

# Redistribution, Time Consistent Fiscal Policy and Representative Democracy

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## Abstract

In a two stage fiscal policy model with heterogeneous wealth holdings, I show that the policymaker's preferences over welfare distributions determine the efficiency loss due to the lack of commitment in policy decisions. For each second best policy, there are policymaker's preferences, given by a certain Pareto weight distribution, such that the policy is time consistent. The time inconsistency problem is severe when poor households are favored; a wealthy friendly policymaker may implement Pareto superior allocations.

Based on these results, I propose a politico-economic model featuring rational voters and representative democracy. Second best fiscal policy can be implemented in a Markov equilibrium if households are sufficiently patient and the expected office tenure long enough. A new redistribution-efficiency trade-off is at work: while contemporaneous distortion-free redistribution is available, it is linked to inefficient future fiscal policy by the policymaker's preferences.

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# 1 Introduction

Absent any commitment device, optimal fiscal policy cannot usually be implemented in equilibrium. This is the classic time inconsistency problem, first noted by Kydland and Prescott (1977) and further developed by Fischer (1980) in the context of fiscal policy. There is an extensive literature studying time consistent fiscal policy, focusing exclusively in the representative agent framework. See, for example, Klein and Rios-Rull (2002), Krusell, Martin and Rios-Rull (2003), Sleet and Yeltekin (2003) and Fernandez-Villaverde and Tsyvinski (2002). The early contribution of Judd (1985) is partly to blame: for a two-class economy, it is shown that the asymptotic optimal fiscal policy is characterized by a zero capital tax and no redistribution, even if the policymaker does not weigh capital owners' welfare at all. Chamley (1986) also concludes that a permanent positive tax in capital income is not an efficient policy for redistribution. Loosely speaking, the resulting common wisdom was that equity considerations play a minor role in shaping optimal fiscal policy.<sup>1</sup>

In this paper, I point out that equity considerations play a key role in the determination of time consistent fiscal policy and its efficiency properties. Hence, while the representative agent model is a good abstraction for the study of fiscal policy with commitment, it is an incomplete framework for the analysis of fiscal policy in absence of commitment.

In a simple two stage model with sufficiently heterogeneous wealth holdings and linear tax rates on capital and labor, I show that any second best (or constrained Pareto efficient) allocations can be implemented without commitment once policy decisions are evaluated according to an utilitarian social welfare function with certain Pareto weights.

The intuition is very simple. Any ex-post policy deviation reducing distortion involves a shift from labor to capital taxation. Wealth heterogeneity implies that such a shift in factor taxation has redistributive consequences as the wealthy households now bear a larger share of the taxation burden. Hence equity considerations can inhibit the incentives to revise the ex-ante optimal fiscal policy if the policymaker's preferences, indexed by a Pareto weight distribution, favor wealthy households. In the model, I show that a policymaker biased towards poor households suffers greatly from the time inconsistency problem; poor households actually prefer a policymaker who assigns a higher relative welfare weight to wealthy households.

These results introduce a politico-economic theory of fiscal policy which constitutes the second part of the paper. There are two key building blocks in the model. First, I assume rational voters in the spirit of Meltzer and Richard (1981). Second, households vote for policymakers, not policies. Hence, the political decision process is based on representative democracy rather than direct democracy. In an infinite horizon, two-class version of the previous fiscal model, I show that political institutions may implement second best fiscal policy in a Markov equilibrium without any exogenous restriction on policy instruments.

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<sup>1</sup>However, transition and policy response to shocks has been shown to depend on the relative welfare weights among groups. See Bassetto (1999).

At the core of the theory there is a new efficiency-redistribution trade-off. Elections are celebrated after the capital supply is inelastic.<sup>2</sup> The median voter, a worker household with no wealth endowment, may choose a worker-friendly policymaker who will tax heavily capital returns, reducing labor taxation and possibly implementing some redistribution. However, if the policymaker elected were to remain in office, the equilibrium fiscal policy in the subsequent periods will be inefficient, with low investment and very high labor taxes, because of the time inconsistency problem. Depending on the worker's intertemporal discount rate and the policymaker's expected office tenure, the median voter may prefer a wealthy-friendly policymaker, forgoing redistribution in order to preserve the efficiency of future fiscal policy.<sup>3</sup>

The politico-economic model is also capable of generating political business cycles driven by investment. Using the standard classification in the literature, the political cycles here are rational partisan.<sup>4</sup> The cycle is induced by the certainty of elections. I show that political instability is related both to more inefficient taxation and higher investment volatility.

## 1.1 Related Literature

This paper makes contact with several research contributions. The results in the fiscal policy model echoes the “conservative central banker” theory of Rogoff (1985). Aside from the context (monetary policy versus fiscal policy) and methodology, there is a substantial difference with this paper. Rogoff (1985) argues that it may be beneficial to appoint an ad hoc policymaker who does not reflect social preferences, in the sense that equilibrium objects (the inflation and output gap) are weighted differently. Appointing a conservative central banker is equivalent to “[giving] the central bank concrete incentives to achieve an intermediate monetary target.”<sup>5</sup> Accordingly to the theory, or maybe because of it, central banks are usually politically independent institutions.

In contrast, the class of policymakers I consider suits much better the desired outcome of a political process: policymaker's preferences are always Paretian and defined over any policy problem. The implementation would be by election rather than by delegation. This is only possible because there is no ex-post deviation from the ex-ante optimal policy which is Pareto superior: I have no presumption that this strict condition is satisfied in the context of monetary policy.

There has been previous literature relating redistribution and the credibility of fiscal policy, specially public debt repayment. Rogers (1986) argues that the government dislike

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<sup>2</sup>Hence, it is not the timing what embodies the representative democracy with “commitment,” as in Torsten and Tabellini (1994).

<sup>3</sup>In the “classic” version of the trade-off, demand for redistribution is bounded by the efficiency costs of distortionary taxation. This trade-off has been used in direct democracy models, from Meltzer and Richard (1981) to Mulligan (2001), among others. However, in order to bound redistribution, one needs to rule out contemporaneous capital tax decisions as it is done in Krusell and Rios-Rull (1999).

<sup>4</sup>Drazen (2000), Chapter 7.4, discusses theory and evidence on the subject.

<sup>5</sup>Rogoff (1985), page 1170.

for inequality affects the incentives to renege on its announced policies, but no time consistent policy is characterized. In the context of public debt, Dixit and Londregan (2000) present an explicit political model and show that if political power and government bond holdings are positively correlated, then government debt repayment is credible. In a recent paper, Sleet and Yeltekin (2002) argue that if debt market participants possess sufficient political influence, they would block debt default, increasing the set of sustainable debt policies.

Bassetto (1999) and Albanesi (2002) also deal with the time inconsistency problem in fiscal policy with heterogeneous agents in absence of direct asset taxation. They show that by manipulating the distribution and maturity structure of government assets, the optimal policy can be made time consistent. These results relate to the work of Lucas and Stokey (1983) and Persson and Svensson (1986), and they can not be extended to the analysis of optimal capital taxation.

There is a large literature on policy credibility that assume some form or another of exogenous commitment ability in political institutions. This has been criticized in McCallum (1995). An exception is Garfinkel and Lee (2000) in the context of political lobbying. However, commitment is assumed in the formation of lobbies who then later influence the policy choice. See also Marceau and Smart (2003).

## 1.2 Organization of the Paper

The rest of the paper is organized as follows. Next section lays the two stage, heterogeneous agents fiscal policy model and it presents the theoretical results. Section 3 proposes a politico-economic recursive equilibrium and Section 4 explores the equilibrium properties. In order to check the robustness of the analytical results, a more complete economy is proposed in Section 5 and numerical results are provided. Section 6 concludes.

# 2 A Simple Fiscal Policy Model

The model is a slightly modified version of Chari and Kehoe (1990), introducing a continuum of households with heterogeneous wealth endowments. The model features two stages, and it intuitively models commitment with different timings of the policy and investment decisions.

## 2.1 Setting and Competitive Equilibrium

The economy is populated by a policymaker and a set  $I = [0, 1]$  of households. There are two stages,  $h = 1, 2$ . In the first stage, each household has an exogenously given wealth endowment,  $w_i \geq 0$ , which the household splits between investment,  $k_i \geq 0$ , and storage,  $w_i - k_i \geq 0$ . The pre-tax return to investment is  $R > 1$ , while the storage has unit return but it is not subject to taxation.

In the second stage  $h = 2$ , each household receives the after-tax returns to investment,  $(1 - \tau^k) Rk_i$ , and decides how much to consume,  $c_i \geq 0$ , how much labor to supply,  $n_i \geq 0$ , and how much leisure to enjoy,  $1 - n_i \geq 0$ , subject to a budget constraint. Labor has an after-tax return of  $(1 - \tau^n)$ , assuming a linear production function in labor with unit productivity.

Preferences over consumption and leisure are given by a standard utility function,  $u(c_i, n_i)$ . The policymaker sets fiscal policy  $\tau = \{\tau^n, \tau^k\}$  to finance an exogenously given government expenditure  $g$ .

Given their atomistic nature, each household  $i$  takes the fiscal policy  $\tau$  as given. Then the household problem is to set  $x_i = \{k_i, n_i, c_i\}$  to solve

$$\max u(c_i, n_i) \quad (1)$$

subject to

$$c_i \leq ((1 - \tau^k) R - 1) k_i + w_i + (1 - \tau^n) n_i$$

and  $0 \leq k_i \leq w_i$ ,  $c_i \geq 0$ ,  $0 \leq n_i \leq 1$ .

