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# **Understanding Mortgage Spreads**

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#### Abstract

Most mortgages in the U.S. are securitized in agency mortgage-backed securities (MBS). Yield spreads on these securities are thus a key determinant of homeowners' funding costs. We study variation in MBS spreads over time and across securities, and document a cross-sectional smile pattern in MBS spreads with respect to the securities' coupon rates. We propose non-interest- rate prepayment risk as a candidate driver of MBS spread variation and present a new pricing model that uses "stripped" MBS prices to identify the contribution of this prepayment risk to the spread. The pricing model finds that the smile can be explained by prepayment risk, while the time-series variation is mostly accounted for by a non-prepayment risk factor that co-moves with MBS supply and credit risk in other fixed income markets. We use the pricing model to study the MBS market response to the Fed's large-scale asset purchase program and to interpret the post-announcement divergence of spreads across MBS.

Key words: agency mortgage-backed securities, option-adjusted spreads, prepayment risk, OAS smile

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"Whoever bought the bonds [...] couldn't be certain how long the loan lasted. If an entire neighborhood moved (paying off its mortgages), the bondholder, who had thought he owned a thirty-year mortgage bond, found himself sitting on a pile of cash instead. More likely, interest rates fell, and the entire neighborhood refinanced its thirty-year fixed rate mortgages at the lower rates. [...] In other words, money invested in mortgage bonds is normally returned at the worst possible time for the lender." — Michael Lewis, Liar's Poker, Chapter 5

## 1 Introduction

At the peak of the financial crisis in the fall of 2008, spreads on residential mortgage-backed securities (MBS) guaranteed by the U.S. government-sponsored enterprises Fannie Mae and Freddie Mac and the government agency Ginnie Mae spiked to historical highs. In response, the Federal Reserve announced that it would purchase MBS in large quantities to "reduce the cost and increase the availability of credit for the purchase of houses."<sup>1</sup> Mortgage rates for U.S. homeowners reflect MBS spread variation as most mortgage loans are securitized. After the announcement, spreads on lower-coupon MBS declined sharply, consistent with the program's objective; however, spreads on higher-coupon MBS widened. This paper shows that the differential response can be explained with an MBS pricing model that features multiple sources of risk. We first characterize the timeseries and cross-sectional variation of MBS spreads in a 15-year sample, and then present a method to disentangle contributions of different risk factors to variation in MBS spreads.

Credit risk of MBS is limited because of the explicit (for Ginnie Mae) or implicit (for Fannie Mae and Freddie Mac) guarantee by the U.S. government. However, MBS investors are uniquely exposed to uncertainty about the timing of cash flows, as exemplified by the quote above. U.S. mortgage borrowers can prepay the loan balance at any time without penalty, and do so especially as rates drop. The price appreciation from rate declines is thus limited as MBS investors are short borrowers' prepayment option. Yields on MBS exceed those on Treasuries or interest rate swaps to compensate investors for this optionality. But even after accounting for the option cost associated with interest rate variability, the remaining *option-adjusted spread* (OAS) can be substantial. Since, as shown in the paper, the OAS is equal to a weighted average of future expected excess returns after hedging for interest rate risk, non-zero OAS suggests that MBS prices reflect compensation for additional sources of risk. We decompose these spreads into risks related to shifts in prepayments

<sup>&</sup>lt;sup>1</sup>http://www.federalreserve.gov/newsevents/press/monetary/20081125b.htm. The term "MBS" in this paper refers only to securities issued by Freddie Mac and Fannie Mae or guaranteed by Ginnie Mae (often called "agency MBS") and backed by residential properties; according to SIFMA, as of 2013:Q4 agency MBS totaled about \$6 trillion in principal outstanding. Other securitized assets backed by real estate property include "private-label" residential MBS issued by private firms (and backed by subprime, Alt-A, or jumbo loans), as well as commercial MBS.

that are not driven by interest rates alone, and a component related to non-prepayment risk factors such as liquidity.

To measure risk premia in MBS, we construct an OAS measure based on surveys of investors' prepayment expectations, and also study spreads collected from six different dealers over a period of 15 years. In both cases, we find that, in the time series, the OAS (to swaps) on a market value-weighted index is typically close to zero but reaches high levels in periods of market stress, such as 1998 (failure of Long-Term Capital Management) or the fall of 2008. We also document important cross-sectional variation in the OAS. At any point in time, MBS with different coupons trade in the market, reflecting disparate rates for mortgages underlying each security. We group MBS according to their "moneyness," or the difference between the rate on the loans in the MBS and current mortgage rates, which is a key distinguishing feature as it determines borrowers' incentive to prepay their loans. In this cross section we uncover an "OAS smile": spreads tend to be lowest for securities for which the prepayment option is at-the-money (ATM), and increase if the option moves out-of-the-money (OTM) or in-the-money (ITM). A similar smile pattern also holds in hedged MBS returns.<sup>2</sup>

The OAS smile suggests that investors in MBS earn risk compensation for factors other than interest rates; in particular, these may include other important systematic drivers of prepayments, such as house prices, underwriting standards, and government policies. While the OAS accounts for the expected path of these non-interest-rate factors, it may still reflect risk premia associated with them, because prepayments are projected under a physical, rather than the risk-neutral, measure. These risk premia, which we refer to as "prepayment risk premia," cannot be directly measured because market instruments that price these individual factors are typically not available.<sup>3</sup>

While prepayment risk premia may give rise to the OAS smile, risk factors unrelated to prepayment, such as liquidity or changes in the perceived strength of the government guarantee, could also lead to such a pattern. For example, newly issued MBS, which are ATM and more heavily traded, could command a lower OAS due to better liquidity. Without strong assumptions on the liquidity component, prices of standard MBS (which pass through both principal and interest payments) are insufficient to isolate prepayment risk premia in the OAS. Instead, we propose a new approach based on "stripped" MBS that pass through only interest payments (an "IO" strip) or

<sup>&</sup>lt;sup>2</sup>Correspondingly, a pure long strategy in deeply ITM MBS earns a Sharpe ratio of about 1.9 in our sample, as compared to about 0.7 for a long-ATM strategy. We also show that OAS predict future realized returns, and that realized returns are related to movements in moneyness in a way consistent with the OAS smile.

<sup>&</sup>lt;sup>3</sup>Importantly, in our usage, "prepayment risk" does not reflect prepayment variation due to interest rates; instead it is the risk of over- or underpredicting prepayments for given rates.

principal payments (a "PO" strip). The additional information provided by separate prices for these strips on a given loan pool, together with the assumption that a pair of strips is fairly valued relative to each other, allows us to identify market-implied risk-neutral ("Q") prepayment rates as multiples of physical ("P") ones. We refer to the remaining OAS when using the Q-prepayment rates as  $OAS^Q$ , while the difference between the standard OAS and  $OAS^Q$  measures a security's prepayment risk premium.

Our pricing model finds that the OAS smile is explained by higher prepayment risk premia for securities that are OTM and, especially, ITM. There is little evidence that liquidity or other non-prepayment risks vary significantly with moneyness, except perhaps for the most deeply ITM securities. In the time series, instead, we document that much of the OAS variation on a valueweighted index is driven by the  $OAS^Q$  component. We show that  $OAS^Q$  on the index is related to spreads on other agency debt securities, which may reflect shared risk factors such as changes in the implicit government guarantee or liquidity. Even after controlling for agency debt spreads, OAS are strongly correlated with credit spreads (Baa-Aaa). Given the different sources of risk in the two markets, this finding may suggest the existence of a common marginal investor in corporates and MBS that exhibits time-varying risk aversion, such as an intermediary subject to time-varying risk constraints (for example, Shleifer and Vishny, 1997; He and Krishnamurthy, 2013). Consistently, we find that a measure of supply of MBS (based on new issuance) is also positively related to  $OAS^Q$ .

The OAS response to the Fed's large-scale asset purchase (LSAP) announcement in November 2008 provides further evidence in line with our findings. According to our model, the OAS<sup>Q</sup> fell across coupons, as investors anticipated that the Fed would absorb much of the near-term MBS supply, thereby relieving private balance sheet constraints. The divergence in OAS across coupons was driven by higher-coupon securities' prepayment risk premia increasing as these securities moved further ITM, reflecting the more general smile pattern.

**Related literature.** Several papers have studied the interaction of interest rate risk between MBS and other markets. This literature finds that investors' need to hedge MBS convexity risk may explain significant variation in interest rate volatility and excess returns on Treasuries (Duarte, 2008; Hanson, 2014; Malkhozov et al., 2016). Our analysis is complementary to this work as we focus on MBS-specific risks and how they respond to changes in other fixed income markets. Closer to this paper, Boudoukh et al. (1997) suggest that prepayment-related risks are a likely candidate for the

component of MBS prices unexplained by the variation in the interest rate level and slope. Carlin et al. (2014) use long-run prepayment projections from surveys, which we also employ, to study the role of disagreement in MBS returns and their volatility.<sup>4</sup>

Gabaix et al. (2007) study OAS on IO strips from a dealer model between 1993 and 1998, and document that these spreads covary with the moneyness of the market, a fact that they show to be consistent with a prepayment risk premium and the existence of specialized MBS investors. Gabaix et al. do not focus on pass-through MBS and, while their conceptual framework successfully explains the OAS patterns of the IOs in their sample, it predicts a linear, rather than a smile-shaped, relation between a pass-through MBS's OAS and its moneyness, since they assume that securities have a constant loading on a single-factor aggregate prepayment shock. We show that the OAS smile is in fact a result of prepayment risk but of a more general form, while also allowing for liquidity or other non-prepayment risk factors to affect OAS. Similarly to this paper's empirical pricing model, Levin and Davidson (2005) extract a market-implied prepayment function from the cross section of pass-through securities.<sup>5</sup> Because they assume, however, that the residual risk premia in the OAS are constant across coupons, the OAS smile in their framework can only be explained by prepayment risk and not liquidity. By using additional information from stripped MBS, this paper relaxes this assumption. Furthermore, we provide a characterization of spread patterns over a long sample period and study risk premia covariates.

