2^m\left(\xi\right)$.

[8] For example, let $\alpha\in\Re^n$ be a vector including all relevant fundamentals of the economy. One can define $\xi=B\alpha\in\Re^m$, $m\leq n$, and $\Xi=\{\xi|\exists\alpha,\xi=A\alpha\}$. In other words, any linear combination of fundamentals can be though as a fundamental by itself. This is important for some of the empirical results presented later.

Then, generically, for any $\xi_1 \neq \xi_2$, $(\xi_1, \xi_2) \in \Xi$,

$$sign\left(\pi_1^m(\xi_1) - \pi_1^m(\xi_2)\right) \neq sign\left(\pi_2^m(\xi_1) - \pi_2^m(\xi_2)\right).$$

Proof. The necessity of the first order condition associated with $\max_{\pi \geq \beta} v\left(\pi, \pi - \pi_j^m(\xi); \xi\right)$ implies that for $j = 1, 2$ and for all $\xi \in \Xi$,

$$v^1\left(\pi_j^m(\xi), 0; \xi\right) + v^2\left(\pi_j^m(\xi), 0; \xi\right) \leq 0 \tag{7}$$

with strict equality if $\pi_j^m(\xi) > \beta$. Without loss of generality, given that $v \in C^2$, let $\left[\frac{\partial^2}{\partial\xi\partial\pi} v\left(\pi, \pi - \hat{\pi}; \xi\right)\right]_{\pi=\hat{\pi}} > 0$ for all $\xi \in \Xi$ and $\hat{\pi} \geq \beta$.

For a given $\xi \in \Xi$, let $\pi_1^m(\xi) < \pi_2^m(\xi)$. If $\beta < \pi_1^m(\xi)$, then, it holds that

$$\frac{\partial}{\partial\pi}\left[v^1\left(\pi_1^m(\xi), 0; \xi\right) + v^2\left(\pi_1^m(\xi), 0; \xi\right)\right] \geq 0 \tag{8}$$

and

$$\frac{\partial}{\partial\pi}\left[v^1\left(\pi_2^m(\xi), 0; \xi\right) + v^2\left(\pi_2^m(\xi), 0; \xi\right)\right] \leq 0. \tag{9}$$

If (8) is not satisfied, then there exists an additional equilibrium in $[\beta, \pi_1^m(\xi))$. Note $v \in C^2$ and (8) would imply that for some $\pi < \pi_1^m(\xi)$, $v^1(\pi, 0; \xi) + v^2(\pi, 0; \xi) < 0$, hence there exists a $\tilde{\pi} \in [\beta, \pi_1^m(\xi))$ such that (7) is satisfied. Condition 2 closes the argument by establishing the sufficiency of (7) for Markov equilibria. See the proof of Theorem 5 for more detailed step by step of the same argument.

If (9) is not satisfied, then there exists an additional equilibrium strictly above $\pi_2^m(\xi)$. The argument again invokes $v \in C^2$, then for some $\pi > \pi_1^m(\xi)$, $v^1(\pi, 0; \xi) + v^2(\pi, 0; \xi) < 0$, and Lemma 6 applies.

Next step is a straightforward application of the implicit function theorem. Take a neighborhood of ξ. Consider first $\beta < \pi_1^m(\xi)$. Generically,

$$\frac{\partial}{\partial\xi}\pi_j^m(\xi) = \frac{-\frac{\partial}{\partial\xi}\left[v^1\left(\pi_j^m(\xi), 0; \xi\right) + v^2\left(\pi_j^m(\xi), 0; \xi\right)\right]}{\frac{\partial}{\partial\pi}\left[v^1\left(\pi_1^m(\xi), 0; \xi\right) + v^2\left(\pi_1^m(\xi), 0; \xi\right)\right]}.$$

The implicit function theorem can be applied generically as

$$\frac{\partial}{\partial\pi^m}\left[v^1\left(\pi_1^m(\xi), 0; \xi\right) + v^2\left(\pi_1^m(\xi), 0; \xi\right)\right] = 0$$

can be dismissed as a non generic pointwise restriction on parameters. The premises on ξ imply that the sign of the numerator is invariant and (8) and (9) sign the denominator differently for $\pi_1^m(\xi)$ and $\pi_2^m(\xi)$.