Fiscal policy must satisfy the government budget constraint

$$g \leq \tau^n n + \tau^k Rk \quad (2)$$

where  $n = \int_I n_i di$  and  $k = \int_I k_i di$ . Fiscal policy must also satisfy  $\{\tau^k, \tau^n\} \leq 1$ . I will assume that  $\bar{g} > g > (R - 1)w$ , where  $w = \int_I w_i di$ . The upper bound  $\bar{g}$  is set such that government consumption is feasible even if  $k = 0$ ; the lower bound  $(R - 1)w$  implies that positive labor taxation is necessary to finance  $g$ .

Finally, an aggregate resource constraint holds:

$$c + g \leq n + (R - 1)k + w \quad (3)$$

where  $c = \int_I c_i di$ .

The household optimality conditions, combined with feasibility, define a private sector competitive equilibrium a given policy  $\tau$ .

**Definition 1** A *private sector competitive equilibrium* is a fiscal policy  $\tau$  and a set of allocations  $x = \{x_i\}_{i \in I}$  such that each household  $i$  solves (1), the government budget constraint (2) is satisfied and the aggregate resource constraint (3) holds.

A quick look at the optimality conditions of the household decision problem reveals an arbitrage condition between the storage and the investment technology,

$$R(1 - \tau^k) \geq 1 \quad (4)$$

as long as  $k_i > 0$  for some  $i \in I$ . If any wealth is invested, then the after-tax return to investment must be at least equal to the unit storage return.

Household  $i$  consumption and leisure decisions can be characterized by allocation rules defined as  $c(\omega, \tau^n)$ ,  $n(\omega, \tau^n)$ , where  $\omega$  is the after-tax wealth given by:

$$\omega_i = w_i + ((1 - \tau^k) R - 1) k_i$$

These allocation rules are obtained by combining the household  $i$  budget constraint with the necessary and sufficient first order condition

$$-\frac{u_i^n}{u_i^c} = (1 - \tau^n) \tag{5}$$

with  $c_i \geq 0$  and  $n_i \in [0, 1]$ . Note that allocation rules  $c(\omega, \tau^n)$  and  $n(\omega, \tau^n)$  are differentiable.

## 2.2 Policy Equilibrium

In this paper, the policy decision is a component of the policy equilibrium. The aim of this section is to analyze the same economy under different policymaker “types” and compare the equilibrium policies for the Ramsey and the Markov policy equilibrium concepts. In the former, the policymaker is able to commit to any policy plan. In the latter, the policymaker can not commit and the policy choice takes private sector expectations as given.

The policymaker types considered differ on their preferences over welfare distributions across households.<sup>6</sup> As it is standard in the literature in optimal fiscal policy, I assume that policymaker’s preferences are given by an utilitarian social welfare function (SWF). However, in a heterogeneous agents model, there are many suitable utilitarian SWF, each corresponding to different Pareto weight distributions. Precisely, the policymaker types are indexed by the Pareto weight distribution.

**Definition 2** *A type  $\lambda$  policymaker has preferences over allocations given by the  $\lambda$ -welfare function:*

$$W^\lambda(x) = \int_I \lambda_i u(c_i, n_i) di$$

I will consider only strictly Paretian policymaker types,  $\lambda_i > 0$  for all  $i \in I$ .<sup>7</sup>

Following the usual nomenclature, the policy equilibrium when the policymaker can commit to any given policy is called a Ramsey equilibrium. It implements the best private sector competitive equilibrium. However, when agents are not identical, the policy choice usually will not be an unanimous choice. Hence, I need to index the equilibrium concept by the policymaker type, as what “best” is depends now on the precise  $\lambda$ -welfare function. The resulting object is called a  $\lambda$ -Ramsey equilibrium.

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<sup>6</sup>I assume households can only be distinguished by their wealth endowment, so the policymaker preferences are effectively over joint welfare and wealth distributions.

<sup>7</sup>A strictly Paretian SWF  $W(x)$  is such that for all  $(x, x')$  if  $x$  is Pareto superior to  $x'$  then  $W(x) > W(x')$ .

**Definition 3** A  $\lambda$ -*Ramsey equilibrium* is a private sector competitive equilibrium  $(x, \tau)$  that maximizes the  $\lambda$ -welfare function,  $W^\lambda$ .

The resulting policy  $\tau$  will be referred to as the  $\lambda$ -Ramsey policy. One can think of the policy equilibrium with commitment as the policymaker setting the policy plan once and for all at date  $h = 0$ , before investment decisions are made.

The arbitrage condition (4) characterizes an important property of the capital tax and investment which holds for all  $\lambda$ -Ramsey equilibria.

**Proposition 1** For all policymaker types  $\lambda$ , the  $\lambda$ -Ramsey equilibrium holds

$$\begin{aligned}\tau^k &\leq \frac{R-1}{R} \\ k &= w\end{aligned}$$

**Proof.** Let  $\tau$  be a  $\lambda$ -Ramsey policy with  $\tau^k > \frac{R-1}{R}$ . Arbitrage condition (4) implies that  $k = 0$ . Consider alternative policy  $\tilde{\tau}^k = \frac{R-1}{R}$ ,  $\tilde{\tau}^n = \tau^n$ , with  $k = w$ . Consumption and labor decisions are left intact. This is a private sector competitive equilibrium, with the particularity that government budget constraint (2) holds with strict inequality. It is straightforward to show that any policy for which the government budget constraint does not bind implements Pareto inferior allocations. Note every household is indifferent between  $\tau$  and  $\tilde{\tau}$ . Therefore  $\tau$  is Pareto inferior as well and it can not be a  $\lambda$ -Ramsey policy. The same argument applies for a  $\lambda$ -Ramsey equilibrium with  $\tau^k = \frac{R-1}{R}$  but  $k < w$ . ■

The fact that the optimal capital tax will not exceed  $\frac{R-1}{R}$  is driven purely by efficiency considerations. A capital tax above  $\frac{R-1}{R}$  reduces the tax base to zero and the resulting labor tax would be higher than under  $\tau^k = \frac{R-1}{R}$ . Indeed, setting the capital tax equal to  $\frac{R-1}{R}$  maximizes capital tax revenues. Whether the capital tax is exactly equal to  $\frac{R-1}{R}$  or it is lower depends on the policymaker's type, i.e., equity considerations. A policymaker with larger weights on the wealthy household welfare is likely to set a capital tax below  $\tau^k < \frac{R-1}{R}$ , as wealthy households will have a larger share of income coming from capital returns.

In the case without commitment, the correspondent policy equilibrium is the  $\lambda$ -Markov equilibrium. Here it is simply equivalent to a Nash equilibrium with a subgame perfection requirement. The timing is as follows: the policymaker decides on the fiscal policy  $\tau$  at  $h = 1\frac{1}{2}$ , i.e., after the households have set investment, but before consumption and leisure are decided. It involves three objects: allocations  $x$ , the fiscal plan  $\tau$  and the household expectations  $\tau^*$  with respect the fiscal plan.

**Definition 4** A  $\lambda$ -*Markov equilibrium* is a triplet  $(x, \tau, \tau^*)$  such that:

1. Allocations and fiscal plan constitute a private sector competitive equilibrium.
2. Household expectations are validated

$$\tau = \tau^*$$

3. Given household expectations  $\tau^*$  and corresponding investment decision  $k$ , fiscal plan  $\tau$  maximizes the  $\lambda$ -welfare function,  $W^\lambda$ .

A policy will be time consistent if there exists the appropriate set of rational expectations supporting it in absence of commitment. In other words, the ex-post policymaker's decision should fulfill the household expectations. As different policymakers will have different incentives at  $h = 1\frac{1}{2}$ , the concept of time consistency needs to be indexed by the policymaker type  $\lambda$  as well.

**Definition 5** A policy  $\tau$  is  $\lambda$ -**time consistent** if there exists an allocation plan  $x$  such that  $(x, \tau, \tau)$  constitutes a  $\lambda$ -Markov equilibrium.

### 2.3 Time Consistency of Optimal Fiscal Policy

In this section I explore conditions such that constrained Pareto efficient policies can be implemented in absence of commitment. In the terminology of the previous section, the twofold result goes like this: first, I will present a simple condition for a  $\lambda$ -Ramsey policy to be  $\lambda$ -time consistent. Second, I will show that, if for any  $\lambda$  the  $\lambda$ -Ramsey policy is not  $\lambda$ -time consistent, there exists a policymaker  $\lambda^*$  such that the  $\lambda$ -Ramsey policy is  $\lambda^*$ -time consistent. Both results indicate that policymakers biased towards wealthy households have a lesser time inconsistency problem.