Two interesting papers subsequent to this work also emphasize the importance of prepayment risk for the cross section of MBS. Chernov et al. (2016) estimate parameters of a simple prepayment function from prices on pass-through MBS. Consistent with our results, they find an important role for a credit/liquidity spread (assumed constant in the cross section) in explaining price variation over time. In terms of prepayment risk, their model implies a dominant role for risks related to turnover independent of refinancing incentives, rather than risks related to refinancing activity of ITM borrowers. Diep et al. (2017) study the cross section of realized MBS excess returns. As in this paper, they find evidence of a smile pattern in their pooled data, with ATM pools earning relatively lower excess returns. However, they argue that different conditional patterns of returns exist

<sup>&</sup>lt;sup>4</sup>Song and Zhu (2016) and Kitsul and Ochoa (2016) study determinants of financing rates implied by MBS dollar rolls, which are generally affected by liquidity, prepayment and adverse selection risks. Dollar rolls are matched purchases/sales of MBS contracts settling in two subsequent months. While implied financing rates partly reflect MBS liquidity, their calculation relies on prepayment rate expectation under the physical measure and therefore should also incorporate prepayment risk premia as discussed in this paper. Furthermore, as Song and Zhu (2016) emphasize, dollar rolls are strongly affected by adverse selection risk.

<sup>&</sup>lt;sup>5</sup>Cheyette (1996) and Cohler et al. (1997) are earlier practitioner papers proposing that MBS prices can be used to obtain market-implied prepayments.

depending on whether the MBS market as a whole is ITM or OTM, suggesting that prepayment risk premia change sign with market moneyness. As we show in Section 3, the smile pattern in expected excess returns as measured by OAS holds irrespective of market type, and cross-sectional patterns in returns are also consistent with the smile when we study their relation with changes in mortgage rates (and thus moneyness).

## 2 Background on agency MBS

This section overviews the agency MBS market (or, for brevity, MBS market), one of the largest and most liquid fixed income markets in the world. In an agency securitization, a mortgage originator pools loans and then delivers the pool to Fannie Mae or Freddie Mac in exchange for an MBS certificate, which can be subsequently sold to investors in the secondary market.<sup>6</sup> Servicers, which are often affiliated with the loan originator, collect payments from homeowners that are passed on to MBS holders after deducting a servicing fee and the agency guarantee fee. In a standard MBS, also known as a pass-through, homeowners' payments (interest and principal) are assigned prorata to all investors. However, cash flow assignment rules can be more complicated with multiple tranches, as is the case for stripped MBS that separate interest and principal payments. We focus on MBS backed by fixed-rate mortgages (FRMs) with original maturities of 30 years on 1-4 family properties, which account for more than two-thirds of all MBS.

In MBS, the risk of default of the underlying mortgages is not borne by investors but by the agencies that guarantee timely repayment of principal and interest. Because of this guarantee, MBS are generally perceived as free of credit risk. While Ginnie Mae securities have the full faith and credit of the U.S. government, assessing credit risk of Fannie Mae and Freddie Mac securities is more complex. Government backing for these securities is only implicit and results from investors' anticipation of government support under a severe stress scenario, as when Fannie Mae and Freddie Mac were placed in federal conservatorships in September 2008.<sup>7</sup>

The bulk of MBS trading occurs in the to-be-announced (TBA) market. The TBA market is a forward market for pass-through MBS where a seller and buyer agree on a number of characteristics of the securities to be delivered (issuer, maturity, coupon, par amount), a transaction price,

<sup>&</sup>lt;sup>6</sup>In addition to these "lender swap" transactions, Fannie Mae and Freddie Mac also conduct "whole loan conduit" transactions, in which the agencies buy loans against cash from (typically smaller) originators, pool these loans themselves, and then market the issued MBS.

<sup>&</sup>lt;sup>7</sup>The conservatorships have resulted in an effective government guarantee of Fannie and Freddie securities since September 2008, but (at least in principle) this guarantee is still temporary and thus not as strong as the one underlying Ginnie Mae securities.

and a settlement date either 1, 2, or 3 months in the future (Vickery and Wright, 2013). The precise securities that are delivered are only announced 48 hours prior to settlement, and are chosen on a "cheapest-to-deliver" basis. Aside from the TBA market, trading also occurs for "specified pools," either for securities that are not TBA-eligible or for ones that are especially valuable.

MBS with different coupons coexist primarily because of disparate loan rates of the mortgages underlying an MBS, resulting from different loan origination dates or other factors such as "points" paid (or received) by the borrowers. The benchmark TBA contract is the "current coupon," a synthetic 30-year fixed-rate MBS obtained by interpolating the highest coupon below par and the lowest coupon above par.<sup>8</sup> While newly originated mortgages are securitized in coupons trading close to par, the current coupon is not representative of the market as a whole because only a small share of all outstanding MBS trades close to par. For example, as shown in Figure 1, the current coupon at the end of 2010 was around 4% (red line, measured on the right y-axis) but securities with a 4% coupon were only about 20% of the total outstanding balance. Rather than focusing on the current coupon, below we compute value-weighted spread averages for the MBS universe and study the cross section of all outstanding traded MBS coupons.<sup>9</sup>

A distinct feature of MBS is the embedded prepayment option: borrowers can prepay the loan balance at par at any time without paying a fee to the current holder of the loan. Borrowers' prepayments are uncertain and depend on changes in their characteristics (e.g. credit scores) and macroeconomic factors such as house prices and, most importantly, interest rates. Borrowers tend to refinance and prepay their mortgages when available rates on new mortgages are below the coupon rate they are currently paying. MBS investors, which are short the borrower's American prepayment option, therefore have limited upside as rates decline. Pricing this embedded option is crucial in the valuation of MBS, as discussed in more detail next.

## **3** Spreads and returns in the MBS market

In this section, we provide formal descriptions and definitions of MBS cash flows, prices, and yield premia (or spreads). In particular, we define the option-adjusted spread (OAS) and a prepayment-risk-neutral version of it (OAS<sup>Q</sup>), and characterize what risk premia they represent. We then pro-

<sup>&</sup>lt;sup>8</sup>Alternatively, it is obtained by extrapolating from the lowest coupon above par in case no coupon is trading below par (which has frequently been the case in recent years). Sometimes the term "current coupon" is used for the actual coupon trading just above par; we prefer the term "production coupon" to refer to that security.

<sup>&</sup>lt;sup>9</sup>Another limitation of the current coupon is that since it is a synthetic contract, variation in its yield or spreads can be noisy because of inter- and extrapolations from other contracts and the required assumptions about the characteristics of loans that would be delivered in a pool trading at par (Fuster et al., 2013).

vide stylized facts about OAS and relate them to MBS returns, both theoretically and empirically.

#### 3.1 MBS cash flows, prices, and spreads

Consider a complete probability space  $(\Omega, \mathcal{F}, \mathbb{P})$ , where  $\Omega$  is the state space,  $\mathcal{F}$  is the  $\sigma$ -algebra of observable events and  $\mathbb{P}$  is a probability measure assigning probabilities to events. The uncertainty in the economy arises from three sources:  $N_r$  factors  $\rho_t$  that drive the evolution of interest rates in the economy,  $N_{\gamma}$  non-interest rate factors  $\gamma_t$  that impact prepayments, and a factor  $l_t$  that affects MBS market liquidity.

For concreteness, we assume that the  $N_r$  interest rate factors evolve according to a vector Brownian motion with (potentially) stochastic drift and stochastic volatility:

$$d\rho_t = \mu_{rt}dt + \sigma_{rt}dZ_{rt},\tag{3.1}$$

where  $Z_{rt}$  is an  $N_r$ -dimensional Brownian motion under the physical probability measure  $\mathbb{P}$ , and  $\mu_{rt}$  and  $\sigma_{rt}$  are a scalar- and a vector-valued function of  $\rho_t$ , respectively. The evolution of the interest rate factors (3.1) is fairly general and includes, as special cases, traditional affine term structure models (e.g. Dai and Singleton, 2000) and the Markovian version of the Heath, Jarrow, and Morton (1992) model employed in the empirical pricing model of Section 4. Under a Markovian factor evolution, the yield  $y_t^{(n)}$  on a risk-free bond with time-to-maturity n only depends on current rate factor realizations:  $y_t^{(n)} = y(\rho_t, n)$ . Below, the instantaneous risk-free rate is also denoted by  $r_t = y_t^{(0)}$ .

Similarly, non-interest-rate prepayment factors  $\gamma_t$  evolve under  $\mathbb{P}$  according to a Brownian motion with (potentially) stochastic drift and stochastic volatility:

$$d\gamma_t = \mu_{\gamma t} dt + \sigma_{\gamma t} dZ_{\gamma t}, \tag{3.2}$$

where  $Z_{\gamma t}$  is an  $N_{\gamma}$ -dimensional standard Brownian motion under  $\mathbb{P}$  uncorrelated with  $Z_{rt}$ , or  $\mathbb{E}_t \left[ dZ_{rt} dZ'_{\gamma t} \right] = 0$ , and  $\sigma_{\gamma t}$  are a scalar- and a vector-valued function of  $(\rho_t, \gamma_t)$ , respectively. For example,  $\gamma_t$  could reflect the strength of the housing market that may affect prepayments due to household relocations. Finally, the liquidity factor  $l_t$  follows a Poisson jump process with jump intensity  $\mu_t$ , counting process  $dJ_t$  and compensated jump process  $dN_t = dJ_t - \mu_t dt$ . When a jump is realized, each MBS security loses a (security-specific) fraction  $\alpha$  of its market value. For example,  $\alpha$  could represent the price impact of an MBS market liquidity shock (as in Acharya and Pedersen, 2005) or a (perceived) weakening of the agency guarantee.