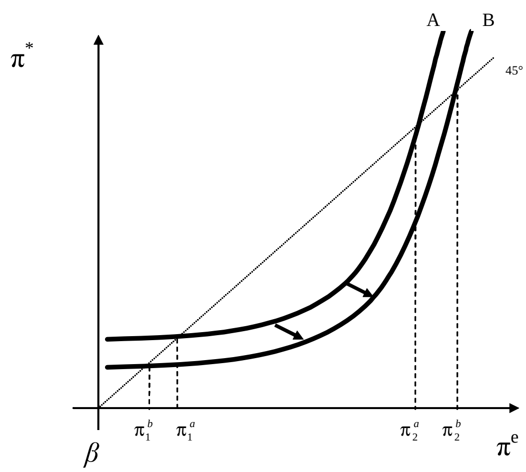

Figure 4: Illustrating the Sign Switch Phenomenon

To complete the proof remains only case $\pi_1^m(\xi) = \beta$. If ,

$$v^1(\beta, 0; \xi) + v^2(\beta, 0; \xi) < 0$$

then for ξ' in a small neighborhood of ξ, $\pi_1^m(\xi) = \pi_1^m(\xi') = \beta$. The previous implicit function result, combined with the assumption on ξ, implies that $\frac{\partial}{\partial \xi}\pi_2^m(\xi) \neq 0$. By continuity, the result extends to the whole set Ξ ■

The logic behind Proposition 8 is illustrated in Figure 4. Consider a perturbation from ξ^A to ξ^B which displaces the best response function to the right as displayed. It is easy to see that the low inflation equilibrium falls $\pi_1^b < \pi_1^a$ while the high inflation equilibrium increases $\pi_2^a < \pi_2^b$.

It is important to emphasize that the additional condition in Proposition 8 is not demanding. In particular, Proposition 8 does not assume any high degree of nonlinearity with respect to the perturbations ξ. For example, the indirect welfare function may be separable in the two arguments and ξ be the weight on the second term,

$$v(\pi, \pi - \pi^e; \xi) = C(\pi) + \xi G(\pi - \pi^e).$$

This is a simple and intuitive functional form, which conforms to the condition in Proposition 8.

5 Two Illustrations

In this section I briefly discuss two prominent examples of multiplicity in the literature on full monetary policy discretion, Albanesi et al. (2003) and King and Wolman (2004). In particular, I show that the welfare costs of inflation are bounded in both environments. The logic is, indeed, quite similar: as inflation is arbitrarily large, some firms' production goes to zero — cash good firms in Albanesi et al. (2003) or firms at the start of the staggered price period in King and Wolman (2004). However, positive production is still sustained by some firms which can insulate against inflation — either credit good firms or firms at the end of the staggered price period.

I also show how unanticipated inflation stimulates employment even if inflation is very large — so Condition 3 is satisfied. The intuition is again quite simple. Sticky price firms protect themselves from anticipated inflation by adjusting to expected inflation. Thus some unanticipated inflation always cuts the markup of a subset of firms and increases employment. Unfortunately, I cannot show that all sufficient conditions are satisfied: it is actually quite difficult to show that Condition 2 holds analytically. Both Albanesi et al. (2003) and King and Wolman (2004) report numerical checks on the sufficiency of their equilibrium conditions.

5.1 A Cash/Credit Goods Model

There is a single period in this economy but some decisions are made sequentially. First, a subset of firms set a nominal price — the sticky price firms. Second, the monetary authority gets to set the inflation rate. Finally, all remaining household and firm decisions are made.

Households. The representative household consumes a continuum of differentiated goods and supplies labor. The household preferences are given by $u(c, n)$ where aggregate consumption c combines the differentiated goods according to

$$c = \left[\int_0^1 c(\omega)^\eta \, d\omega\right]^{1/\eta}$$

with $0 < \eta < 1$; and n is the labor supplied. Preferences u satisfy the standard conditions as well as the Inada condition $\lim_{c \to 0} u(c, n) = -\infty$. Hence welfare can be unbounded low.

Each good in this continuum is one of three types. Half of the goods are cash goods, denoted by $c_1(\omega)$. An unspecified payment technology leaves only a fraction $1 - \varphi(\pi)$ of produced cash goods available for consumption, where π is the inflation rate and $\varphi(\pi)$ is a strictly increasing function, with $\varphi(\beta) = 0$ and $\lim_{\pi \to \infty} \varphi(\pi) = 1$.[9] Let $y_1(\omega)$ be the

[9] Given the choice of preferences, it does not matter whether the output loss is incurred at the level of the households (as in Albanesi et al. (2003)) or at the level of production (as in Armenter and Bodenstein (forthcoming)).

production of a differentiated cash good. Aggregate cash-good consumption is given by

$$c_1 = (1 - \varphi(\pi)) \left[\int_0^{1/2} y_1(\omega)^\eta \, d\omega \right]^{1/\eta}.$$