To motivate the formal results, consider the following exercise. I assume household expectations  $\tau^*$  are such that all wealth has been invested,  $k = w$ . In other words, household expect a  $\lambda$ -Ramsey policy to be implemented, without specifying which one. Then I evaluate the first order welfare change associated with a marginal increase in the capital tax  $\Delta\tau^k$ , which finances a budget balancing decrease in labor taxation  $\Delta\tau^n$ . This is done at  $h = 1\frac{1}{2}$ , i.e., when investment decisions  $k$  cannot be modified. If the first order welfare is 0, I will conclude that the policy  $\tau^*$  is (at the margin) time consistent.<sup>8</sup>

In the Appendix A I show that the first order effect on the household  $i$  welfare is given by

$$D^i = u^c(c_i, n_i) \left( \frac{1}{\psi} \left( \frac{n_i}{n} \right) - \left( \frac{w_i}{w} \right) \right) \quad (6)$$

where

$$\psi = \frac{1 + \tau^n n^\tau}{1 - \tau^n n^\omega}$$

with  $n^\tau = \int_I \frac{\partial n(\omega_i, \tau^n)}{\partial \tau^n} \frac{1}{n} di$  and  $n^\omega = \int_I \frac{\partial n(\omega_i, \tau^n)}{\partial \omega} \frac{w_i}{w} di$ . Loosely speaking, the term  $\psi$  captures the fact that labor taxation is distortionary but capital taxation is not once investment is

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<sup>8</sup>In order to conclude whether a policy is time consistent or not, it is necessary to check the global optimality properties as the first order conditions are usually not sufficient.

inelastic. Consider the case  $\psi < 1$ : an “average” household, i.e.,  $n_{ave} = n$  and  $w_{ave} = w$ , will have a marginal welfare *gain*,

$$D^{ave} = u^c(c_{ave}, n_{ave}) \left( \frac{1}{\psi} - 1 \right) > 0$$

The conclusion of a representative agent model would be that any capital tax  $\tau^k \leq \frac{R-1}{R}$  would not be time consistent, as a marginal increase on the capital tax would induce net welfare gains,  $D^{ave} > 0$ . These are pure efficiency gains, associated with substituting a distortionary tax (the labor tax) by a lump sum tax (de facto, the capital tax).<sup>9</sup>

However, wealth heterogeneity implies that shifts in factor taxation have redistributive consequences. If a household  $i$  is wealthy with respect to the average, then (6) shows that the marginal welfare effect can be negative,  $D^{wealthy} < 0$ , as the ratio  $\frac{w_i}{w}$  is large enough. Wealthy households are likely to be “losers,” and poor households “winners,” from a capital tax increase once investment decisions have been made. Hence, no (marginal) deviation from any  $\lambda$ -Ramsey policy is Pareto superior if there is enough wealth heterogeneity.<sup>10</sup>

For a policymaker of type  $\lambda$ , the aggregate marginal welfare effect is

$$D^\lambda = \int_I \lambda_i D^i di$$

It is easy to see that if there exists a non-zero measure subset of households with  $D^i < 0$ , there exists  $\lambda$  such that  $D^\lambda = 0$ .

The following proposition provides a sufficient condition to identify policymaker types  $\lambda$  who are able to sustain their respective  $\lambda$ -Ramsey policy in absence of commitment. For the remaining of the section I assume that policymakers, regardless of the type, have single-peaked preferences over the tax choice given  $\{k_i\}_I$ .<sup>11</sup>

**Proposition 2** *For any policymaker  $\lambda$  such that the  $\lambda$ -Ramsey policy holds  $\tau^k < \frac{R-1}{R}$ , the  $\lambda$ -Ramsey policy is  $\lambda$ -time consistent.*

<sup>9</sup>It is well known that preferences can be specified such that Ramsey policy is time consistent. In the present environment, this is captured by the possibility that  $\psi = 1$ . For example, if preferences are given by

$$u(c, n) = \log c + \log(1 - n)$$

leisure decisions by a household with zero wealth endowment are left undisturbed by labor taxation. Hence, there are no pure efficiency gains for this household.

<sup>10</sup>This is unlikely to be an issue in any reasonable calibration, at least for United States: Rodriguez, Diaz-Gimenez, Quadrini and Rios-Rull (2002) report that the top wealth is over one thousand times the average wealth.

<sup>11</sup>This is weaker than assuming that the first order conditions of the policymaker’s problem are sufficient, which is commonly done. See, for example, Bassetto (1999) and Aiyagari, Marcet, Sargent and Seppala (2002), who then check the assumption with computational exercises. With standard household preferences and a thick wealth distribution, I failed to generate economies where the single-peaked property failed. However, in some cases the first order conditions had several solutions.

**Proof.** See Appendix A. ■

The intuition is simple. A policymaker willing to reduce capital tax revenues at date  $h = 0$  cares enough about the wealthy household's welfare to consent increased distortion from higher labor taxes. As the arbitrage condition (4) holds with strict inequality, additional (small) capital taxation will behave as lump sum taxes at  $h = 0$ . Hence, at  $h = 1\frac{1}{2}$ , these policymakers have no incentives to modify the  $\lambda$ -Ramsey policy: if they wanted to reduce tax distortion, they could have done it (marginally) at  $h = 0$ .

One would like to complete Proposition 2 and find a condition such that the policy  $\tau^k = \frac{R-1}{R}$  can be implemented. Recall that  $\tau^k = \frac{R-1}{R}$  maximizes capital tax revenues under commitment. Proposition 3 states that the only necessary condition is the presence of wealthy enough households, as hinted in the previous discussion. Then there exists a policymaker type  $\lambda^*$  for whom  $\tau^k = \frac{R-1}{R}$  is  $\lambda^*$ -time consistent.

**Proposition 3** *Assume there is a non-zero measure subset  $\tilde{I}(\delta) = \{i : w_i \geq \delta w\} \subset I$  for  $\delta$  large enough. Then there exists a policymaker type  $\lambda^*$  such that  $\tau^k = \frac{R-1}{R}$  is  $\lambda^*$ -time consistent.*

**Proof.** See Appendix A. ■

Obviously, the  $\lambda^*$ -Ramsey policy sets  $\tau^k = \frac{R-1}{R}$  as well. This exhausts all possible  $\lambda$ -Ramsey policies – recall Proposition 1. However, there are still many more policymaker types  $\lambda'$ , whose  $\lambda'$ -Ramsey policy would maximize capital tax revenues as well, but the  $\lambda'$ -Ramsey policy would not be  $\lambda'$ -time consistent. The last proposition of this section further characterizes the time consistency properties of this economy.

**Proposition 4** *Let  $\lambda$  be more progressive than  $\lambda'$  if for any wealth endowment level  $\omega \in \mathfrak{R}$ ,*

$$\int_{I(\omega)} \lambda_i di \geq \int_{I(\omega)} \lambda'_i di$$

where  $I(\omega) = \{i \in I | w_i \leq \omega\}$ , with strict sign for some  $\omega$ .

Then, for any  $\lambda$  more progressive than  $\lambda^*$ :

1. The  $\lambda$ - and  $\lambda^*$ -Ramsey policy are identical.
2. The  $\lambda$ -Ramsey policy is  $\lambda^*$ -time consistent but it is not  $\lambda$ -time consistent.

**Proof.** See Appendix A. ■

Hence, any policymaker  $\lambda$  who assigns a larger weight to poor households than policymaker  $\lambda^*$  suffers from the time inconsistency problem, in the sense that the policy choice with commitment can not be implemented by policymaker  $\lambda$  without commitment. However, it can be implemented without commitment by policymaker  $\lambda^*$ .

Since the equity considerations must offset the net efficiency gains, policymaker  $\lambda^*$  must be biased towards wealthy households. One way to see this is to recall the “average” household exercise above. Also, one can think of a policymaker  $\tilde{\lambda}$  such that the  $\tilde{\lambda}$ -Ramsey equilibrium allocation satisfies

$$\tilde{\lambda}_i u^c(c_i, n_i) = \tilde{\lambda}_j u^c(c_j, n_j)$$

i.e., the Pareto weight distribution  $\tilde{\lambda}$  conforms to the market welfare distribution under commitment. It is easy to see that, in the exercise above, the first order effect on aggregate welfare will be positive,  $D > 0$ , so policymaker type  $\tilde{\lambda}$  is “too” progressive and the  $\tilde{\lambda}$ -Ramsey policy would not be  $\tilde{\lambda}$ -time consistent.

Finally, the next lemma characterizes the welfare properties of Markov equilibria, establishing that every household will prefer policymaker  $\lambda^*$  than a more progressive policymaker  $\lambda$ . Note that even poor households, who see their relative welfare weight decrease, are strictly better.

**Lemma 5** *There exists a  $\lambda^*$ -Markov equilibrium that Pareto dominates any  $\lambda$ -Markov equilibrium for any  $\lambda$  more progressive than  $\lambda^*$ .<sup>12</sup>*

It is possible to illustrate the results by plotting the constrained Pareto efficient set and comparing it to the time consistent utility possibility frontier. Figure 1 plots the welfare distributions associated with  $\lambda$ -Ramsey and  $\lambda$ -Markov equilibria for all  $\lambda$ , for a two-class economy, where  $\lambda$  is the welfare weight associated with the wealthy household.<sup>13</sup> Not surprisingly, spanning  $\lambda$ -Ramsey equilibria for all policymaker types traces the Pareto utility frontier with commitment. This is the highlighted section in Figure 1.

Spanning all  $\lambda$ -Markov equilibria has a very different result, since the policymaker is a “player” in the economy without commitment. The welfare distributions associated with some selected  $\lambda$ -Markov equilibria,  $\lambda \in \{0.5, 1, 2\}$ , are shown, as well as the policymaker  $\lambda^* = 1.31$  associated with Proposition 2. Note that the utility possibility frontier includes the highlighted section but then it bends backwards. Thus, the constrained Pareto efficient and the time consistent constrained Pareto efficient set are identical. An economy without commitment can not be automatically ranked as Pareto inferior to an economy with commitment. In this sense, the representative agent model is misleading. It is also clear from Figure 1 that  $\lambda$ -Markov equilibrium outcomes where  $\lambda < \lambda^*$  are inefficient: most of the utility possibility frontier without commitment can be Pareto ranked, being the  $\lambda^*$ -Markov equilibrium Pareto superior to any  $\lambda$ -Markov equilibrium with  $\lambda < \lambda^*$ .

<sup>12</sup>Because for any policymaker there exists a Markov equilibrium with no investment, it can not be said that a policymaker  $\lambda^*$  is strictly superior. There are some equilibrium refinements to rule out the trivial Markov equilibrium with no investment.

<sup>13</sup>The illustration departs from the analyzed economy in several aspects. There are just 2 types of households of identical measure: poor (no wealth) and wealthy (all wealth but no time endowment) households. Moreover, a Cobb-Douglas technology is used and wealth can be either invested or consumed in the first stage. For each policymaker  $\lambda$ , only the best Markov equilibrium is plotted.