Consider a mortgage pool with original balance of 1, maturity date T and coupon rate c. Let  $\tilde{c}$ 

be the fixed mortgage rate paid by the mortgagor, which is about 50 bps higher than *c* in order to cover servicing and guarantee fees. As shown in Appendix A, the remaining principal balance  $\theta_t$  at time *t* is:

$$\theta_t = e^{-\int_0^t s_u^* du} \times \frac{e^{\tilde{c}T} - e^{\tilde{c}t}}{e^{\tilde{c}T} - 1}.$$

where  $s_t^*$  represents instantaneous unscheduled principal payments. Unscheduled payments  $s_t^*$  depend on a borrower's refinancing incentive  $\tilde{c} - y_t^{(T)}$ , where  $y_t^{(T)}$  is the interest rate on new loans, and on the vector of non-interest risk factors  $\gamma_t$ :

$$s_t^* = f\left(\gamma_t, \tilde{c}_j - y_t^{(T)}\right).$$

In this section, we consider standard pass-through MBS that pay out cash flows,  $dX_t$ , equal to all principal,  $-d\theta_t$ , and interest payments net of fees,  $c\theta_t dt$ :  $dX_t = c\theta_t dt - d\theta_t$ . As shown in Appendix A, the evolution of the principal balance  $d\theta_t$  is locally-deterministic, so  $d\theta_t = -\mu_{\theta t} dt$ , where  $\mu_{\theta t}$  is known at time *t*. Then the cash flows to the pass-through are also locally-deterministic, with  $dX_t = \mu_{Xt} dt$  and  $\mu_{Xt}$  known at time *t*.

Below, we will use different risk-neutral measures to define OAS and  $OAS^{\mathbb{Q}}$ . To streamline the exposition, we consider a generic measure  $\mathbb{R}$  that is absolutely continuous with respect to the physical probability measure  $\mathbb{P}$ . Under no-arbitrage, there exists a pricing kernel  $M_t^{\mathbb{R}}$  under  $\mathbb{R}$ such that the time *t* price of the pass-through security is given by

$$P_{t} = \mathbb{E}_{t}^{\mathbb{R}} \left[ \int_{0}^{T-t} \frac{M_{t+s}^{\mathbb{R}}}{M_{t}^{\mathbb{R}}} (1-\alpha)^{\int_{0}^{s} dJ_{t+u}} \mu_{X,t+s} ds \right].$$
(3.3)

Since the pass-through security loses a fraction  $\alpha$  of its market value every time that a market liquidity event occurs,  $(1 - \alpha)^{\int_0^s dJ_{t+u}}$  is the fraction of market value that remains after  $\int_0^s dJ_{t+u}$  liquidity events occur between time *t* and time *t* + *s*. Under  $\mathbb{R}$ , the risk-free rate, *r*<sub>t</sub>, is

$$r_t dt = -\mathbb{E}_t^{\mathbb{R}}\left[\frac{dM_t^{\mathbb{R}}}{M_t^{\mathbb{R}}}
ight]$$

and an investor in the pass-through security earns an instantaneous excess return

$$dr x_t^{\mathbb{R}} = \frac{dP_t^{\mathbb{R}} + \mu_{Xt} dt}{P_{t^-}} - r_t dt, \qquad (3.4)$$

where  $P_{t-}$  is the price the instant before the uncertainty about a jump occurring at date *t* is resolved.

Under no arbitrage, excess returns satisfy the fundamental asset pricing equation:

$$\mathbb{E}_{t}^{\mathbb{R}}\left[drx_{t}^{\mathbb{R}}\right] \equiv rp_{t}^{\mathbb{R}}dt = -\mathbb{E}_{t}^{\mathbb{R}}\left[\frac{dM_{t}^{\mathbb{R}}}{M_{t}^{\mathbb{R}}}drx_{t}^{\mathbb{R}}\right];$$
(3.5)

that is, the expected excess return compensates an investor in the MBS for decreases in cash flows that occur when the equilibrium marginal discount rate is high. Iterating (3.4) forward, the price of the pass-through is

$$P_t = \mathbb{E}_t^{\mathbb{R}} \left[ \int_0^{T-t} \exp\left( -\int_0^s \left( r_{t+u} du + dr x_{t+u}^{\mathbb{R}} \right) \right) (1-\alpha)^{\int_0^s dJ_{t+u}} \mu_{X,t+s} ds \right].$$
(3.6)

MBS are often valued in terms of a yield premium, or spread, which is a parallel shift in the riskfree yield curve that equalizes the market price and the expected discounted value of cash flows:

**Definition 3.1** ( $yp^{\mathbb{R}}$ ). The yield premium on a mortgage-backed security under measure  $\mathbb{R}$ ,  $yp_t^{\mathbb{R}}$ , at time t is implicitly defined as

$$P_t = \mathbb{E}_t^{\mathbb{R}} \left[ \int_0^{T-t} \exp\left( -\int_0^s \left( r_{t+u} + y p_t^{\mathbb{R}} \right) du \right) (1-\alpha)^{\int_0^s dJ_{t+u}} \mu_{X,t+s} ds \right].$$
(3.7)

This is the continuous-time analog of a standard spread measure used in industry (see Hayre, 2001, for example), where the cash flows that accrue to an investor include the payments made into the pass-through,  $\mu_{Xt}dt$ , but also allow for losses of market value due to liquidity shocks,  $\alpha dJ_t$ . The yield premium is increasing in the discounted value of cash flows relative to the market price, so that an MBS trading below the model price under  $\mathbb{R}$  has a positive yield premium.

The standard OAS is constructed under an interest-rate-risk-neutral measure  $Q_r$ , which differs from the usual asset pricing (fully) risk-neutral measure Q. Crucial to our analysis is also the interest-rate-and-prepayment-risk neutral measure  $Q_{r,\gamma}$ . The three measures (summarized in Table 1) are all absolutely continuous with respect to the physical probability measure  $\mathbb{P}$  and are distinguished by the subset of securities having martingale discounted gains processes under each measure.<sup>10</sup> To define the three measures, consider the price  $P_{\chi t}$  of a claim to generic cash flows

$$d\chi_t = \mu_{\chi t} dt + \sigma_{\chi r, t} dZ_{rt} + \sigma_{\chi \gamma, t} dZ_{\gamma t},$$

$$dG_{\chi t}^{\mathbb{R}} = d\left(\exp\left(-\int_{0}^{t} r_{u} du\right) P_{\chi t}\right) + \exp\left(-\int_{0}^{t} r_{u} du\right) d\chi_{t}^{\mathbb{R}}.$$

<sup>&</sup>lt;sup>10</sup>For a security with price  $P_{\chi t}$  and cash flows  $d\chi_t$ , the discounted gains process  $dG_{\chi t}^{\mathbb{R}}$  under measure  $\mathbb{R}$  is (Duffie, 2010a)

that loses a fraction  $\alpha_{\chi}$  of market value each time a liquidity shock if realized. Under the standard risk-neutral measure  $\mathbb{Q}$ , the discounted gains process is a martingale, the yield premium is zero  $(yp_t^{\mathbb{Q}} \equiv 0)$ , and the price  $P_{\chi t}$  of a stream of cash flows  $\{d\chi_{t+s}\}_{s\geq 0}$  that has exposure  $\alpha_{\chi}$  to the liquidity shock is given by

$$P_{\chi t} = \mathbb{E}_{t}^{\mathbb{Q}} \left[ \int_{0}^{T-t} \exp\left( -\int_{0}^{s} r_{t+u} du \right) (1 - \alpha_{\chi})^{\int_{0}^{s} dJ_{t+u}} d\chi_{t+s} \right].$$
(3.8)

In practice, MBS investors actively hedge interest rate risk but are exposed to liquidity risk and non-interest factors that affect prepayments, such as changes in lending standards, house prices and government policies. As a result, MBS are typically valued in terms of the option-adjusted spread (OAS), which is a yield premium that takes into account variability in the future path of interest rates and the resulting variation in prepayment due to interest rates only.<sup>11</sup> The practical reason why the OAS does not incorporate non-interest factors is that market instruments priced off each of these factors are not available; thus, risk premia associated with these factors cannot be measured directly.<sup>12</sup> Formally, the industry-standard OAS is the yield premium under the interest rate-risk-neutral measure  $Q_r$ :

$$yp_t^{\mathbb{Q}_r} \equiv OAS_t.$$

Under the interest-rate-risk-neutral  $Q_r$ , the discounted gains processes of securities not exposed to either prepayment or liquidity risk ( $\mathbb{E}_t [d\chi_t d\gamma_t] = 0$  and  $\alpha_{\chi} = 0$ ) are martingales. The price of such securities is:

$$P_{\chi t} = \mathbb{E}_t^{\mathbb{R}} \left[ \int_0^{T-t} \exp\left( -\int_0^s r_{t+u} du \right) d\chi_{t+s} \right],$$
(3.9)

with  $\mathbb{R} = \mathbb{Q}_r$ . For securities that are exposed to either prepayment or liquidity risk, the discounted gains process is no longer a martingale under  $\mathbb{Q}_r$ , and the price is given by (3.3) for  $\mathbb{R} = \mathbb{Q}_r$ . When liquidity and prepayment risks are priced, the OAS for these securities will be non-zero.

To separate prepayment risk from other risks (such as liquidity), we introduce the rate-and-

<sup>&</sup>lt;sup>11</sup>Earlier versions of this paper also studied spreads that do not account for interest rate volatility (often called zero-volatility spreads, or ZVS) as well as the difference between these spreads and the OAS.