Remaining goods are credit goods, denoted by $c_2(\omega)$. Some credit goods are produced by sticky price firms and some by flexible price firms. Sticky price firms must set their nominal price before the monetary authority's decision. Aggregate credit good consumption is given by

$$c_2 = \left[\int_0^{1/4} \left[y_2^f(\omega) \right]^\eta d\omega + \int_0^{1/4} \left[y_2^s(\omega) \right]^\eta d\omega \right]^{1/\eta}$$

where $y_2^s(\omega)$ and $y_2^f(\omega)$ are the production of sticky-price and flexible-price credit goods respectively. I have assumed there is an equal measure of each. All cash goods are produced by flexible price firms.

Firms. Each firm is the sole producer of one differentiated good, using an identical linear production technology, $y(\omega) = n(\omega)$. Because the household demand curves have a constant elasticity, all firms set prices as a fixed markup, $1/\eta$, over marginal cost. In terms of consumption, the nominal price of a cash good is

$$P_1 = (1 - \varphi(\pi)) \frac{1}{\eta} W$$

where W is the nominal wage rate. I have dropped the index ω: all firms within a type are assumed to be symmetric.

The optimal price of a credit good produced by a flexible price firm is

$$P_2^f = \frac{1}{\eta} W.$$

Finally, sticky price firms have to set the price without knowing the actual inflation rate. Their nominal price then reflects their expectation on the nominal wage W^e,

$$P_2^s = \frac{1}{\eta} W^e.$$

Inflation and relative prices. For simplicity, the monetary authority sets the general price level, or equivalently, the inflation rate.[10] I write the equilibrium price conditions as

$$\begin{aligned} p_1 &= (1-\varphi(\pi))\frac{1}{\eta}w, \\ p_2^f &= \frac{1}{\eta}w, \\ p_2^s &= \frac{1}{\eta}\frac{w^e\pi^e}{\pi}, \end{aligned}$$

where lower case letters denote real prices. The real price of sticky price firms requires further discussion. The term $w^e\pi^e$ is the expected nominal wage. The real price, though, is deflated by the actual inflation rate π. Unless $\pi \neq \pi^e$, sticky price firms will not have their desired real price.

The characterization of a competitive equilibrium would be closed by the market clearing conditions as well as the demand of each good type.

Welfare costs of inflation are bounded. I show here that Condition 1 holds. Whenever inflation expectations are confirmed, $\pi = \pi^e$, sticky and flexible price firms are identical and the only relative price difference is between cash and credit goods,

$$\frac{p_1}{p_2^f} = 1 - \varphi(\pi).$$

Using the demand for both goods we can solve for the wage rate

$$w = \eta\left[.5 + .5\left(1-\varphi(\pi)\right)^{\frac{\eta}{1-\eta}}\right]^{\frac{1-\eta}{\eta}} \tag{10}$$

for any value of $\pi \geq \beta$.

There are a couple of welfare properties of inflation which are immediate from (10). First, along the locus $\pi = \pi^e$ the wage rate is always below the first best, i.e., even if $\pi = 0$, $w = \eta < 1$. In other words, employment is inefficiently low in any rational expectations equilibrium. Second, anticipated inflation is no good: the wage rate is decreasing in π, and since the wage rate was already too low, welfare is decreasing with inflation as well. The Ramsey equilibrium is thus to set inflation at its lower bound $\pi = \beta$.

Despite welfare being strictly decreasing in inflation, the welfare costs of inflation are bounded. As inflation grows, cash goods vanish from the economy, i.e., $c_1 \to 0$. However, the credit good sector sustains a positive wage rate in the limit

$$\lim_{\pi\to\infty} w = \eta 2^{\frac{\eta-1}{\eta}}.$$

[10] Albanesi, Chari and Christiano (2002) and Armenter and Bodenstein (forthcoming) show the equivalence with an specification in which the monetary authority chooses the nominal interest rate or the money growth rate.

It follows that there is a positive amount of labor being supplied and aggregate consumption is bounded away from zero — indeed, it is just equal to the production of credit goods. Hence both leisure and consumption remains bounded away from zero and welfare is bounded from below. Condition 1 naturally holds.

Unexpected inflation raises employment. In this economy, the monetary authority can stimulate employment by setting inflation higher than expected and cutting the markup the sticky price firms effectively charge. To evaluate this claim, I have to consider the wage rate given any pair $\{\pi, \pi^e - \pi\}$.