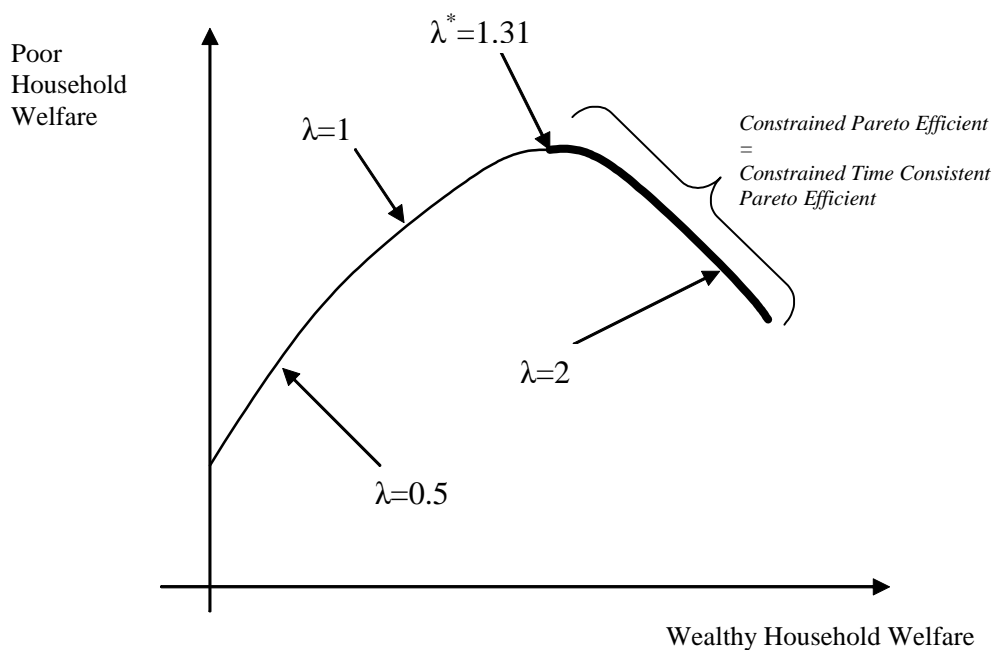


Figure 1: Utility Possibility Frontier for Ramsey and Markov Equilibria

### 3 Political Economy Model

In this section, I explicitly model a political decision framework in an infinitely repeated, two-class version of the economy presented above. First, I detail the political decision process and define a recursive politico-economic equilibrium, with some additional structure that imposes a Markov requirement on equilibrium functions. Then I provide some equilibrium properties and I show how efficient fiscal policy can be implemented in equilibrium given a sufficiently low intertemporal discount rate and some structure on the exogenous occurrence of elections. I also discuss the possibility of political cycles.

#### 3.1 Setting

The economy is an infinitely repeated version of the two-stage model presented above. Intertemporal preferences are given by

$$U_{it} = \sum_{j=t}^{\infty} \beta^{j-t} u_{ij}$$

with  $0 \leq \beta < 1$ .

For tractability, household population is divided in two types: the capitalist household ( $i = 1$ ) and the worker household ( $i = 2$ ). Capitalist households own all wealth but have no time endowment; worker households have no wealth. There is a measure of capitalist households  $\phi < 1$  and the measure of worker households is normalized to 1.

The timing is slightly modified to include the political decision mechanism as follows: every period, after investment decisions have been set but before any fiscal policy decision has been taken, a possibly random process  $s$  determines whether elections are celebrated. In absence of elections, the previous policymaker remains in office. In the event of elections, voters elect a new representative who then sets policy. The pool of candidates is the set of policymaker types  $\lambda \geq 0$ . Each type indexes the policymaker's preferences over private sector allocations given by an utilitarian SWF, as in the previous section:

$$W^\lambda(c_1, c_2, n) = \lambda u_1(c_1) + u_2(c_2, n)$$

Note the pool of candidates is restricted to have preferences defined by a strictly Paretian SWF, with the exception of type  $\lambda = 0$ , whose SWF is weakly Paretian.<sup>14</sup> Moreover, each policymaker's preferences are defined for any policy problem. Both seem desirable properties for the pool of candidates in a representative democracy model.

I assume that voters are rational in the spirit of Meltzer and Richard (1981). Despite households being atomistic and hence their vote irrelevant, each voter makes an optimal vote

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<sup>14</sup>A weakly Paretian SWF  $W(x)$  is such that for all  $(x, x')$  if  $x$  is Pareto superior to  $x'$  then  $W(x) \geq W(x')$ .

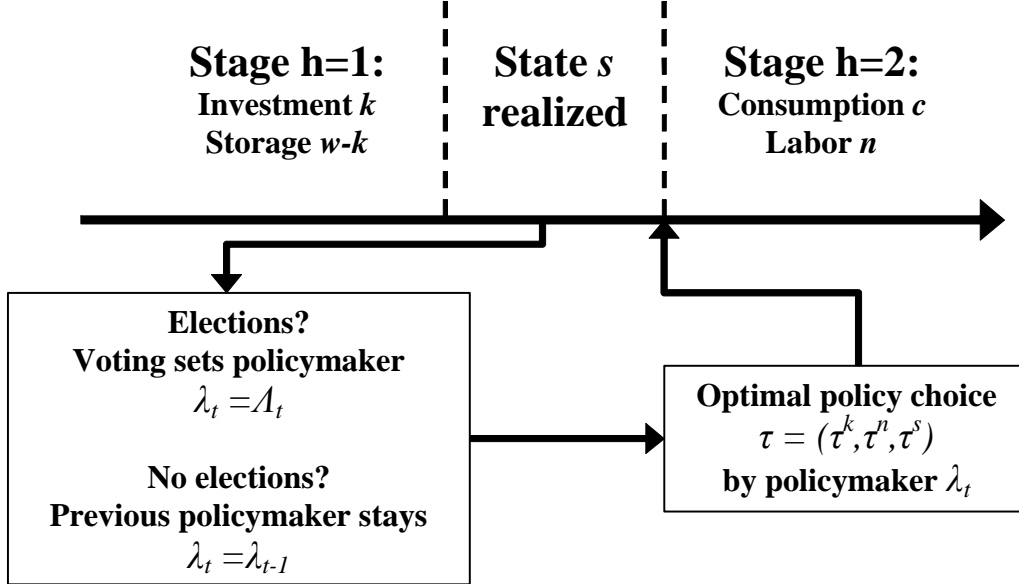


Figure 2: Timing of Economic and Political Decisions

decision, forming rational expectations about the resulting fiscal policy and the impact on the outcome of future elections. The decisive voter is the median voter, a worker household by assumption since  $\phi < 1$ . Hence, elections are equivalent to let a representative worker household set the policymaker type of her choice: the capitalist households are actually disenfranchised.

Note that investment decisions are made before elections, so they are taken as given by the median voter, and the newly elected policymaker chooses the current fiscal policy with no restriction from past periods.<sup>15</sup> This implies that the private sector must form expectations about the outcome of the elections and resulting after-tax investment return. Hence, investment is done under political uncertainty. The timing within each period  $t$  is summarized in Figure 2.

### 3.2 Recursive Politico-economic Equilibrium

The proposed equilibrium concept is a recursive politico-economic equilibrium in the spirit of the Krusell and Rios-Rull (1999). I have already stressed that voters are rational and forward-looking in predicting the effects on current policy and future political outcomes. Moreover,

<sup>15</sup>This is in contrast of Torsten and Tabellini (1994), where elections occur before investment decisions, and Krusell and Rios-Rull (1999) where the capital tax is determined one period in advance.

investors must forecast the current political outcome. The considerable complexity of this problem usually calls for a computational analysis, but the simple model analyzed here, where the only physical variable is the policymaker type, allows for some analytical results. Section 5 performs some computational exercises and show that analytical results can be extended to more complete economies.

The possibility of elections is driven by an exogenous first order Markov process  $\{s_t\}_{t=0}^{\infty}$  given by Markov matrix  $\Pi$  and initial state  $s_{-1}$ . Function  $e(s) : S \rightarrow \{0, 1\}$  determines whether there are elections,  $e(s) = 1$ , or not,  $e(s) = 0$ , at state  $s$ . In absence of elections, the previous period policymaker remains in office. Hence, the state of the economy at the beginning of the period is given by  $(s_{t-1}, \lambda_{t-1})$ .<sup>16</sup>

Now I describe each household problem. The **capitalist household**  $i = 1$  chooses  $k$  and  $\{c_1(s')\}_{s' \in S}$  to solve

$$\max \sum_{s' \in S} \pi(s'|s) u_1(c_1(s')) \quad (7)$$

subject to

$$c_1(s') \leq (1 - \tau^k(s')) Rk + w - k$$

for all  $s' \in S$  and non-negativity constraints on consumption, investment and storage.

As the realization of state  $s'$  influences the policy choice, the capitalist household investment decision involves a policy schedule  $\{\tau(s')\}_{s' \in S}$  rather than a single tax  $\tau$ . Rationality implies that households correctly forecast the policy schedule, which requires knowledge of the economy state  $(s, \lambda)$ , and functions mapping the state into the political outcome and policy choice (to be detailed below). I will denote the optimal investment decision as  $\kappa(\lambda, s)$ , only making the dependence on the economy state explicit.

Capitalist household consumption, though, it is just a function of the investment decision  $k$  and the policy choice  $\hat{\tau}$ , so it can be written  $c_1(k, \hat{\tau})$ . Similarly, one can characterize an ex-post indirect period utility function

$$v_1(k, \hat{\tau}) = u_1(c_1(k, \hat{\tau}))$$

The **worker household**  $i = 2$  chooses  $\{c_2(s'), n(s')\}_{s' \in S}$  to solve

$$\max \sum_{s' \in S} \pi(s'|s) u_2(c_2(s'), n(s')) \quad (8)$$

subject to

$$c_2(s') \leq (1 - \tau^n(s')) n(s')$$

for all  $s' \in S$  and non-negativity constraints on consumption, labor and leisure.