<sup>&</sup>lt;sup>12</sup>House prices are an important factor driving prepayments. While they are themselves dependent in part on interest rates, empirically the link is surprisingly weak (e.g., Kuttner, 2014). Risk premia for the residual component could potentially be measured if there was a liquid market for house price futures; however, despite various efforts this is not the case (see e.g. https://www.inman.com/2012/03/23/market-housing-futures-has-yet-take/; accessed December 1, 2017). There exist other assets that could potentially be used to partially hedge home price risk (e.g. stocks of construction companies), but this is associated with basis risk and complexity.

prepayment-risk-neutral measure  $\mathbb{Q}_{r,\gamma}$ . Under this measure, only the discounted gains processes of securities not exposed to liquidity risk ( $\alpha_{\chi} = 0$ ) are martingales and the price of these securities is given by (3.9) with  $\mathbb{R} = \mathbb{Q}_{r,\gamma}$ . For securities exposed to liquidity risk, the discounted gains process is not a martingale under  $\mathbb{R} = \mathbb{Q}_{r,\gamma}$ , and the price is given by (3.3) with  $\mathbb{R} = \mathbb{Q}_{r,\gamma}$ . To distinguish the yield premium under  $\mathbb{Q}_{r,\gamma}$  from the traditional OAS and to simplify notation, we denote the prepayment-risk-neutral yield premium by  $OAS^{\mathbb{Q}}$  (rather than  $OAS^{\mathbb{Q}_{r,\gamma}}$ ):

$$yp_t^{\mathbb{Q}_{r,\gamma}} \equiv OAS_t^{\mathbb{Q}}$$

### 3.2 Risk premia in OAS and OAS<sup>Q</sup>

Which risk premia do  $OAS_t$  and  $OAS_t^{\mathbb{Q}}$  reflect? To study this, we express the evolution of cash flows to the pass-through in terms of interest and principal payments,  $c\theta_t dt$  and  $-d\theta_t$ . Furthermore, we use of the fact that, for a security with fractional market value recovery in the case of a jump, the jump component modulates the discount rate, given the measure-appropriate jump intensity (e.g., Lando, 2009, Ch. 5.6). Thus, the price of the pass-through is

$$P_{t} = \mathbb{E}_{t}^{\mathbb{R}} \left[ \int_{0}^{T-t} \exp\left( -\int_{0}^{\tau} \left( r_{t+u} + yp_{t}^{\mathbb{R}} + \alpha \mu_{t+u} \pi_{l,t+u}^{\mathbb{R}} \right) du \right) \left( c\theta_{t+\tau} d\tau - d\theta_{t+\tau}^{\mathbb{R}} \right) \right],$$

where  $\mu_t \pi_{lt}^{\mathbb{R}}$  is the intensity of the liquidity jump process under measure  $\mathbb{R}$ , and the evolution of the remaining principal balance,  $d\theta_t^{\mathbb{R}}$ , is also taken under  $\mathbb{R}$ . Integrating by parts and substituting the evolution of the remaining principal balance:

$$P_{t} = \theta_{t} \left\{ 1 + \mathbb{E}_{t}^{\mathbb{R}} \left[ \int_{0}^{T-t} \exp\left( -\int_{0}^{\tau} \left( r_{t+u} + yp_{t}^{\mathbb{R}} + \alpha\mu_{t+u}\pi_{t+u}^{\mathbb{R}} + s_{t+u}^{*} \right) du \right) \times \frac{e^{\tilde{c}T} - e^{\tilde{c}\tau}}{e^{\tilde{c}T} - e^{\tilde{c}t}} \left( c - \left( r_{t+\tau} + yp_{t}^{\mathbb{R}} + \alpha\mu_{t+\tau}\pi_{l,t+\tau}^{\mathbb{R}} \right) \right) d\tau \right] \right\}.$$

To simplify the exposition, we next assume that there are no shocks to the interest-rate factors, thus  $y_t^{(T)} = r_t \equiv r$ . We assume further that the arrival rate of liquidity shocks is constant, so that  $\mu_t \equiv \mu$ , and that the price of liquidity risk is constant as well, so that  $\pi_{lt} \equiv \pi_l$ .<sup>13</sup> Then the price of

<sup>&</sup>lt;sup>13</sup>In Appendix D, we show under more general assumptions that holders of MBS securities are compensated for both prepayment and liquidity risks.

the pass-through simplifies to

$$P_{t} = \theta_{t} \left\{ 1 + \left( c - \left( r + y p_{t}^{\mathbb{R}} + \alpha \mu \pi_{l}^{\mathbb{R}} \right) \right) \int_{0}^{T-t} e^{-\left( r + y p_{t}^{\mathbb{R}} + \alpha \mu \pi_{l}^{\mathbb{R}} \right) \tau} \frac{e^{\tilde{c}T} - e^{\tilde{c}(\tau+t)}}{e^{\tilde{c}T} - e^{\tilde{c}t}} S_{t,\tau}^{\mathbb{R}} d\tau \right\},$$
(3.10)

where  $S_{t,\tau}^{\mathbb{R}}$  is the expectation under  $\mathbb{R}$  of the cumulative unscheduled principal prepayment between dates *t* and *t* +  $\tau$ :

$$S_{t,\tau}^{\mathbb{R}} = \mathbb{E}_{t}^{\mathbb{R}} \left[ \exp \left( - \int_{0}^{\tau} s_{t+u}^{*} du \right) \right].$$

We are now ready to evaluate what the OAS (the yield premium under the interest-rate-riskneutral measure,  $Q_r$ ) and the OAS<sup>Q</sup> (the yield premium under the prepayment-and-interest-raterisk-neutral measure,  $Q_{r,\gamma}$ ) on the pass-through represent. Under the fully risk-neutral measure Q, based on (3.10), the equilibrium price of the pass-through is given by:

$$P_{t} = \theta_{t} \left\{ 1 + (c - (r + \alpha \mu \pi_{l})) \int_{0}^{T-t} e^{-(r + \alpha \mu \pi_{l})\tau} \frac{e^{\tilde{c}T} - e^{\tilde{c}(\tau+t)}}{e^{\tilde{c}T} - e^{\tilde{c}t}} S_{t,\tau}^{\mathbb{Q}} d\tau \right\}.$$
(3.11)

That is, the equilibrium price in this case reflects both the risk-neutral probability of a liquidity shock  $\mu \pi_l$  and the expected prepayment path  $S_{t,\tau}^{\mathbb{Q}}$  under the risk-neutral measure.

Next, the standard OAS is given as the solution to

$$P_{t} = \theta_{t} \left\{ 1 + (c - (r + OAS_{t} + \alpha \mu)) \int_{0}^{T-t} e^{-(r + OAS_{t} + \alpha \mu)\tau} \frac{e^{\tilde{c}T} - e^{\tilde{c}(\tau+t)}}{e^{\tilde{c}T} - e^{\tilde{c}t}} S_{t,\tau} d\tau \right\}.$$
(3.12)

Comparing (3.11) and (3.12), we see that the standard OAS compensates the investor both for differences between the risk-neutral and the physical expected cumulative prepayment (differences between  $S_{t,\tau}^{\mathbb{Q}}$  and  $S_{t,\tau}$ ) and for the difference between the risk-neutral and the physical probability of a liquidity shock (the difference between  $\alpha\mu\pi_l$  and  $\alpha\mu$ ).

Finally, the prepayment-risk-neutral OAS, OAS<sup>Q</sup>, is given as the solution to

$$P_t = \theta_t \left\{ 1 + \left( c - \left( r + OAS_t^{\mathbb{Q}} + \alpha \mu \right) \right) \int_0^{T-t} e^{-\left( r + OAS_t^{\mathbb{Q}} + \alpha \mu \right) \tau} \frac{e^{\tilde{c}T} - e^{\tilde{c}(\tau+t)}}{e^{\tilde{c}T} - e^{\tilde{c}t}} S_{t,\tau}^{\mathbb{Q}} d\tau \right\}.$$
(3.13)

Thus, the prepayment-risk-neutral OAS only compensates the investor for the difference between the risk-neutral and the physical probability of a liquidity shock. Indeed, comparing (3.11) and (3.13), we see that OAS<sup>Q</sup> is given by

$$OAS_t^{\mathbb{Q}} = \alpha \mu \left( \pi_l^{\mathbb{Q}} - 1 \right). \tag{3.14}$$

Substituting into (3.12), we have

$$P_t = \theta_t \left\{ 1 + \left( c - \left( r + OAS_t - OAS_t^{\mathbb{Q}} + \alpha \mu \pi_l \right) \right) \int_0^{T-t} e^{-\left( r + OAS_t - OAS_t^{\mathbb{Q}} + \alpha \mu \pi_l \right) \tau} \frac{e^{\tilde{c}T} - e^{\tilde{c}(\tau+t)}}{e^{\tilde{c}T} - e^{\tilde{c}t}} S_{t,\tau} d\tau \right\}.$$

Thus, the difference between OAS and OAS<sup>Q</sup> only reflects the difference between the physical  $S_{t,\tau}$  and the risk-neutral  $S_{t,\tau}^{Q}$  expectation of the prepayment path. We therefore define:

**Definition 3.2.** *The prepayment risk premium on a pass-through security is equal to*  $OAS - OAS^{\mathbb{Q}}$ *.* 

#### 3.3 OAS variation in the time series

To provide a sense of variation in the standard OAS (which the yield premium that market participants generally focus on and report) we next study it both in the time series (using a market value-weighted index) and in the cross section (in terms of MBS moneyness). We then return to the model expressions above, and in Section 4 explain and implement our method to isolate prepayment risk premia, as just defined.

Because OAS are model-dependent, we collected end-of-month OAS on Fannie Mae securities in the TBA market from six different major dealers over the period 1996 to 2010.<sup>14</sup> As a result, the stylized facts we present are robust to idiosyncratic modeling choices of any particular dealer and, through data-quality filters we impose, issues arising from incorrect or stale price quotes. Whenever possible our analysis uses OAS relative to swaps, rather than Treasuries, since these instruments are more commonly used for hedging MBS (see e.g. the discussion in Duarte, 2008) and also because interest rate volatility measures, used to calibrate the term structure model, are more readily available for swaps.<sup>15</sup> Throughout the paper, OAS are expressed in basis points (bps) per year, following market convention. Further details on the sample, the data-quality filters, and descriptive statistics are provided in Appendix B.

To characterize time-series OAS variation, we follow the methodology of fixed income indices (such as Barclays and Citi, which are main benchmarks for money managers) and construct a mar-

<sup>&</sup>lt;sup>14</sup>Freddie Mac securities are generally priced relatively close to Fannie Mae's, reflecting the similar collateral and implicit government backing. The prices of Ginnie Mae securities can differ significantly (for the same coupon) from Fannie and Freddie MBS, reflecting the difference in prepayment characteristics (Ginnie Mae MBS are backed by FHA/VA loans) and perhaps the explicit government guarantee. Throughout this paper, we focus on Fannie Mae MBS.

<sup>&</sup>lt;sup>15</sup>Feldhütter and Lando (2008) study the determinants of spreads between swaps and Treasuries and find that they are mostly driven by the convenience yield of Treasuries, though MBS hedging activity may also play a role at times.

ket value-weighted index (the "TBA index") based on the universe of outstanding pass-through MBS. In contrast to other indices we do not rely on any particular dealer's pricing model; instead, we average the OAS for a coupon across the dealers for which we have quotes on a given day, and then compute averages across coupons using the market value of the remaining principal balance of each coupon in the MBS universe.