It turns out it is useful to express the difference between expected and actual inflation as a ratio, $q = \pi/\pi^e$. By combining the relative prices of each good type, the wage rate is given by

$$w = \eta \left[\frac{.5 + (1 - \varphi(\pi))^{\frac{\eta}{1-\eta}}}{2 - .5\,(q/w^e)^{\frac{\eta}{1-\eta}}} \right]^{\frac{1-\eta}{\eta}}$$

for any pair $\{\pi, q\}$ and expectation w^e. The wage rate is always increasing in q: a small unexpected component of inflation raises the wage rate and stimulates employment. Because the wage rate is inefficiently low, this is welfare increasing. Thus Condition 3 holds.

Note that the wage is increasing in a neighborhood of $q = 1$ *for any inflation level* π. Even if anticipated inflation is arbitrarily large, sticky prices still charge a markup over marginal cost: some further unexpected inflation will raise employment.

5.2 Staggered Pricing

King and Wolman (2004) have shown that staggered pricing can lead to expectation traps. Here I argue that the cost of inflation are naturally bounded under staggered pricing. Hence the economy in King and Wolman (2004) satisfies the sufficient conditions introduced here. Indeed, the logic of the argument is very similar to the previous model: at worst, high inflation simply shuts down a fraction of the firms with staggered prices.

To see this, consider a sector composed by a continuum of differentiated goods which combine into consumption according to

$$y_t = \left[\int y_t(\omega)^\eta \, d\omega \right]^{1/\eta}.$$

As before, each firm produces one of these differentiated goods according to a linear technology $y(\omega) = n(\omega)$. All firms have to set the same nominal price for two periods: half of them do it in even periods, the other half in odd periods. For simplicity, I assume firms do not discount future periods and inflation is constant at rate π.[11]

A firm that sets the price at date t will choose a nominal price such that the real price at date t is

$$p_t^t = \frac{1}{\eta} \frac{y_t w_t + y_{t+1} w_{t+1} \pi^{\frac{1}{1-\eta}}}{y_t + y_{t+1} \pi^{\frac{\eta}{1-\eta}}}.$$

At date $t+1$ the same nominal price would remain: the real price is thus discounted by π,

$$p_{t+1}^t = \pi^{-1} p_t^t.$$

I now bring inflation to be arbitrarily high in all periods, $\pi \to \infty$. Let me take aggregate demand and the wage rate as exogenously constant for now. Simple algebra shows that a staggered price firm will set an arbitrarily large price in the first period,

$$\lim_{\pi \to \infty} p_t^t = \infty,$$

i.e., it is essentially shutting down for the first period as the demand for its good will approximate zero. In contrast, the limit for the real price in the second period is well defined,

$$\begin{aligned} \lim_{\pi \to \infty} p_{t+1}^t &= \lim_{\pi \to \infty} \left(\pi^{-1} p_t^t \right) \\ &= \frac{1}{\eta} w. \end{aligned}$$

Thus in each period half of the staggered price firms have the same price as a flexible price firm – the other half has a price so high that there is zero demand for their goods. Thus

$$y_t = .5^{1/\eta} y_t^{t-1}.$$

Using the demand for goods of firms which set the price at date $t-1$,

$$w = 5^{1/\eta} \eta.$$

Hence, even if $\pi \to \infty$, the wage rate and production are positive. The welfare costs of inflation are then bounded.

[11] Since I only discuss the condition that welfare costs of inflation are bounded, I look only at the rational expectations locus $\pi = \pi^e$.

6 Some Evidence

I complement the theoretical results of this paper with a primer on empirical evidence supporting the hypothesis that the U.S. monetary policy was in an expectation trap in the 70s. Table 1 summarizes the basic descriptive statistics for different inflation rate measures. It is clear that the U.S. inflation experience was significantly different in the 70s.

I focus in two well documented observations: the large persistence of inflation and its qualitatively different dynamics before and after the end of the 70s. Multiplicity of equilibria provides an explanation for the former as shifts in private sector expectations can be non-stationary. But the theory also imposes a restriction. The local uniqueness of equilibria implies that the non-stationary component of inflation must arise from shifts in the inflation mean. This is exactly the main finding in the recent literature on inflation mean breaks.

The instability of the estimates of the response of inflation, I will argue, is evidence of the sign-switch phenomenon. As discussed in Section 4, the inflation response can be qualitatively different in each equilibrium. Hence, it is not enough that estimates differ: they must switch their sign. Again, this is what I find using the Generalized Method of Moments to estimate the inflation response to shocks in the output gap.