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<sup>16</sup>In order to make the political process significant from the very first period, I assume that  $\Pr(e(s_0) = 1 | s_{-1}) = 1$ , so policymaker type  $\lambda_{-1}$  is irrelevant.

Consumption and labor allocations solving (8) can be written as a function of aggregate investment  $k$  and actual policy  $\hat{\tau}$ ,

$$c_1(k, \hat{\tau}), c_2(k, \hat{\tau}), n(k, \hat{\tau})$$

And the ex-post indirect period utility functions can also be expressed as

$$v_2(k, \hat{\tau}) = u_2(c_2(k, \hat{\tau}), n(k, \hat{\tau}))$$

The policy choice depends on the policymaker type in office  $\lambda$ , as well as investment level  $k$ . The **optimal fiscal policy** function  $\tau(k, \lambda)$  satisfies

$$\tau(k, \lambda) = \arg \max_{\hat{\tau} \in \Gamma(k)} \lambda v_1(k, \hat{\tau}) + v_2(k, \hat{\tau}) \quad (9)$$

where  $\Gamma(k)$  is the set of tax rates  $\hat{\tau} = (\tau^k, \tau^n)$  that satisfy the government budget constraint given  $k$  and constraints  $0 \leq (\tau^k, \tau^n) \leq 1$ . Note that the optimal fiscal policy problem is intra-temporal as the policy decision has no impact on future periods.

The **median voter decision** is given by the political function  $\Lambda(k, s)$ :

$$\Lambda(k, s) = \arg \max_{\lambda \geq 0} v_2(k, \tau(k, \lambda)) + \beta V_2(\lambda, s) \quad (10)$$

where  $V_2$  is the continuation value for the worker. In contrast with the policy choice problem, the median voter decision is an intertemporal one: the elected policymaker may remain in office several periods, affecting future policy choices.

Finally, the **law of motion** of the policymaker type is given by  $\lambda' = \tilde{\Lambda}(\lambda, k, s)$ , where

$$\tilde{\Lambda}(\lambda, k, s) = \begin{cases} \Lambda(k, s) & \text{if } e(s) = 1 \\ \lambda & \text{otherwise} \end{cases} \quad (11)$$

All equilibrium objects have been introduced and only the formal definition of the recursive politico-economic equilibrium remains to be given.

**Definition 6** *A Recursive Politico-economic Equilibrium consists of:*

1. *Investment function*  $\kappa(\lambda, s)$ ,
2. *Private Sector Competitive Allocation functions*  $x(k, \hat{\tau}) = \{c_1(k, \hat{\tau}), c_2(k, \hat{\tau}), n(k, \hat{\tau})\}$ ,
3. *Optimal Policy function*  $\tau(k, \lambda)$ ,
4. *Political function*  $\Lambda(k, s)$ ,
5. *Value functions*  $V_i(\lambda, s), i = 1, 2$ .

such that

1. Functions  $x(k, \hat{\tau})$  and  $\kappa(\lambda, s)$  solve (7) and (8), with  $\{\hat{\tau}(s')\}_{s' \in S}$  given by  $\tau$  and  $\Lambda$ .
2. Function  $\tau(k, \lambda)$  satisfies (9).
3. Function  $\Lambda(k, s)$  satisfies (10).
4. Value functions  $V_i$  satisfy

$$V_i(\lambda, s) = \sum_{s' \in S} \pi(s'|s) \left( v_i \left( k, \varphi \left( k, \tilde{\Lambda}(\lambda, k, s') \right) \right) + \beta V_i \left( \tilde{\Lambda}(\lambda, k, s'), s' \right) \right)$$

where

$$k = \kappa(\lambda, s)$$

and  $\tilde{\Lambda}$  given by (11)

The recursive politico-economic equilibrium can be described as a functional fixed point involving the investment function, the political function and the worker's value function. The investment decision  $\kappa(\lambda, s)$  depends on the political rule,  $\Lambda$ , to compute the after tax return to investment. The median voter  $\Lambda(k, s)$  decision depends on the continuation value  $V_2$  associated with the policymaker type chosen. Both equilibrium functions map back into the value function  $V_2(\lambda, s)$ .

Remaining equilibrium objects,  $\{x, v_1, v_2, \tau, V_1\}$ , are straightforward as their computation does not require the knowledge of functions  $\{\kappa, \Lambda, V_2\}$ , only of the realizations  $\{k, \lambda, \tau\}$ . From now on, I will denote a recursive politico-economic equilibrium by triple  $\{\kappa, \Lambda, V_2\}$ .

### 3.3 Markov Equilibrium Restriction

The recursive formulation rules out history dependent equilibria, but it is still possible to embed trigger strategies in the equilibrium investment function if no additional structure is imposed. It results from the fact that for any policymaker type  $\lambda$  there is always a trivial equilibrium with zero investment. This can easily be seen in the two stage fiscal policy model presented above: if there is no investment, the policymaker is indifferent between any capital tax and expectations inducing zero investment can be trivially validated. This multiplicity of equilibrium investment levels can operate as a reward function for the political outcome. For example, a possible equilibrium function would be a positive investment whenever the policymaker type is  $\lambda'$ ; and zero otherwise.

I want to avoid this type of equilibria driving the results. In order to do so, I will impose additional structure on the investment equilibrium decision. This is not standard but there are several reasons for this choice. First, sometimes the unique equilibrium is characterized by zero investment, therefore it can not be arbitrarily ruled out. Second, as continuity and

differentiability may not be satisfied by any equilibrium function, one should not proceed by restricting equilibrium function properties in this framework.

In particular, the equilibrium investment function must satisfy:

$$\kappa(\lambda, s) = \sup \left\{ k \leq w : \sum_{s' \in S} \pi(s'|s) q(\lambda, k, s') r(\lambda, k, s') \geq 0 \right\} \quad (12)$$

where

$$\begin{aligned} q(\lambda, k, s) &= u_1^c \left( c_1 \left( k, \tau \left( k, \tilde{\Lambda}(\lambda, k, s) \right) \right) \right) \\ r(\lambda, k, s) &= \left( 1 - \tau^k \left( k, \tilde{\Lambda}(\lambda, k, s) \right) \right) R - 1 \end{aligned}$$

Since any  $\tau^k$  solves (9) when  $k = 0$  for any  $\lambda$ ,  $\kappa(\lambda, s) \geq 0$ . Note that condition (12) makes sure  $\kappa(\lambda, s) \neq 0$  if for some  $k > 0$  the expected return to investment is above the storage unit return. In this sense, the structure imposed picks the best investment level compatible with equilibrium (which does not imply it selects the best recursive politico-economic equilibrium).

One property of recursive politico-economic equilibria that satisfy (12) is that resulting equilibrium functions are Markov in the sense proposed by Maskin and Tirole (2001): allocation and policy functions depend only on payoff-relevant state variables. Note that the policymaker type  $\lambda$  may or not may be payoff-relevant given some  $s$ . For example, if  $\Pr(e(s') = 1|s) = 1$ , i.e., there are elections with total certainty, the policymaker type  $\lambda$  is not payoff-relevant. Hence, the set of payoff-relevant state variables is not easily explicitly characterized.

## 4 Equilibrium Properties

In this section I characterize some of the properties associated with Markov recursive politico-economic equilibria. In particular, I explore under which conditions the worker household optimal fiscal policy may be implemented in equilibrium. I also study the possibility of political business cycles. Special attention is given to the relationship between political stability and policy efficiency.

### 4.1 Efficient Fiscal Policy in Equilibrium

Rational voters and representative democracy introduce a new trade-off between redistribution and efficiency.<sup>17</sup> At the time of the elections, contemporaneous tax distortion does not bound the demand for redistribution as investment is inelastic and contemporaneous

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<sup>17</sup>Redistribution takes place first by shifting the financial burden of government spending to the capitalist households. One can also introduce the possibility of a lump sum positive transfer to the worker households, so further redistribution can be pursued, without affecting any of the results.

capital taxation is available. However, in order to be implemented, redistribution requires a policymaker type with a low welfare weight in the capitalist household. If the policymaker is to remain in office, future fiscal policy will be inefficient: the time inconsistency problem will be severe, as shown in Section 2. Hence, the trade-off is between redistribution today and inefficient policy in future periods.

For this trade-off to be relevant, worker households must value positively the future and the incumbent policymaker should have a positive probability of remaining in office in the next periods. Next proposition states that if these conditions do not hold, there is no investment at all: any positive investment  $k > 0$  would lead to the policymaker type  $\lambda = 0$  being elected and all capital returns taxed away,  $\tau^k(k, 0) = 1$ .

**Proposition 6** *Consider an economy where:*

1. *The intertemporal discount rate is 0,  $\beta = 0$ ; or*
2. *Elections occur each period with absolute certainty,  $e(s) = 1, \forall s \in S$ ;*

*then the only Markov recursive politico-economic equilibrium  $\{\kappa, \Lambda, V_2\}$  has zero investment.*

**Proof.** Consider first  $\beta = 0$ . Since  $v_2(k, \tau(k, \lambda))$  is weakly decreasing in  $\lambda$ ,  $\Lambda(k, s) = 0$  for  $\forall k > 0$  follows from (10). Therefore, only  $\kappa(\lambda, s) = 0$  is compatible with equilibrium as  $\tau^k(k, 0) = 1$ . If  $e(s) = 1, \forall s \in S$ , the state variable and the policymaker type  $\lambda$  are irrelevant for the investment decision because of the Markov restriction imposed by (12). Therefore  $V_2(\lambda, s) = V_2$  and it is straightforward to show that (10) implies  $\Lambda(k, s) = 0$  for all  $k > 0$ . ■

This negative result introduces the opposite possibility as well. If the worker household intertemporal discount is sufficiently low and the expected tenure of a policymaker is long enough, then maybe full investment can be supported. Worker households may forego redistribution in order to preserve the efficiency of future fiscal policy. From Proposition 3, it is known that there exists a policymaker type  $\lambda^*$  such that the worker's optimal fiscal policy (the  $\lambda$ -Ramsey policy with  $\lambda = 0$ ) would be implemented in a  $\lambda^*$ -Markov equilibrium. The question is whether the median voter prefers policymaker  $\lambda^*$  to any other policymaker type under full investment  $k = w$ .