The resulting time series of spreads on the TBA index is shown in the top panel of Figure 2. The OAS on the value-weighted index is typically close to zero, consistent with the view that credit risk of MBS is generally limited; however, the OAS spiked to more than 150 bps in the fall of 2008, and also rose significantly around the 1998 demise of the Long-Term Capital Management fund. To provide initial evidence on potential drivers of this time-series variation we study the relation between the OAS on the TBA index and commonly used fixed income risk factors. Table 2 shows estimated coefficients from a regression of the OAS on: (i) the convenience yield on Treasury securities (reflecting their liquidity and safety) as measured by the Aaa-Treasury spread; (ii) credit spreads as measured by the Baa-Aaa spread; (iii) the slope of the yield curve (measured by the yield difference between 10-year Treasury bonds and 3-month Treasury bills); and (iv) the swaption-implied volatility of interest rates.<sup>16</sup>

The OAS on the TBA index is strongly related to credit spreads (and to a lesser extent to the Aaa-Treasury spread) both over the full sample (column 1) and the pre-crisis period (ending in July 2007, column 2), and is largely unaffected by the other risk measures. This suggests the existence of common pricing factors between the MBS and corporate bond markets.<sup>17</sup> In contrast, implied rate volatility does not explain the OAS variation, which is to be expected since the OAS adjusts for interest rate risk and thus should not reflect interest rate uncertainty. The slope of the yield curve, often used as a proxy for term premia, is also not systematically related to the OAS. In Section 5, we return to the determinants of the time-series variation in spreads, focusing on mortgage-specific risk factors.

<sup>&</sup>lt;sup>16</sup>All right-hand-side variables are standardized so that each coefficient estimate can be interpreted as the spread impact in basis points of a unit standard deviation increase. As in Krishnamurthy and Vissing-Jorgensen (2012) the Aaa-Treasury spread is the difference between the Moody's Seasoned Aaa corporate bond yield and the 20-year constant maturity Treasury (CMT) rate. The Baa rate is also from Moody's, and bill rates and 10-year Treasury yields are CMTs as well. All rates were obtained from the H.15 release. Swaption quotes are basis point, or normal, volatility of 2-year into 10-year contracts, from JP Morgan. We choose the Newey-West lag length based on the Stock-Watson rule-of-thumb measure  $0.75 * T^{1/3}$ .

<sup>&</sup>lt;sup>17</sup>Brown (1999) relates the OAS to Treasuries of pass-through MBS over the period 1993–1997 to other risk premia and finds a significant correlation of the OAS with spreads of corporate bonds to Treasuries. He interprets his findings as implying a correlation between the market prices of credit risk or liquidity risk on corporates and that of prepayment risk on MBS, but notes that it could also be driven by time variation in the liquidity premium on Treasuries.

#### 3.4 OAS variation in the cross section: the OAS smile

While variation in OAS on the TBA index is informative of the MBS market as a whole, it masks significant variation in the cross section of securities. As discussed above, this cross section is composed of MBS with different underlying loan rates. This rate variation across MBS leads to borrower heterogeneity in their monetary incentives to refinance. We refer to this incentive as a security's "moneyness" and define it (for security j at time t) as

$$Moneyness_{i,t} = Coupon_i + 0.5 - FRMrate_t$$

where FRMrate<sub>t</sub> is the mortgage rate on new loans at time *t*, measured by the end-of-month value of the Freddie Mac Primary Mortgage Market Survey rate on 30-year FRMs. We add 0.5 to the coupon rate because the mortgage loan rates are typically around 50 bps higher than the MBS coupon.<sup>18</sup> When moneyness is positive, a borrower can lower his monthly payment by refinancing the loan—the borrower's prepayment option is "in-the-money" (ITM)—while if moneyness is negative, refinancing (or selling the home and buying another home with an equal-sized mortgage) would increase the monthly payment—the borrower's option is "out-of-the-money" (OTM).

Aside from determining the refinancing propensity of a loan, moneyness also measures an investor's gains or losses (in terms of coupon payments) if a mortgage underlying the security prepays (at par) and he reinvests the proceeds in a "typical" newly originated MBS (which will approximately have a coupon equal to FRMrate<sub>t</sub> minus 50 bps). MBS based on ITM loans make higher coupon payments than newly-issued MBS at the current market rate and therefore typically trade at a premium; MBS based on OTM loans conversely trade at a discount. An investor in premium MBS loses when prepayments increase, while an investor in discount MBS benefits.

The bottom panel of Figure 2 shows the (pooled) variation of spreads as a function of security moneyness. OAS display a smile-shaped pattern: they are lowest for at-the-money (ATM) securities and increase moving away in either direction, especially ITM. OAS on deeply ITM securities on average exceed those on ATM securities by 50 bps or more.

In Table 3 we report results from regressions that allow us to more precisely quantify this pattern and assess its statistical significance. Instead of imposing parametric restrictions, we simply regress OAS on 50-basis-point moneyness bin dummies, with [-0.25, 0.25) as the omitted category.

<sup>&</sup>lt;sup>18</sup>The difference gets allocated to the agency guarantee fee as well as servicing fees (see Fuster et al., 2013, for details). We could alternatively use a security's "weighted average coupon" (WAC) directly, but the WAC is not known exactly for the TBA securities studied in this section.

Column 1 of the top panel studies the pooled variation over the full sample (as displayed in Figure 2), confirming a statistically and economically significant smile pattern. Column 2 shows that this pattern remains almost unchanged if we control for month fixed effects (meaning that only cross-sectional variation is exploited). In columns 3 and 4, we repeat the same regressions but end the sample before the onset of the financial crisis in August 2007. For ITM coupons, this somewhat reduces the relative spread differences to ATM securities, but the differences remain monotonic in absolute moneyness and highly statistically significant.

In panel (b) of the table, we split the sample in terms of the moneyness of the agency MBS market as a whole, in order to investigate changes in cross-sectional OAS patterns. As we discuss in more detail below, the model in Gabaix et al. (2007) predicts that the relationship of OAS with moneyness is linear at a point in time, but can be either upward sloping (if the market is ITM) or downward sloping (if the market is OTM). More recently Diep et al. (2017) present a similar theory which also predicts that prepayment risk premia change sign with market moneyness.<sup>19</sup> Instead, our findings indicate that the OAS smile is present irrespective of whether the market is ITM or not, even though coefficient estimates vary somewhat and are not always very precise due to limited number of observations (e.g. for ITM bins in OTM markets). In Appendix C, we present further robustness evidence on the smile pattern in OAS, showing for instance that it holds when excluding outliers or coupons with low remaining balance.

What explains this smile pattern? In the framework of Section 3.1, the OAS reflects both the difference between the physical  $S_{t,\tau}$  and the risk-neutral  $S_{t,\tau}^{\mathbb{Q}}$  expectation of the prepayment path, and the exposure to liquidity risk,  $\alpha$ . Consequently, the OAS smile may arise if the fraction of market value lost in case of a liquidity shock,  $\alpha$ , has a smile-shape—for example, if more recently issued securities (which tend to be at-the-money) are more liquid than older securities. Alternatively, the OAS smile can arise as compensation for prepayment risk.

We now illustrate how prepayment risk premia could vary in the cross section, using three stylized representations of borrowers' prepayment behavior, and for simplicity assuming that there is no liquidity risk ( $\alpha = 0$ ). In each case, we assume that the non-interest prepayment factors are constant, but unknown to the investors in the security.<sup>20</sup> Under the physical measure, assume the factors  $\gamma_i$  are log-normally distributed with mean  $\mu_i$  and variance  $\Sigma_i$ :  $\gamma_i \sim \log \mathcal{N}(\mu_i, \Sigma_i)$ . Under the risk-neutral measure, the factors  $\gamma_i$  are log-normally distributed with mean  $\mu_i$  and variance  $\Sigma_i$ :  $\gamma_i \sim \log \mathcal{N}(\mu_i, \Sigma_i)$ .

<sup>&</sup>lt;sup>19</sup>They document patterns in realized returns that appear consistent with this prediction; we further discuss the relationship between our findings and theirs in the next subsection.

<sup>&</sup>lt;sup>20</sup>One can also think of the non-interest factors as parameters of the prepayment function.

 $\Sigma_i^{\mathbb{Q}}$ :  $\gamma_i \sim \log \mathcal{N}\left(\mu_i^{\mathbb{Q}}, \Sigma_i^{\mathbb{Q}}\right)$ . Figure 4 plots the OAS as a function of the prepayment incentive with a positive prepayment risk premium ( $\mu_1^{\mathbb{Q}} > \mu_1, \Sigma_1^{\mathbb{Q}} > \Sigma_1$ ) (top panel) and a negative prepayment risk premium ( $\mu_1^{\mathbb{Q}} < \mu_1, \Sigma_1^{\mathbb{Q}} < \Sigma_1$ ) (bottom panel) for each of the three cases.

**Case 1:**  $s_t = \gamma_1 \beta$ . This is essentially the framework studied by Gabaix et al. (2007). Each pool has a constant exposure  $\beta$  to a single market-wide prepayment shock  $\gamma_1$ . The solid blue lines in Figure 4 show that regardless of the sign of the prepayment risk premium, the OAS in this case is monotone in moneyness. The intuition for the linearity is that OTM securities and ITM securities act as hedges for one another (as premiums and discounts have opposite exposure to  $\gamma_1$ ), such that their risk premia must have opposite signs. This case is thus inconsistent with the OAS smile.

**Case 2:**  $s_t = \bar{s} + \gamma_1(c - r)$ . Like in the previous case, a single factor drives prepayment behavior, but the security's exposure to the prepayment shock now depends on its moneyness, which varies over time. This functional form implies that when ITM securities prepay faster than expected (a positive shock to  $\gamma_1$ ), OTM securities prepay slower than expected. This may arise because of mortgage originators' capacity constraints in (larger than expected) refinancing waves.<sup>21</sup> The OAS for this case is plotted as the dashed red lines in Figure 4. When the prepayment risk premium is positive (which is the natural case since every security has a positive exposure to  $\gamma_1$ ), the OAS exhibits a smile-shape in moneyness. When the prepayment risk premium is negative, the OAS exhibits an inverse smile-shape in moneyness.