These two features are distinctive of the theory. A unique equilibrium economy will be hard-pressed to match both. A non-stationary inflation rate is at odds with most of the monetary models used in the literature, as discussed in Clarida, Gali and Gertler (1999). Even assuming a large, persistent shock driving the inflation, it would require steep nonlinearities to match the sign switch phenomenon.[12] Of course, it is easy to come up with arbitrary changes in the monetary authority's objective function which reproduce both features.

The bottom line of this section is that expectation traps constitute a positive theory of regime changes in monetary policy. As a result, expectation traps can reap several of the findings of a growing literature which claims that regime changes are essential to explain monetary policy in the U.S.[13]

6.1 Inflation Rate Persistence

It is well known that the inflation rate displays a large amount of persistence. Indeed, the unit root hypothesis is often hard to reject in the data. Expectation traps provide a theoretical basis for large inflation persistence. Under the expectation trap hypothesis, beliefs govern inflation dynamics as the economy coordinates either in the low or the high inflation equilibrium. There is no reason for beliefs to follow a stationary process.

[12] Learning or incomplete information are alternatives, too. In both cases, though, the sign-switch phenomenon does not arise naturally.

[13] See Sims and Zha (2004) and Owyang and Ramey (2004) and many others.

	GDP Deflator		**PPI**		**PCE**		**CPI**	
Sample	*Mean*	*Std.Desv*	*Mean*	*Std.Desv*	*Mean*	*Std.Desv*	*Mean*	*Std.Desv*
1948:1-2004:3	3.41	0.024	3	0.037	3.38	0.024	3.75	0.029
1948:1-1970:1	2.41	0.019	1.62	0.029	2.19	0.018	2.3	0.023
1970:2-1982:2	6.82	0.018	7.74	0.039	6.77	0.022	7.59	0.03
1982:3-2004:3	2.55	0.009	1.81	0.018	2.73	0.011	3.09	0.011

Table 1: **Descriptive Statistics for Inflation Across Different Samples**

On the other hand, the theory does impose structure on the type of persistence to be observed. Since each Markov equilibrium is locally unique, expectation shifts should induce breaks in the inflation mean. These breaks should be the only nonstationary component of inflation. In other words, the "inflation regime switching process" should be the main source of persistence; controlling by the regime, the unit root hypothesis should be safely rejected.

A recent line of research has pointed out that ignoring breaks in the inflation mean induces a upward bias in the persistence estimates of inflation. Corvoisier and Mojon (2004) uses the Altissimo-Corradi test of multiple breaks for several OECD countries and Benati (2004) does a similar exercise using the Bai-Perron test instead.[14] Evidence of a unit root becomes tenuous at best once inflation mean breaks are allowed. Corvoisier and Mojon (2004) states that "the roots of autoregressive models of inflation are actually far below unity." Benati (2004) concludes that "conditional on estimated breaks, inflation shows little persistence" and "the null of the unit root can be strongly rejected for a vast majority of series." These results have not been without criticism. As the authors acknowledge, it is difficult to distinguish between breaks in the intercept or in the coefficient of an autoregressive process—specially when it is close to nonstationary. Pivetta and Reis (2004), for example, take the opposite view and conclude that inflation has indeed an unit root.

Figure 5 shows the breaks reported in Corvoisier and Mojon (2004) for the GDP Deflator growth rate. It is not clear whether the first (67:3) and the second (73:1) break correspond to different expectation shifts. I will treat them as lower and upper bound estimate for the start of the high inflation equilibrium regime, ending in 82:2. Results in Benati (2004), using the Bai-Perron test instead, do not differ substantially.

It is noteworthy how changes in the Fed chairmanship coincide with the estimated inflation mean breaks. Figure 6 again displays the growth rate of the GDP Deflator. Some key events in the Federal Reserve System are indicated with a vertical line. Most of the high inflation episode in the U.S. happened mainly under the tenure of Arthur Burns and G.W.

[14] See Bai and Perron (1998) and Altissimo and Corradi (2003). These tests often find weak evidence of further breaks. See Levin and Piger (2004).