To explore this possibility, I impose some further structure into the election process. I assume elections occur with an i.i.d. probability  $p$ , which can be captured with a very simple Markov process

$$\begin{aligned} S &= \{1, 2\} \\ e(s = 1) &= 1; e(s = 2) = 0 \\ \Pi &= \begin{bmatrix} p & 1 - p \\ p & 1 - p \end{bmatrix} \end{aligned}$$

This Markov process implies that the expected tenure is  $\frac{1}{p}$ . One can interpret  $p$  as capturing the stability of the legislature. Because it is payoff-irrelevant, the exogenous state  $s$  will not be an argument of the investment function or the political function.

Next Proposition confirms the conjecture just discussed, highlighting the role of the intertemporal discount rate and the degree of political stability.

**Proposition 7** *There exists  $\bar{\beta} < 1$  such that for  $\beta$  and  $p$  with  $\beta(1-p) \geq \bar{\beta}$ , the worker's optimal fiscal policy can be implemented in a recursive politico-economic equilibrium.*

**Proof.** I conjecture the following candidate equilibrium :

$$\hat{\kappa}(\lambda) = \begin{cases} w & \text{if } \lambda = \lambda^* \\ 0 & \text{otherwise} \end{cases}$$

$$\hat{\Lambda}(k) \leq \hat{\Lambda}(w) = \lambda^*$$

where  $\lambda^*$  holds  $\tau^k(w, \lambda^*) = \frac{R-1}{R}$ . The existence of policymaker type  $\lambda^*$  is given by Proposition 3. Note that the wealth heterogeneity is extreme in the two-class economy.

First I check that  $\hat{\kappa}$  is an equilibrium given  $\hat{\Lambda}$ . When  $\lambda = \lambda^*$ , the return on investment when  $k = w$  is 1 in all events. So  $\hat{\kappa}(\lambda^*) = w$  is an optimal investment decision. It is clear the the tax policy function  $\tau^k(k, \lambda)$  can not be decreasing with the policymaker type. Moreover, at  $\tau^k = \frac{R-1}{R}$ , the capitalist household is actually indifferent between any level of  $k$ . Hence, if  $\lambda < \lambda^*$ ,  $\tau^k(k, \lambda) > \frac{R-1}{R}$  for all  $k > 0$ .

Given a positive probability  $1-p > 0$  that  $\lambda$  stays in office, the return on investment is less than 1, so for  $k > 0$

$$r(\lambda, k, s) < 0$$

and

$$\sum_{s' \in S} \pi(s'|S) q(\lambda, k, s) r(\lambda, k, s) < 0$$

To see the last step, note that  $u^c(rk)r$  is increasing in  $r$  for  $r \leq 0$ . Hence,  $\hat{\kappa}$  is an equilibrium function. Incidentally, note that it satisfies (12).

To check for  $\hat{\Lambda}(w) = \lambda^*$ , first note that  $v_2(w, \tau(w, \lambda))$  peaks at  $\lambda = 0$ , and  $V_2(\lambda) = V_2(\lambda')$  for  $(\lambda, \lambda') < \lambda^*$  as  $\hat{\kappa}(\lambda, s) = 0$  for  $\lambda < \lambda^*$  and the policymaker type is irrelevant if  $k = 0$ . Hence, the median voter decision is either  $\lambda = \lambda^*$  or  $\lambda = 0$ . If

$$v_2(w, \tau(w, \lambda^*)) + \beta V_2(\lambda^*) \geq v_2(w, \tau(w, 0)) + \beta V_2(0) \quad (13)$$

then  $\hat{\Lambda}(w) = \lambda^*$  would be validated. Rewrite  $V_2(0)$  as follows

$$V_2(0) = \frac{v_2(0, \tau(0, 0))}{1 - \beta(1-p)} + \frac{\beta p}{1 - \beta(1-p)} V_2(\lambda^*)$$

where I have used the fact that  $\hat{\Lambda}(0) = \lambda^*$  necessarily since  $V_2(\lambda^*) > V_2(0)$  since  $v_2(0, \tau(0, \lambda))$  is invariant with  $\lambda$ . Using  $V_2(\lambda^*) = v_2(w, \tau(w, \lambda^*)) (1 - \beta)^{-1}$ , one can show that (13) is equivalent to

$$\beta (v_2(w, \tau(w, \lambda^*)) - v_2(0, \tau(0, 0))) \geq (1 - \beta(1 - p)) (v_2(w, \tau(w, 0)) - v_2(w, \tau(w, \lambda^*)))$$

Since  $v_2(w, \tau(w, 0)) > v_2(w, \tau(w, \lambda^*)) > v_2(0, \tau(0, 0))$ , there exists  $\bar{\beta} < 1$  such that the candidate is an equilibrium if  $\beta(1 - p) \geq \bar{\beta}$ . ■

The rationale behind (and the proof) of Proposition 7 is surprisingly similar to folk theorem like results. But recall that no history dependent equilibria and no trigger strategies are involved in the result. Indeed, one could argue that the sufficient degree of “history dependence” is given by the intertemporal link created by the policymaker type. A deviation, i.e., a choice of a policymaker type  $\lambda < \lambda^*$ , is automatically punished by the welfare loss associated with the time inconsistency problem.

## 4.2 Political Business Cycles

Markov recursive politico-economic equilibria can display an array of different interesting phenomena. Using a different, deterministic, Markov first order chain for the exogenous elections occurrence process, I show how political business cycles can be generated. These cycles correlate investment with the uncertainty inherent in the political process.

Consider the following Markov chain, involving a term of  $N$  periods,  $S = \{s_1, s_2, \dots, s_N\}$ :

$$\begin{aligned} e(s_1) &= 1 \\ e(s_i) &= 0; i = 2, 3, \dots, N \\ \pi(s_{i+1}|s_i) &= 1; i = 1, 2, 3, \dots, N - 1 \\ \pi(s_1|s_N) &= 1 \end{aligned}$$

Now the political process is totally deterministic, given by office tenures of length  $N$  periods. The longer the elected policymaker will stay in office, the more likely that a capitalist friendly type is elected. But the term length also determines the frequency of cycles. As the end of the office term approaches, investment is hold back by the uncertainty about the political outcome. Too much investment around elections time would trigger the median voter to choose a more worker friendly policymaker, taxing capital returns. Low investment on election dates decreases the value of capital taxation by means of a smaller tax base. Then the median voter votes for a more capitalist friendly policymaker, who then will ensure investment in the non-election periods.

Next proposition formally states the conditions such that investment displays cycles. Again, the intertemporal discount rate and the length of office tenure are the key parameters.

**Proposition 8** *For any  $N$ , there is an intertemporal discount rate  $\beta(N) < 1$  such that for all  $\beta < \beta(N)$  investment displays cycles under a Markov recursive politico-economic*

equilibrium. If  $\beta \geq \beta(N)$ , the worker's optimal fiscal policy can be sustained. Moreover,  $\beta(N)$  is decreasing in  $N$  and bounded below by  $\hat{\beta} > 0$ .

**Proof.** To show the possibility of cycles, consider the following conjectured equilibria  $\hat{\kappa}$  and  $\hat{\Lambda}$ .

$$\begin{aligned}\hat{\kappa}(s_N) &= 0 \\ \hat{\kappa}(\lambda^*, s_i \neq s_N) &= w \\ \hat{\kappa}(\lambda < \lambda^*, s_i \neq s_N) &= 0 \\ \hat{\Lambda}(0) &= \lambda^* \\ \hat{\Lambda}(k \neq 0) &= 0\end{aligned}$$

It is straightforward to show that the investment function  $\hat{\kappa}$  is an equilibrium object given  $\hat{\Lambda}$ . Also,  $\hat{\Lambda}(0)$  is easily validated: the policymaker has no impact on  $v_2(0, \tau(0, \lambda))$ , and  $V_2(\lambda^*) \geq V_2(\lambda)$  under the conjectured equilibrium. For  $\hat{\Lambda}(k)$  when  $k \neq 0$ , note that

$$V_2(\lambda^*, s_1) = \frac{(1 - \beta^{N-1})}{1 - \beta} v_2(w, \tau(w, \lambda^*)) + \beta^{N-1} V_2(s_N)$$

and for  $\lambda < \lambda^*$ ,

$$V_2(\lambda, s_1) = \frac{(1 - \beta^{N-1})}{1 - \beta} v_2(0, \tau(0, \lambda)) + \beta^{N-1} V_2(s_N)$$

Thus the median voter decision is either policymaker type  $\lambda = \lambda^*$  or  $\lambda = 0$ . If

$$\begin{aligned}& v_2(w, \tau(w, \lambda^*)) + \frac{(\beta - \beta^N)}{1 - \beta} v_2(w, \tau(w, \lambda^*)) + \beta^N V_2(s_N) \\ & \leq v_2(w, \tau(w, 0)) + \frac{\beta - \beta^N}{1 - \beta} v_2(0, \tau(0, 0)) + \beta^N V_2(s_N)\end{aligned}$$

then  $\hat{\Lambda}(k \neq 0) = 0$  is validated. The previous expression can be rewritten as

$$(\beta - \beta^N) (v_2(w, \tau(w, \lambda^*)) - v_2(0, \tau(0, 0))) \leq (1 - \beta) (v_2(w, \tau(w, 0)) - v_2(w, \tau(w, \lambda^*)))$$