**Case 3:**  $s_t = \bar{s} + \gamma_1 \mathbf{1}_{c < r} + \gamma_2 \mathbf{1}_{c \ge r}$ . In this multi-factor formulation, OTM and ITM prepayments are driven by different shocks (which for simplicity we assume to be orthogonal). For instance,  $\gamma_1$  might represent the pace of housing turnover while  $\gamma_2$  might be the effective cost of mortgage refinancing (which varies with underwriting standards and market competitiveness). In equilibrium, the signs of the prices of risk  $\pi_{\gamma i}$  are determined by the average exposure of the representative investor. Holding a portfolio of ITM and OTM securities, this investor will have a negative exposure to  $\gamma_1$  risk (since OTM securities benefit from fast prepayment) and a positive exposure to  $\gamma_2$  risk (since the price of ITM securities declines with faster prepayments). Thus  $\pi_{\gamma 1} < 0$  and  $\pi_{\gamma 2} > 0$ , resulting in a positive risk premium for both ITM and OTM securities and a (v-shaped) OAS smile, as shown by the crossed green lines in Figure 4. If the prepayment risk premium is instead nega-

<sup>&</sup>lt;sup>21</sup>When capacity is tight, mortgage originators may be less willing to originate purchase loans (which are more labor intensive), and they may reduce marketing effort targeted at OTM borrowers (for instance, to induce them to cash out home equity by refinancing their loan). Fuster et al. (2017) show that originators' pricing margins are strongly correlated with mortgage application volume, consistent with the presence of capacity constraints.

tive, the OAS has an inverted v-shape.

In sum, the risk premium displays a smile pattern in moneyness if prepayments are driven by the specification in case 2 or 3, but not in the single-factor representation of case 1. More generally, prepayment risk premia can explain the OAS smile whenever OTM securities are not a hedge for ITM ones (as they would be in case 1).<sup>22</sup> To separate liquidity and prepayment risk premia, in the next section we provide a method to identify the prepayment-risk-neutral OAS<sup>Q</sup>. This will allow us to identify the prepayment risk premium as the difference between the OAS and  $OAS^Q$ .

#### 3.5 OAS and MBS returns

The OAS is a valuation measure that is widely tracked by financial market participants but that has also been called into question for its model dependence (Kupiec and Kah, 1999). In this section, we address this issue by first deriving the formal relationships between OAS (and changes in OAS) and expected excess returns, and then testing these relationships in the data.<sup>23</sup> We show that the cross-sectional smile pattern exhibited by OAS is also present in realized returns, but also demonstrate that realized returns to an important extent depend on realized rate changes, which in our view makes it preferable to study ex-ante expected returns (as measured by OAS) rather than ex-post realized ones.

In what follows, we derive the relationship between returns and yield premia under  $Q_r$ , i.e. the standard OAS, since that is what we will test in the data; however, the derivations apply more broadly for the generic measure  $\mathbb{R}$ . Comparing expression (3.6) for the price of the pass-through to the expression (3.7), both with  $\mathbb{R} = Q_r$ , the yield premium on the security is related to the path of instantaneous expected excess returns:

$$\mathbb{E}_{t}^{\mathbb{Q}_{r}}\left[\int_{0}^{T-t}\exp\left(-\int_{0}^{s}\left(r_{t+u}du+drx_{t+u}^{\mathbb{Q}_{r}}\right)\right)(1-\alpha)^{\int_{0}^{s}dJ_{t+u}}\mu_{X,t+s}ds\right]$$
(3.15)  
=  $\mathbb{E}_{t}^{\mathbb{Q}_{r}}\left[\int_{0}^{T-t}\exp\left(-\int_{0}^{s}\left(r_{t+u}+yp_{t}^{\mathbb{Q}_{r}}\right)du\right)(1-\alpha)^{\int_{0}^{s}dJ_{t+u}}\mu_{X,t+s}ds\right].$ 

Linearizing the right hand side of (3.15) around zero OAS (=  $yp^{Q_r}$ ), we obtain:

<sup>&</sup>lt;sup>22</sup>This is also pointed out by Levin and Davidson (2005), who note that "[a] single-dimensional risk analysis would allow for hedging prepayment risk by combining premium MBS and discount MBS, a strategy any experienced trader knows would fail."

<sup>&</sup>lt;sup>23</sup>Shiller, Campbell, and Schoenholtz (1983) derive first-order approximations of the yield to maturity on couponbearing Treasury securities in terms of future bond excess returns and of contemporaneous returns and yield changes. Despite the analogy, our derivations (shown in Appendix A) differ from theirs because MBS cash flows are uncertain.

**Result 3.1.** To a first-order approximation, the OAS is a weighted average of expected excess returns over the lifetime of the security under  $Q_r$ :

$$OAS_t = \mathbb{E}_t^{\mathbb{Q}_r} \left[ \int_0^{T-t} \left( 1 - \exp\left( - \int_0^s dr x_{t+u}^{\mathbb{Q}_r} \right) \right) w_{t,s} ds \right],$$
(3.16)

where the weights  $w_{t,s}$  are declining in horizon *s*.

*Proof.* Follows from taking a first-order Taylor expansion of the right hand side of (3.15) around  $yp_t^{\mathbb{R}} = 0$  and solving for  $yp_t^{\mathbb{R}}$ ; the above is for  $\mathbb{R} = \mathbb{Q}_r$ . See Appendix A for details.

We next turn to the relationship between realized excess returns, the level of the OAS and changes in the OAS.

**Result 3.2.** Realized excess returns under measure  $Q_r$  are, to a first-order approximation, equal to the sum of income (or carry) and capital gains/losses resulting from duration-weighted changes in spreads (where  $D_t$  is a security's modified spread duration):

$$drx_t^{\mathbb{Q}_r} = OAS_t dt - D_t dOAS_t.$$
(3.17)

*Proof.* Follows from approximating the evolution of  $P_t$  in (3.7) under the measure  $\mathbb{R}$  around locally small changes in the yield premium over time and substituting that evolution into the definition of an excess return under  $\mathbb{R}$ , (3.4); above is for  $\mathbb{R} = \mathbb{Q}_r$ . See Appendix A for details.

Relationship (3.17) should not be surprising as analogous expressions have been shown for coupon-bearing Treasury securities (Shiller et al., 1983) or defaultable corporate bonds (Campello, Chen, and Zhang, 2008). They are also actively used by practitioners (Lehman Brothers, 2008).

In order to test the results derived above in the data, we conclude by noting that expected excess hedged returns under  $Q_r$  coincide with expected excess hedged returns under the physical measure  $\mathbb{P}$ :

**Result 3.3.** The expected excess return on an interest-rate-hedged portfolio under the interest-rate-riskneutral measure coincides with the expected excess return on the portfolio under the physical measure.

*Proof.* Follows from the fund separation theorem. See Appendix A for details.  $\Box$ 

Breeden (1994) shows that the OAS predicts future MBS excess returns between 1988 and 1994, consistent with Result 3.1. In the remainder of this section, we confirm this finding in a longer sample, then test the contemporaneous relation between excess returns and OAS implied by Result 3.2,

and show that excess returns also exhibit a smile pattern with respect to moneyness. Finally, we show that the relationship between MBS returns in the cross section and changes in interest rates is also consistent with the OAS smile, but not with alternative theories.

Constructing MBS returns is complex because of the large number of securities, different pricing conventions, and security-specific prepayments. We rely on monthly return data from the MBS sub-components by coupon of the Bloomberg Barclays Aggregate Bond Index, which is the leading benchmark for fixed income index funds. MBS returns are available both unhedged (that is, as measured from MBS prices and prepayments alone) and interest-rate-hedged (relative to a duration-matched portfolio of Treasury securities).<sup>24</sup> We are primarily interested in expected hedged returns, since they correspond to expected excess returns under  $Q_r$  in our derivations above, but also analyze unhedged returns for robustness.

Bloomberg Barclays MBS returns have recently been analyzed by Diep, Eisfeldt, and Richardson (2017), and we match their 1994-mid 2016 sample period and size cutoff (excluding coupons with less than \$1 billion in outstanding principal). Unlike in the previous subsections, here we rely on the Barclays OAS relative to Treasuries that covers this entire sample period.

The first two columns of panel (a) of Table 4 report estimates of a regression of future 1-year hedged MBS returns ( $t \rightarrow t + 12$ ) on OAS<sub>t</sub> either including or excluding time fixed effects; panel (b) shows corresponding coefficients for unhedged returns. Estimated loadings on the OAS range between 0.8 and 1.5 and are highly statistically significant (based on Newey-West standard errors with 18 lags).<sup>25</sup> Columns 3 and 4 repeat the same exercise using 1-month returns. The loading of excess returns on the lagged OAS is about 1.8 both with and without time fixed effects for hedged returns (p < 0.01 based on standard errors clustered by month), and it is similar for unhedged

<sup>&</sup>lt;sup>24</sup>The return on an MBS is equal to the sum of price appreciation, coupon yield and paydown return. Because more seasoned securities often trade at a premium to the TBA price, the Barclays index adjusts the capital gain return component with "payup" information. In addition, the calculation of the index incorporates cusip-specific prepayment information to compute the paydown return. The index uses same-day settlement as opposed to standard PSA settlement (fixed monthly dates) as it is typically the case in TBA trading, which is associated with discrete price drops on PSA dates, which is when the attribution of prepayments and coupons is determined (see Chapter 29 in Fabozzi, 2016, for more detail). The excess return for an MBS is calculated as the difference between its total return and that of the equivalent Treasury position, where the equivalent position is obtained from key-rate durations, which are sensitivities to the movement of specific parts of the yield curve. For additional information see, for example, Lehman Brothers (2008). We have also verified that qualitative patterns are similar when using data from a different dealer (results available upon request).

<sup>&</sup>lt;sup>25</sup>As is well known in the literature (Hansen-Hodrick), the overlapping return sample generates an MA(12) component in the error term. We use NW with 18 lags to guarantee a positive definite variance covariance matrix and counteract the underweighting of higher covariances from the NW kernel function. Realized returns in the sample move about one-for-one with the OAS; we do not have a sharp prediction on the size of the coefficient since the right-hand side of (3.16) features declining weights whereas in our empirical implementation we apply equal weights and truncate at one year.

returns. In sum, OAS predict realized hedged and unhedged returns at the 1-month and 1-year horizon consistent with the prediction of Result 3.1.