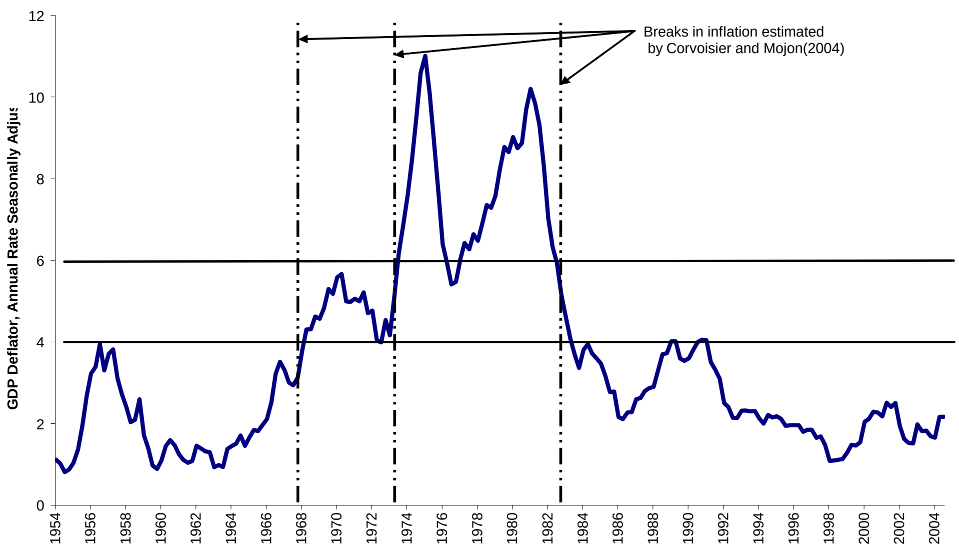

Figure 5: **U.S. Inflation over 1954-2004** Vertical lines indicate breaks in the inflation mean as reported in Corvoisier and Mojon (2004).

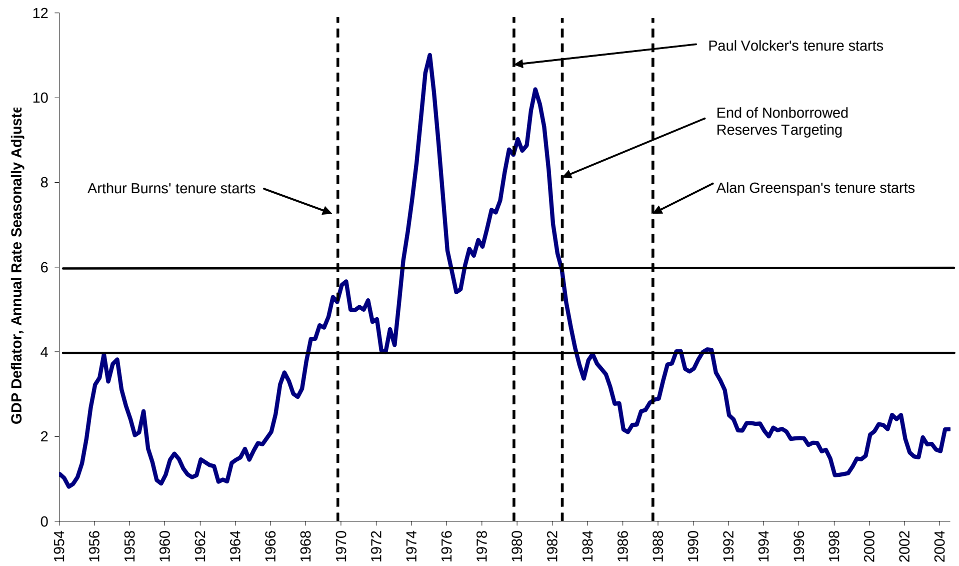

Figure 6: **U.S. Inflation over 1954-2004** Vertical lines indicate several events in the Federal Reserve System

Miller as well as the first years of Paul Volcker's tenure. In particular, the start of Paul Volcker's tenure and the end of nonborrowed reserves targeting have been used in Clarida et al. (2000), Rudebusch and Svensson (1999) and elsewhere as indicators of monetary policy regime changes. It must be noted, though, that the unit root in inflation is not always rejected when chairmanship's changes are used as subsample breaks.

6.2 Inflation Response

Using the estimated breaks in the inflation mean from Corvoisier and Mojon (2004), I proceed to test the sign switch phenomenon. To estimate an inflation rate policy equation, I proceed in the spirit of the nominal interest rate policy rule estimation in Clarida et al. (2000).

I assume the Fed targets inflation with one period lag and incorporates feedback from the real interest rate r_t and the output gap y_t:

$$\pi^*_{t+1} = \pi^*_i + (r_t - r^*_i)\,\psi_i + E\left\{y_{t+1}|\Omega_t\right\}\eta_i \tag{11}$$

where $i = 1, 2$ denote the monetary policy regime and Ω_t is the set of information available to the monetary authority at date t. It is important to emphasize that I do not pursue the best policy response function, an out-of-the-equilibrium object. In the notation of Section 2, I seek an estimate of $\pi^m(\xi)$ and not of $\pi^*(\pi^e;\xi)$

Actual inflation π_t behaves according to

$$\pi_t = \rho_i(L)\,\pi_{t-1} + (1-\rho_i)\,\pi_t^* + \varepsilon_t \tag{12}$$

where ε_t is distributed i.i.d. Hence I allow for a certain degree of inflation inertia as well as some implementation noise. Let $\rho_i = \rho_i(1)$.