For the existence of cycles, the sufficient condition is

$$(\beta - \beta^N) (v_2(w, \tau(w, \lambda^*)) - v_2(0, \tau(0, 0))) < (1 - \beta) (v_2(w, \tau(w, 0)) - v_2(w, \tau(w, \lambda^*)))$$

which holds for  $\beta < \beta(N) < 1$ . It is also straightforward that if  $\beta \geq \beta(N)$ , the worker's optimal fiscal policy can be sustained. Also note that  $\beta(N)$  is decreasing in  $N$ , with

$$\lim_{N \rightarrow \infty} \beta(N) = \frac{v_2(w, \tau(w, 0)) - v_2(w, \tau(w, \lambda^*))}{v_2(w, \tau(w, 0)) - v_2(0, \tau(0, 0))} > 0$$

■

Note how the political process shapes the equilibrium dynamic properties. An economy with a deterministic term of  $N$  is not equivalent to another economy with an i.i.d. probability  $p$  of elections such that  $N = \frac{1}{p}$ . However, if the intertemporal discount rate is high enough, the worker's optimal fiscal policy can be sustained in equilibrium for both economies. Moreover, as  $N \rightarrow \infty$  and  $p \rightarrow 0$ , the requirement on the intertemporal discount rate converges in both economies.

From Proposition 7, it is clear that political instability captured by an i.i.d. probability of elections is equivalent to a lower intertemporal discount rate. Hence, higher political instability not only dampens the possibility of efficient fiscal policy, but increases the likelihood of political business cycles.

## 5 A More Complete Model

Analytical results in the previous section correspond to a simple economy. In this section, I solve a more complete version of the economy presented above and show numerically that the previous results are robust.

First, I introduce a constant returns to scale, strictly concave production function of a Cobb-Douglas form

$$y = Ak^\alpha n^{1-\alpha}$$

with  $A > 0$ ,  $\alpha > 0$ . The fact that marginal product of labor is increasing in capital enhances the worker's welfare gains on investment. It also allows to rule out policymaker type  $\lambda = 0$  without loss of generality as long as the intertemporal discount rate is positive,  $\beta > 0$ .

Second, the capitalist's alternative to investment is now to consume in the first stage rather than storage. This formalizes rigorously non-taxability and it ensures that investment is a continuous function of the policymaker type. Hence, the capitalist preferences are given now by  $u_1(c_1^1, c_1^2)$  where  $c_1^1$  and  $c_1^2$  are the capitalist's consumption in the first and second stage respectively.

In order to proceed to numerical analysis, utility functions must be specified and parameter values chosen. With respect to the former, I have used

$$\begin{aligned} u_1(c_1^1, c_1^2) &= \frac{(Rc_1^1 + c_1^2)^{1-\theta}}{1-\theta} \\ u_2(c_2, n) &= \frac{c_2^{1-\sigma}}{1-\sigma} + \psi_0 \frac{(1-n)^{1-\psi}}{1-\psi} \end{aligned}$$

which are standard forms.

The period term is set equal to one quarter. Parameters are calibrated such that, under the worker's optimal fiscal policy, the government to output ratio is equal to 15%,  $\frac{g}{y} = 0.15$ , time not supplied is around 5%,  $n = 0.95$ , and the annual marginal return to capital is equal

<b>Equilibrium Investment</b>		
<i>Intertemporal Discount Rate</i>	$p = 0.0625$	$p = 0.1$
$\beta = 0.3$	0.3823	0.3587
$\beta = 0.33$	0.5442	0.4947
$\beta = 0.36$	0.8656	0.8602
$\beta = 0.4$	1	1

Table 1: **Sustainability of Full Investment**

to 5%. Parameter  $R$  is set such that the capitalist household is indifferent between early consumption and investment if the annual after-tax capital return is equal to 2%. The share of capital is one third,  $\alpha = .33$ , which implies a capital to output ratio of approximately 3. Remaining preferences parameters are set  $\theta = 0.25$  and  $\sigma = 0.5$ . Wealth is normalized to 1, so the full investment equilibrium is characterized by  $k = 1$ .

For an economy with an i.i.d. probability  $p$  of an election, equilibrium investment as share of the wealth endowment is shown for different intertemporal discount rates  $\beta$  in Table 1.<sup>18</sup> The result in Proposition 7 holds: there exists a probability of an election  $p$  and an  $\beta$  intertemporal discount rate such that full investment is implemented in equilibrium. Moreover, the equilibrium investment monotonically increases with  $\beta$  and decreases with the  $p$ , as expected.

The intertemporal discount rate needed to achieve the efficient level of investment is very low. Obviously, even the extended version of the economy used here is not ready for any serious calibration exercise. Note that reducing the average term length from 16 ( $p = 0.0625$ ) to 10 ( $p = 0.1$ ) has a sizeable impact on the equilibrium investment level, specially in the relatively impatient economies. One can interpret this result as the cost of political instability.

The cycle phenomena is also replicated by numerical exercises. I set  $N = 16$ , so elections occur with perfect regularity every four years. Figure 3 plots several allocation and policy decisions along the time path. Four economies with annual intertemporal discount rates  $\beta \in \{0.3, 0.3175, 0.335, 0.3525\}$  are considered. Investment (as share of total wealth), output, labor and the labor tax are displayed.

The result political business cycles are driven by investment, and they have a significant impact upon output. Note that the labor tax decreases during the elections period. But this should not be regarded as fiscal stimulus at all; recall labor tax decreases are inefficient ex-ante and they are result of the time inconsistency problem.

The numerical exercise unveils another interesting property of the political business cycles generated by the model: the volatility associated with the political business cycle increases with the severity of the time inconsistency problem.<sup>19</sup> Note that before I established that

<sup>18</sup>Intertemporal discount rates are shown at the annual frequency.

<sup>19</sup>This feature was robust across several alternative parametrizations and functional forms.

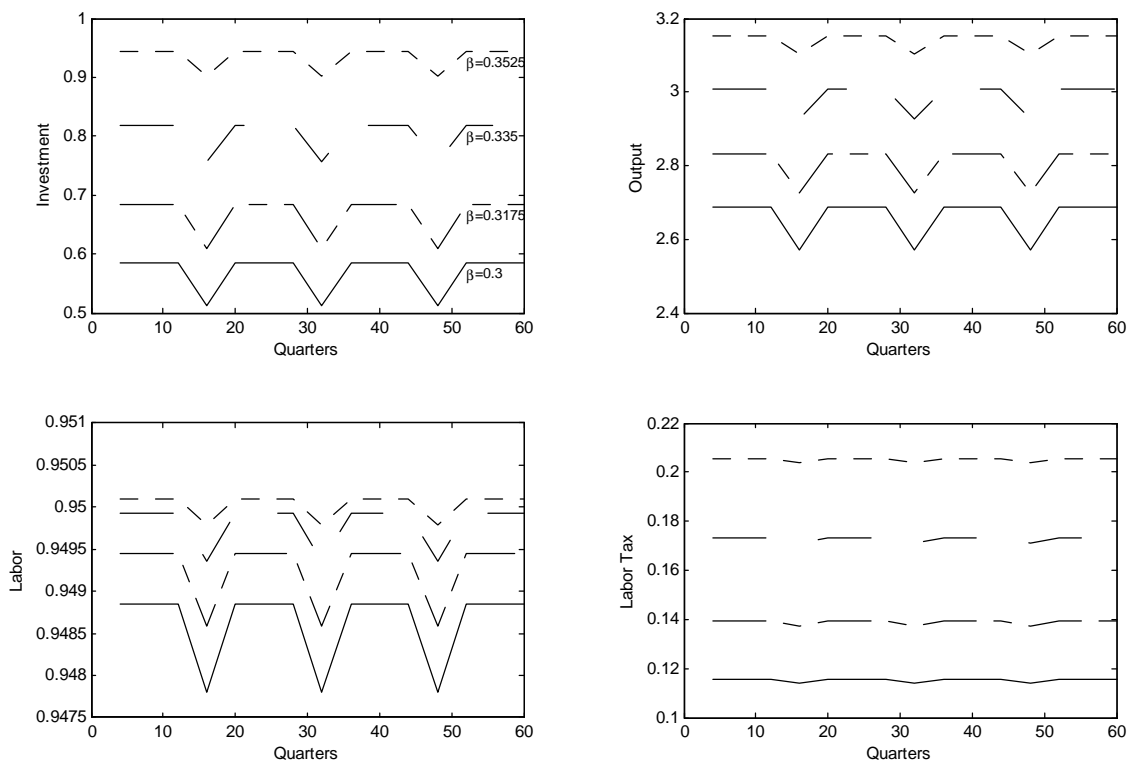


Figure 3: Recursive Politico-Economic Equilibrium Dynamics, Several Intertemporal Discount Rates

<i>Intertemporal Discount Rate</i>	<i>Average Investment</i>	<i>Std.Desv.Output</i>
$\beta = 0.31$	0.66	0.0462
$\beta = 0.33$	0.8	0.0344
$\beta = 0.35$	0.93	0.0207
$\beta = 0.37$	1	0

Table 2: **Average Investment and Output Volatility**

the likelihood of cycles was increasing with political instability. Numerical results suggest that the magnitude of the cycles also increases. Table 2 summarizes the average investment upon total wealth and the output standard deviation for different annual intertemporal discount rates. Even when most of the wealth is invested in average ( 93% ) the volatility is considerable, an output standard deviation slightly above 0.02.