To study Result 3.2, we extend the 1-month regression to include contemporaneous changes in the OAS. As predicted, the coefficient on the spread change is always negative (p < 0.01) across the four specifications with point estimates that range between -1.8 and -3.3. The adjusted  $R^2$  in the hedged return regression when omitting time effects exceeds 50%, meaning that changes in OAS explain much of the realized variation in hedged returns.

A key feature of the cross section of OAS is the smile pattern with respect to moneyness (see Section 3.4). Figure 3 shows that 1-month excess returns display a similar pattern, which is also reflected in differential Sharpe ratios (SRs) across moneyness levels. We compute (annualized) SRs based on monthly Barclays index returns for portfolios of ITM, ATM and OTM MBS.<sup>26</sup> Relative to ATM securities, SRs are much larger for non-ATM securities: using unhedged returns (minus the risk-free rate from Ken French's website), the SR of a long-ITM (long-OTM) portfolio is 1.86 (1.15) compared to a SR of 0.66 for a long-ATM position. The pattern in hedged returns is similar, though more pronounced for ITM securities (ITM SR = 0.68; OTM SR = 0.12; ATM SR = 0.04).

The above analysis established that there is a tight link between realized hedged returns and changes in OAS: when OAS fall, realized returns tend to be high. In addition, we have documented a smile-shaped pattern both in expected and realized returns. We now combine these relations to test an additional prediction about the link between realized returns and changes in mortgage rates, without relying on OAS directly. Movements in mortgage rates change the moneyness of MBS by moving them along the smile, and the OAS smile predicts an opposite effect of rate changes on hedged returns depending on whether an MBS is ITM or not. For an OTM MBS, changes in OAS and changes in rates are positively related, as the MBS moves closer to being ATM when rates fall. This implies a *negative* relationship between rate changes and contemporaneous hedged returns on OTM securities. Conversely, for ITM securities, hedged returns and rate changes should be positively related.<sup>27</sup>

Panel (c) of Table 4 shows that as predicted, hedged returns are negatively (positively) related to changes in mortgage rates for OTM (ITM) securities, irrespective of whether fixed effects are

<sup>&</sup>lt;sup>26</sup>We compute the ATM portfolio using the return on the coupon that is closest to zero moneyness in each period, as long as it is not more than 25 bps ITM or OTM. For the ITM (OTM) portfolio we use the most ITM (OTM) coupon as long as it is at least 100 bps ITM (OTM).

<sup>&</sup>lt;sup>27</sup>The OAS smile implies that OTM securities tend to outperform ITM securities when rates fall, as for example was the case following the November 2008 LSAP announcement, discussed in Section 5.3.

included or not (columns 1 and 2).<sup>28</sup> In columns 3 and 4, we re-estimate this relation in a split sample based on whether the market as a whole is ITM or not. This allows us to test whether prepayment risk premia change sign as a function of the market overall moneyness as predicted by Gabaix et al. (2007) and Diep et al. (2017).<sup>29</sup> Contrary to what is implied by these theories, but consistent with our OAS smile evidence in Section 3.4, we do not find that the relationship between returns and rate changes flips sign with market moneyness.

In sum, our analysis of MBS returns shows that, while model-dependent, the OAS is related to realized returns, and that a smile pattern is also evident in the cross section of hedged returns. In the remainder of the paper, we focus on the OAS rather than realized returns, since it is a more direct and less noisy measure of expected excess returns. Also, as the concluding part of the analysis above illustrated, realized returns across securities are systematically affected by changes in interest rates, which in finite sample can bias the measurement of expected returns when using average realized returns.<sup>30</sup>

## 4 Pricing model: Decomposing the OAS

In this section, we propose a method to decompose the standard OAS into a prepayment risk premium component and a remaining risk premium ( $OAS^Q$ ). We then implement this method using a pricing model, which consists of an interest rate and a prepayment component. In contrast to standard approaches, such as Stanton (1995) or practitioner models, we employ information from stripped MBS to identify a market-implied prepayment function and the contribution of prepayment risk to the OAS.

#### 4.1 Identification of OAS<sup>Q</sup>

As discussed in Section 3, the OAS only adjusts for interest rate uncertainty (and only interest rates are simulated in empirical pricing models) and ignores other sources of prepayment risk, such as

<sup>&</sup>lt;sup>28</sup>When we add month fixed effects, we can only test whether the relationship between returns and rate changes is more positive for ITM securities; the uninteracted coefficient on the mortgage rate change is not identified.

<sup>&</sup>lt;sup>29</sup>In Gabaix et al. (2007) the relationship of OAS with moneyness is linear and upward sloping if the market is ITM or downward sloping if the market is OTM. This theory thus predicts that in an ITM market, the relationship between hedged returns and mortgage rate changes should be positive for *all* securities, while in an OTM market, the relationship should be negative for all securities.

<sup>&</sup>lt;sup>30</sup>This may explain why Diep et al. (2017) find linear relationships between realized returns and moneyness that change sign depending on whether the market as a whole is ITM or not, which contrasts with the OAS smile that is present in both market types, as shown earlier. For instance, over the period they (and we) study, rates have tended to fall when the market is OTM, which according to the OAS smile leads to negative returns for ITM securities as they move further ITM and their OAS increases.

uncertainty about house prices or lending standards. In this section we propose a method to identify a risk-neutral prepayment function obtained from market prices, then compute an OAS using this function ( $OAS^Q$ ) and finally obtain the contribution of prepayment risk to the OAS. Pricing information on a standard pass-through MBS alone is insufficient to identify the risk-neutral prepayment probability, because a single observable (the price) can only determine one unknown (the spread) in the pricing model, leaving the price of prepayment risk unidentified. To resolve this identification problem, we use additional pricing information from "stripped" MBS, which separate cash flows from pass-through securities into an interest component ("interest only" or IO strip) and a principal component ("principal only" or PO strip). That is, for a pool with remaining principal balance  $\theta_t$ , the IO strip receives interest payments net of fees,  $dX_{t,IO} = c\theta_t dt$ , as cash flows, while the PO strip receives cash flows equal to all principal payments,  $dX_{t,PO} = -d\theta_t$ .

Cash flows of these strips depend on the same underlying prepayment path and therefore face the same prepayment risk, but are exposed to it in opposite ways, as illustrated in Figure 5. As prepayment rates increase (top to bottom panel), total interest payments shrink (since interest payments accrue only as long as the principal is outstanding) and thus the value of the IO strip declines. Conversely, principal cash flows (the gray areas) are received sooner, and therefore the value of the PO strip increases.

More formally, consider first the IO strip. The yield premium  $yp_{t,IO}^{\mathbb{R}}$  under measure  $\mathbb{R}$  on the IO is given by

$$P_{t,IO} = \mathbb{E}_{t}^{\mathbb{R}} \left[ \int_{0}^{T-t} \exp\left( -\int_{0}^{\tau} \left( r_{t+u} + yp_{t,IO}^{\mathbb{R}} + \alpha\mu_{t+u}\pi_{l,t+u}^{\mathbb{R}} \right) du \right) c\theta_{t+\tau} d\tau \right]$$
$$= c\theta_{t} \mathbb{E}_{t}^{\mathbb{R}} \left[ \int_{0}^{T-t} \exp\left( -\int_{0}^{\tau} \left( r_{t+u} + yp_{t,IO}^{\mathbb{R}} + \alpha\mu_{t+u}\pi_{l,t+u}^{\mathbb{R}} + s_{t+u}^{*} \right) du \right) \frac{e^{\tilde{c}T} - e^{\tilde{c}(\tau+t)}}{e^{\tilde{c}T} - e^{\tilde{c}t}} d\tau \right].$$

Thus, an increase in the prepayment path  $\{s_{t+u}\}_{u=0}^{T-t}$  has to be offset by a decrease in the yield premium  $yp_{t,IO}^{\mathbb{R}}$  for the model price (the RHS of the equality) to match the market price ( $P_{t,IO}$ ). That is, the yield premium on the IO is decreasing in the speed of prepayment.

Similarly, the yield premium  $yp_{t,PO}^{\mathbb{R}}$  under measure  $\mathbb{R}$  on the PO is given by

$$\begin{split} P_{t,PO} &= \mathbb{E}_{t}^{\mathbb{R}} \left[ \int_{0}^{T-t} \exp\left( -\int_{0}^{\tau} \left( r_{t+u} + yp_{t,PO}^{\mathbb{R}} + \alpha \mu_{t+u} \pi_{l,t+u}^{\mathbb{R}} \right) du \right) (-d\theta_{t+\tau}) \right] \\ &= \theta_{t} \left\{ 1 - \mathbb{E}_{t}^{\mathbb{R}} \left[ \int_{0}^{T-t} \exp\left( -\int_{0}^{\tau} \left( r_{t+u} + yp_{t,PO}^{\mathbb{R}} + \alpha \mu_{t+u} \pi_{l,t+u}^{\mathbb{R}} + s_{t+u} \right) du \right) \times \right. \\ & \left. \times \left( r_{t+\tau} + yp_{t,PO}^{\mathbb{R}} + \alpha \mu_{t+\tau} \pi_{l,t+\tau}^{\mathbb{R}} \right) \frac{e^{\tilde{c}T} - e^{\tilde{c}(\tau+t)}}{e^{\tilde{c}T} - e^{\tilde{c}t}} d\tau \right] \right\}. \end{split}$$

Thus, an increase in the prepayment path  $\{s_{t+u}\}_{u=0}^{T-t}$  has to be offset by an increase in the yield premium  $yp_{t,PO}^{\mathbb{R}}$  for the model price (the RHS of the equality) to match the market price ( $P_{t,PO}$ ). That is, the yield premium on the PO is increasing in the speed of prepayment.<sup>31</sup>

Since the yield premium on the PO is increasing in the speed of prepayment and the yield premium on the IO is decreasing, there is a single point of intersection between the two. In the simple setting of Section 3.2, we have that the prepayment-risk-neutral OAS on the IO equals that on the PO:

$$OAS_{t,IO}^{\mathbb{Q}} = OAS_{t,PO}^{\mathbb{Q}} = \alpha \mu \left( \pi_l - 1 \right)$$
,

so that the yield premium of the IO equals the yield premium on the PO if and only if the yield premium is calculated under a measure that is risk-neutral with respect to both prepayment and interest rate risks.