Finally, the nominal and the real interest rate hold

$$R_t = r_t + E\{\pi_{t+1}|\Omega_t\}. \tag{13}$$

Combining (11), (12) and (13)

$$\pi_t = \rho_i(L)\,\pi_{t-1} + (1-\rho_i)\left(\pi^* + (R_{t-1} - \pi_t - r^*)\,\psi_i + y_t\eta_i\right) + w_t$$

where

$$w_t = \varepsilon_t + (1-\rho_i)\left(\eta_i\left(E\{y_t|\Omega_{t-1}\} - y_t\right) - \psi_i\left(E\{\pi_t|\Omega_{t-1}\} - \pi_t\right)\right).$$

I will estimate the coefficients of the inflation rate policy rule (11) in each regime $i = 1, 2$ using the orthogonality conditions

$$E_t\{w_t z_t|\Omega_{t-1}\}$$

for instruments $z_t \in \Omega_{t-1}$. I use the Generalized Method of Moments (GMM) with an optimal weighting matrix. I follow Clarida et al. (2000) and let r_i^* be equal to the average real interest rate in the period. This allows to identify π_i^*. I take the lag polynomial $\rho_i(L)$ to be of second order for both periods.

I use the annualized growth rate of the GDP implicit price deflator as preferred measure of inflation. The nominal interest rate is the Federal funds effective rate, in annualized percent. Both series are at the quarterly frequency and drawn from the Federal Reserve Board Database. The output gap is computed using the potential real GDP as estimated by the Congressional Budget Office.

Commodity price inflation, money supply growth and the long-short U.S. bond spread, as well as lags of the output gap and nominal interest, are used as instruments.[15] The Commodity Research Bureau provides a spot commodity price index. I use M2 to compute the money supply growth, given by the Federal Reserve Board. The long-short U.S. bond spread is the difference between the 10-year U.S. bond yield and the market yield on the

[15] This list of instruments very much coincides with Clarida et al. (2000).

	Parameters			
Period	π^*	ρ	ψ	η
1973:1 - 1982:2	8.20**	0.60**	0.26**	0.53**
	(0.51)	(0.09)	(0.13)	(0.25)
1982:3 - 2004:4	2.22**	0.31**	0.23**	-0.12**
	(0.14)	(0.10)	(0.07)	(0.06)

** indicates significance at a 5% level. * indicates significance at a 10% level.

The set of instruments includes four lags of inflation, the federal funds rate, the output gap, commodity price inflation, and the M2 money supply.

Table 2: **Baseline Specification**

U.S. Treasury securities at a 3-month constant maturity, quoted on investment basis. Both series are from the Federal Reserve Board.

The sample ranges from 1973:1 to 2004:4. The first quarter of 1973 is the point estimate for an inflation mean break in the 70s reported in Corvoisier and Mojon (2004). Benati (2004) and Rapach and Wohar (2005) have very similar dates. I split the sample taking the second quarter of 1982 as the end of the high inflation period. This is the point estimate in Corvoisier and Mojon (2004) but this time there is less agreement, with estimates ranging from 1979 to 1984.

Table 2 displays the estimated coefficients for both periods. The standard deviation is shown in parenthesis. For the baseline case, the instruments used are four lags of inflation, the federal funds rate, the output gap, commodity price inflation, and the M2 money supply.

All estimates are significant at the 5% level, but most importantly, the coefficient for the output gap η switches signs between periods. That is, the response of inflation to changes in the output gap was qualitatively different under low and high inflation. In the high inflation period, the estimated rule indicates that inflation largely accommodated to aggregate demand. In contrast, from 1982:3 inflation "leans against the wind," cooling down inflation when the economy is in expansion. Using Gaussian disturbances, the hypothesis that η has the same sign across sample periods is rejected at the 5% confidence level.

Note how the inflation inertia coefficient ρ is safely below the unit root in both subsamples. This is, of course, expected given that I used the estimated inflation mean breaks. But it is

Case	*Unit Root?*	*Sign Switch?*
Baseline	No	Yes
Added Instruments		
Short-Long Spread	No	Yes
Removed Instruments		
Commodity Price Inflation	No	Yes
M2 Growth Rate	No	Yes
Alternative Breaks		
Volcker's Nomination	Yes	

Unit Root: Dickey-Fuller Augmented test at 5%

Sign Switch: Test at 5% assuming Gaussian disturbances.