Again, a note of caution should be included to interpret magnitudes. Careful calibration of the political process is not possible, as politics is everything but simple enough to be captured by few parameters. A more appropriate representation of the political process would be a combination of the *iid* economy and the electoral cycle feature. Moreover, there are several shortcomings. The absence of government debt is specially significant.

## 6 Conclusions

This paper started by exploring the implications of wealth heterogeneity for time consistent fiscal policy. It has proved not to be a trivial extension. In particular, equity considerations are shown to play a key role for the determination of the time inconsistency problem and thus the efficiency of fiscal policy. In absence of commitment, it is misleading to abstract from equity considerations even if the focus is only on policy efficiency.

Then I show how a simple political economy model can sustain the second best fiscal policy in equilibrium under the assumption of representative democracy and rational voters. The link between the time inconsistency problem and the political decision process is a step forward in the politico-economic analysis in several dimensions.

First, I do not assume any exogenous commitment ability in the policy decision. The commitment needed is embedded in the rules of the political process. It is worth to note that constitutions often govern political institutions but rarely fiscal policy.

Second, the political process is modeled after representative democracy rather than direct democracy. This is not only a better representation of the actual political process in most democracies, but it allows to drop any exogenous restriction on the set of policy instruments available. Note that the time inconsistency problem can be solved without leaving the set of benevolent policymakers, with preferences defined for any policy problem.

Finally, the theory implies that benevolent policymakers can be ranked in terms of policy efficiency. This helps to explain why there is fewer redistribution than explained by a pure

median voter model.

An interesting question is whether the same rationale presented here applies to the time inconsistency problem in monetary policy. A first consideration is that the redistributive consequences of monetary policy are not that straightforward as for factor taxation. Recall it is necessary that no ex-post deviation from the optimal fiscal policy is Pareto superior. This condition is satisfied naturally in the context of fiscal policy once there is enough wealth heterogeneity. Interestingly, the extensive discussion of independent central bank and explicit targets has not translated into similar proposals for fiscal policy.

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## A Fiscal Policy Model

Without loss of generality (given Proposition 1), the  $\lambda$ -Ramsey policy can be characterized by solving

$$\max_{\tau} \int_I \lambda_i u(c_i, n_i) di \quad (R-P)$$

subject to

$$\begin{aligned} \tau^k &\leq \frac{R-1}{R} \\ \tau^n &\leq 1 \\ g &\leq \tau^k R w + \tau^n n \end{aligned}$$

and  $c_i = c(\omega_i, \tau^n)$  and  $n_i = n(\omega_i, \tau^n)$  as defined in the main text. The arbitrage condition (4) imposes the constraint that  $k = w$  if and only if  $\tau^k \leq \frac{R-1}{R}$ .

Following the exercise in the main text, one can think of the  $\lambda$ -Markov policy problem. It differs from the  $\lambda$ -Ramsey policy because it takes the private sector investment decisions as given and the arbitrage condition (4) does not impose any constraint in the capital tax.

Formally, the  $\lambda$ -Markov policy problem, given any  $\{k_i\}_I$ , can be characterized by solving

$$\max_{\tau} \int_I \lambda_i u(c_i, n_i) di \quad (M-P)$$

subject to

$$\begin{aligned} \tau^k &\leq 1 \\ \tau^n &\leq 1 \\ g &\leq \tau^k R k + \tau^n n \end{aligned}$$

and  $c_i = c(\omega_i, \tau^n)$  and  $n_i = n(\omega_i, \tau^n)$ .

Expression (6) in the main text is obtained by combining the first order conditions associated with  $M-P$  when  $k = w$ . Note that for any  $\{k_i\}_I$ ,

$$\begin{aligned} \frac{\partial \omega_i}{\partial \tau^k} &= -R k_i \\ c_i^\omega &= 1 + (1 - \tau^n) n_i^\omega \\ c_i^\tau &= -n_i + (1 - \tau^n) n_i^\tau \end{aligned}$$

where  $x^y$  is the derivative of variable  $x$  with respect  $y$ . The second and third expressions are derived from the household budget constraint.

The first order condition with respect to  $\tau^k$  is

$$-R \int \lambda_i k_i u_i^c di + \gamma R \left( k - \tau^n \int k_i n_i^\omega di \right) \geq 0$$

with strict equality if  $\tau^k < 1$ . The Lagrange multiplier for the government budget constraint is denoted by  $\gamma$ . Assuming  $k > 0$  (otherwise the problem is trivial),

$$\int \lambda_i \left( \frac{k_i}{k} \right) u_i^c di \leq \gamma \left( 1 - \tau^n \int \left( \frac{k_i}{k} \right) n_i^\omega di \right)$$

The first order condition with respect to  $\tau^n$  is

$$\gamma \left( n + \tau^n \int n_i^\tau di \right) \leq \int \lambda_i u_i^c n_i di$$

since  $n > 0$ ,

$$\gamma \left( 1 + \tau^n \int \left( \frac{n_i^\tau}{n} \right) di \right) \leq \int \lambda_i u_i^c \left( \frac{n_i}{n} \right) di$$

Using the following notation

$$\begin{aligned} n^\tau &= \int \left( \frac{n_i^\tau}{n} \right) di \\ n^\omega &= \int \left( \frac{k_i}{k} \right) n_i^\omega di \end{aligned}$$

and assuming an interior solution for  $\tau^k$  and  $\tau^n$ ,

$$\begin{aligned} \int \lambda_i \left( \frac{k_i}{k} \right) u_i^c di &= \gamma (1 - \tau^n n^\omega) \\ \int \lambda_i \left( \frac{n_i}{n} \right) u_i^c di &= \gamma (1 + \tau^n n^\tau) \end{aligned}$$

and hence

$$D^\lambda (\{k_i\}) = \int \lambda_i u_i^c \left( \frac{1}{\psi} \frac{n_i}{n} - \frac{k_i}{k} \right) = 0 \quad (14)$$

where  $\psi = \frac{1+\tau^n n^\tau}{1-\tau^n n^\omega}$ , and expressions for  $D^i$ ,  $D^{ave}$  and  $D$  in the main text follow by substituting  $k_i = w_i$ .

Note how problems  $R-P$  and  $M-P$  in the case  $k = w$  only differ on the constraint on the capital tax. In short,  $R-P$  is a constrained version of  $M-P$  when  $k = w$ . The key observation is that for some policymaker type  $\lambda'$ , the solution to  $M-P$  when  $k = w$  satisfies the constraint

in  $R-P$ . Hence, the same policy solves the constrained and unconstrained version. Or, in other words, the  $\lambda'$ -Ramsey policy is shown to  $\lambda'$ -time consistent as it is the solution to  $M-P$  when  $k = w$  as well.

Proofs of propositions 2-4 are based in this insight.

**Proof of Proposition 2.** If for any  $\lambda$ , the solution to problem  $R-P$  satisfies  $\tau^k < \frac{R-1}{R}$ , then it solves problem  $M-P$  with  $k = w$  as well as policymaker preferences are assumed to be single-peaked. Hence the  $\lambda$ -Ramsey policy constitutes a  $\lambda$ -Markov equilibrium. ■

**Proof of Proposition 3.** Take  $R-P$  for  $\lambda$  such that the  $\lambda$ -Ramsey policy holds  $\tau^k = \frac{R-1}{R}$ . With  $D^\lambda$  given by (14), the necessary first order condition is

$$D^\lambda(\{w_i\}) = \theta$$

with  $\theta \geq 0$  being the Lagrangian multiplier associated with restriction  $\tau^k \leq \frac{R-1}{R}$ . If a set  $\tilde{I}(\delta) = \{i : w_i \geq \delta w\}$  has positive measure for an arbitrarily large  $\delta > 0$ , then it exists  $\lambda^*$  such that  $D^{\lambda^*}(\{w_i\}) = 0$ , since  $D_i < 0$  for all  $i \in \tilde{I}(\delta)$ . Because of the single-peaked property and the continuity of problem  $R-P$ , for arbitrarily close  $\lambda$  and  $\lambda'$  the  $\lambda$ - and  $\lambda'$ -Ramsey policies are arbitrarily close as well. Hence it is possible to establish a sequence of policymaker types  $\tilde{\lambda}_t$  from  $\lambda$  to  $\lambda^*$  such that  $\tau^k = \frac{R-1}{R}$  is the  $\lambda_t$ -Ramsey policy. Note that one does not need to assume the sufficiency of first order conditions to conclude that  $\tau^k = \frac{R-1}{R}$  must solve the  $\lambda^*$ -Ramsey problem.

Finally, following the same reasoning than in the proof of proposition 2,  $\tau^k = \frac{R-1}{R}$  also solves problem  $M-P$  for  $k = w$  under  $\lambda^*$  and hence  $\tau^k = \frac{R-1}{R}$  is  $\lambda^*$ -time consistent. ■

**Proof of Proposition 4.** Note that for all  $\lambda$  more progressive than  $\lambda^*$ ,  $D^\lambda(\{w_i\}) > 0$  as set  $\tilde{I}(\delta)$  is the upper tail of the endowment distribution and necessarily

$$\int_{\tilde{I}(\delta)} \lambda_i di < \int_{\tilde{I}(\delta)} \lambda_i^* di$$

This implies that  $\tau^k = \frac{R-1}{R}$  solves both the  $\lambda$ -Ramsey and  $\lambda^*$ -Ramsey policy problems. Note that for all  $\lambda$  such that  $D^\lambda(\{w_i\}) = \theta > 0$  at  $\tau^k = \frac{R-1}{R}$ , the  $\lambda$ -Ramsey policy would not be  $\lambda$ -time consistent, as  $D^\lambda(\{w_i\}) \neq 0$ , violating the necessary first order condition for problem  $M-P$  when  $k = w$ . ■