Table 3: **Summary Results**

worth to emphasize that inflation appears to have a unit root in different subsamples—most notably, in the pre-Volcker period (1960:1-1979:2) used in Clarida et al. (2000) and other studies.

I perform a battery of robustness exercises. Table 3 reports first whether the augmented Dickey-Fuller test rejects the unit root hypothesis and then whether the sign switch phenomenon is significant at the 5% confidence level. I find that the results are robust to different instrument specifications. However, the unit root reappears with alternative subsamples.

Table 4 performs the same exercise with the long-short spread as additional instrument. The sign switch is significant. Only the coefficient in the real interest rate loses the significance at the 5%.Further robustness checks are performed in Tables 5 and 6, where commodity price inflation and M2 money supply are excluded as instruments respectively. In both cases, the coefficient η_i loses its 5% significance level for the high inflation period, although it retains it for the low inflation period. In both cases, though, the hypothesis that $sign\left(\eta_1\right) = sign\left(\eta_2\right)$ is rejected at the 5% level.

7 Conclusions

The main point of this paper is that, under very general conditions, full policy discretion implies the existence of expectation traps. If the welfare costs of inflation are bounded, the monetary authority can be caught into validating high inflation expectations. This is not to claim that there are no reasonable monetary models featuring a single policy equilibrium. But the general perception on expectation traps should change: if anything, they are the rule rather than the exception.

Period	π *	ρ	ψ	η
1973:1 - 1982:2	8.54**	0.62**	0.18*	0.51**
	(0.49)	(0.07)	(0.11)	(0.25)
1982:3 - 2004:4	2.22**	0.30**	0.22**	-0.13**
	(0.14)	(0.10)	(0.07)	(0.06)

** indicates significance at a 5% level. * indicates significance at a 10% level. The set of instruments includes four lags of inflation, the federal funds rate, the output gap, commodity price inflation, the M2 money supply, and the short-long spread.

Table 4: **Robustness check with the short-long spread added as an instrumental variable**

Period	π *	ρ	ψ	η
1973:1 - 1982:2	7.87**	0.66**	0.28*	0.56*
	(0.68)	(0.10)	(0.16)	(0.31)
1982:3 - 2004:4	2.25**	0.35**	0.21**	-0.13**
	(0.16)	(0.10)	(0.07)	(0.06)

** indicates significance at a 5% level. * indicates significance at a 10% level. The set of instruments includes four lags of inflation, the federal funds rate, the output gap, and the M2 money supply.

Table 5: **Robustness check without commodity price inflation as an instrumental variable**

Period	π *	ρ	ψ	η
1973:1 - 1982:2	8.38**	0.64**	0.33*	0.69*
	(0.70)	(0.11)	(0.19)	(0.37)
1982:3 - 2004:4	2.22**	0.30**	0.24**	-0.14**
	(0.15)	(0.10)	(0.07)	(0.06)

** indicates significance at a 5% level. * indicates significance at a 10% level. The set of instruments includes four lags of inflation, the federal funds rate, the output gap, and commodity price inflation.

Table 6: **Robustness check without M2 money supply as an instrumental variable**

Researchers should be aware of the generality of expectation traps. For example, any welfare evaluation of the lack of commitment in monetary policy can not be restricted to the time consistency properties of the optimal policy. Even if the optimal monetary policy is time consistent, it may be just one of many equilibria.[16]

I think it is no stretch to say that policymakers are more concerned about avoiding a high inflation episode like the 70s than about shaving half a percentage point of the long run inflation rate. The theory provides a unsettling prediction: the current success against inflation may hang on something as ephemeral as the beliefs of the private sector. After all, there have been no major institutional changes since the high inflation of the 70s.

Moreover, expectation traps are a promising avenue for empirical research. Indeed, this paper can be seen as a first step towards a positive theory of regime changes in monetary policy. Accordingly, expectation traps reap several of results of the empirical literature on regime changes in monetary policy. The next step in this agenda is to elaborate on a theory of expectation shifts.

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[16] Another example is found in the exchange rate regime literature. At least since Obstfeld (1996), a perceived cost of a fixed exchange rate is the increased volatility arising from self-fulfilling equilibria. Yet, as discussed in Armenter and Bodenstein (2005), expectation traps imply that independent monetary policy is also subject to self-fulfilling equilibria.

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