More generally, Proposition D.1 in Appendix D states that the risk premium on the IO equals the risk premium on the PO if and only if the risk premium is calculated under a measure that is risk-neutral with respect to both prepayment and interest rate risks. Since yield premia are weighted averages of instantaneous risk premia, the OAS on the IO and the PO are equalized under the prepayment-and-interest-rate-risk-neutral measure  $Q_{r,\gamma}$ . Intuitively, since the IO and the PO have opposite exposures to the same source of risk (prepayment on the pool), the yield premium of the IO equals the yield premium on the PO if and only if the yield premia are calculated under a measure that is risk-neutral with respect to this source of risk.

We now turn to the empirical implementation of this idea. Since we only have observations of monthly physical prepayment rates, rather than instantaneous prepayment speeds, we focus on identifying the risk-neutral monthly prepayment rate (known as "single month mortality rate")

<sup>&</sup>lt;sup>31</sup>This follows since  $(r_{t+\tau} + yp_{t,PO}^{\mathbb{R}} + \alpha\mu_{t+\tau}\pi_{l,t+\tau}^{\mathbb{R}}) \exp\left(-\int_{0}^{\tau} \left(r_{t+u} + yp_{t,PO}^{\mathbb{R}} + \alpha\mu_{t+u}\pi_{l,t+u}^{\mathbb{R}}\right) du\right)$  is increasing in  $yp_{t,PO}$  for all  $\tau$ .

 $SMM_t^Q$ . In terms of the instantaneous unscheduled prepayment rate  $s_t^*$ , the SMM is given by:

$$SMM_t = \exp\left(-\int_0^{\frac{1}{12}} s_{t+u}^* du\right)$$

Using the risk-neutral evolution of  $s_t^*$  in the above gives the risk-neutral SMM. Let  $\tilde{\Lambda}_{jt}$  be the ratio at time *t* between the risk-neutral and the physical SMM on pool *j*:

$$\tilde{\Lambda}_{jt} \equiv \frac{SMM_{jt}^{Q}}{SMM_{jt}}.$$

To price a security on pool *j* at date *t*, we would then need to identify the full future path of  $\tilde{\Lambda}_{j,t+h}$  until the maturity of the security. Since we only have one pair of observations per pool on each date *t*, identification requires restrictions on the shape of  $\tilde{\Lambda}_{j,t+h}$ .

Instead, we approximate the period- and pool-specific multiplier  $\tilde{\Lambda}_{j,t+h}$  with a pool-specific multiplier  $\Lambda_{j,t}$ . This specification allows us to be flexible in how the prepayment-risk-neutral SMM changes in the cross section but comes at the cost of being able only to identify a life-time average  $\tilde{\Lambda}_{j,t+h}$ . However, we believe that this approach captures the first-order differences between pools. In fact, even after controlling for moneyness, prepayments are known to be driven by pool-specific factors, such as the state in which the underlying mortgages are located. As we discuss below, our key finding that the smile pattern in OAS is due to prepayment risk premia while  $OAS^Q$  is flat with respect to moneyness is preserved in an alternative framework where the Q prepayment is estimated from the cross section of MBS prices only.

Figure 6 illustrates our empirical approach graphically. In this example, at  $\Lambda = 1$ , the physical prepayment speed, the OAS on the IO strip (shown in black) is about 200 bps and the OAS on the PO (shown in gray) is about zero. As  $\Lambda$  increases, the OAS on the IO declines while the spread on the PO increases because of their opposite sensitivities to prepayments. The sensitivity of the OAS on the pass-through ("recombined" as the sum of IO and PO) is also negative (red line), because in this example it is assumed to be a premium security and so its price declines with faster prepayments.

For each IO/PO pair, we identify  $\Lambda$  as the crossing of the OAS schedules of IO and PO at the point where the residual risk premium ( $OAS^Q$ ) on the two strips is equalized.<sup>32</sup> By the law of one price, the residual risk premium on the pass-through will also be equalized at this point; thus,

<sup>&</sup>lt;sup>32</sup>MBS market participants sometimes calculate "break-even multiples" similar to our  $\Lambda$  but, to our knowledge, do not seem to track them systematically as measures of risk prices.

the OAS schedule on the pass-through intersects the other two schedules at the same point. The difference between the OAS on the pass-through at the physical prepayment speed (OAS) and at the market-implied one ( $OAS^{\mathbb{Q}}$ ) is then equal to the prepayment risk premium.

We apply this method to each IO/PO pair in our sample, thereby identifying pool- and datespecific  $\Lambda$  and  $OAS^Q$ . This allows us to study time-series and cross-sectional variation in the  $OAS^Q$  without imposing parametric assumptions and we can thus remain agnostic as to whether prepayment risk or other risks are the source of the OAS smile.<sup>33</sup> Notice that while the above discussion (and the theoretical derivations in Appendix D) has emphasized the case of prepayment risk premia due to non-interest rate factors, our empirical setting also subsumes the possibility of a prepayment event risk premium (which by itself would lead to date-specific  $\Lambda$ ).

The key to this identification is the assumption that  $OAS^Q$  are equal across IO and PO strips on the same pool. One could relax this assumption by imposing a parametric form linking  $OAS^Q$  (or  $\Lambda$ ) across pools. That said, the impact on the prepayment risk premium and  $OAS^Q$  on the passthrough will be limited for reasonable liquidity differences between IOs and POs. For example, we find that assuming  $OAS^Q_{PO}$  to be 50 bps higher than  $OAS^Q_{IO}$  never changes  $OAS^Q$  by more than 5 bps relative to the baseline specification with  $OAS^Q_{IO} = OAS^Q_{PO}$ . Intuitively, as shown in Figure 6, the slope of the OAS schedule for the pass-through is less steep in  $\Lambda$  than the slopes of the IO and PO schedules, and thus  $OAS^Q_{IO} - OAS^Q_{PO}$  differences will have a limited effect on the recombined pass-through.

#### 4.2 Stripped MBS data

To implement the identification described above, we start with an unbalanced panel of end-of-day price quotes on all IO/PO pairs ("trusts") issued by Fannie Mae, obtained from a large dealer, for the period January 1995 to December 2010.<sup>34</sup> We merge these with characteristics of the underlying pools, using monthly factor tape data describing pool-level characteristics obtained from the data provider eMBS. We use end-of-month prices, which we also subject to a variety of screens, as described in Appendix B. Following these data-quality filters, our data include 3713 trust-month

 $<sup>^{33}</sup>$ An alternative way to identify  $\Lambda$  would be to assume that the OAS reflects only prepayment risk. With this approach, Levin and Davidson (2005) obtain a Q prepayment function by equalizing the OAS (relative to agency debt) on all pass-through coupons to zero. By construction, both the time-series and cross-sectional variation in the OAS will then be the result of variation in prepayment risk.

<sup>&</sup>lt;sup>34</sup>We end our sample on that date because, according to market participants, IO/PO strips became less liquid after 2010, as trading started focusing on Markit's synthetic total return swap agency indices IOS, POS and MBX instead. These indices mimic the cash flows of strips on a certain coupon-vintage (e.g. Fannie Mae 30-years with coupon 4.5% originated in 2009). The methods in this paper could easily be extended to those indices.

observations, or about 19 per month on average, from 95 trusts total. The year with the lowest number of observations is 1999, where we have an average of 10 trusts per month, while after 2005 we have at least 20 trusts in all but one month.

The original face value of securities in our sample ranges from \$200 million to about \$4.5 billion, with a median of \$2 billion. The median remaining principal balance of trusts in months in our dataset is \$1.13 billion. In the cross-sectional analysis, we average spread measures to the coupon level (weighting by market value of the trusts), resulting in 1005 coupon-month pairs that cover most of the outstanding coupons in the Fannie Mae fixed-rate MBS universe (on average, 91% of remaining face value).<sup>35</sup> A potential concern is that the IO/PO strips we have are not necessarily representative of securities traded in the TBA market, to which we are comparing our model output. As we will see, however, we obtain similar spread patterns based on IO/PO prices, both in the time series and cross section. One advantage of the stripped MBS that we are using relative to TBAs, which trade on a forward "cheapest-to-deliver" basis, is that we do not need to make assumptions about the characteristics of the security.

#### 4.3 Interest rate and prepayment model

A standard MBS pricing model has two main components: an interest rate model and a prepayment model. The two are combined to simulate interest rate paths and corresponding prepayment flows to obtain model prices and spreads. We use a three-factor Heath et al. (1992) interest rate model, calibrated at month-end to the term structure of swap rates and the interest rate volatility surface implied by the swaption matrix, by minimizing the squared distance between the modelimplied and the observed volatility surface. We obtain swap zero rates from an estimated Nelson-Siegel-Svensson curve. Details on the interest and yield curve model are provided in Appendix E.<sup>36</sup>

The academic literature has considered either structural/rational prepayment models (e.g., Dunn and McConnell, 1981a,b; Stanton, 1995) or reduced-form statistical prepayment models estimated on historical data (e.g., Richard and Roll, 1989; Schwartz and Torous, 1989). While structural

<sup>&</sup>lt;sup>35</sup>As in Figure 1, this means that the range of trust coupons in which the remaining face value is concentrated shifts downward over time. For instance, in January 1995, about 90% of the face value of securities for which we have quotes is in 7, 7.5, or 8% coupon securities. In January 2003, over 90% are in 5.5, 6, 6.5, or 7% securities. Finally, in December 2010, the last month in our data, the most prominent coupons are 4, 4.5, 5, and 5.5, which together account for 88% of face value.

<sup>&</sup>lt;sup>36</sup>A potential concern with using the risk-neutral evolution of interest rates inferred from the swaption matrix for pricing MBS is limits-to-arbitrage between the interest rate swap and MBS markets. Such differences could translate into differential  $OAS^{\mathbb{Q}}$  across moneyness. Since we do not impose cross-moneyness restrictions on  $OAS^{\mathbb{Q}}$ , our empirical specification is sufficiently flexible to capture these effects. As we will see, however, the cross-section of  $OAS^{\mathbb{Q}}$  is flat in moneyness, both in the full sample and in the pre-crisis period, suggesting that swaptions and MBS are priced fairly relative to each other.

models are more appealing, MBS investors favor reduced-form models (see, e.g., Fabozzi, 2016), for example, because in tranched CMOs, cash flows depend on prior prepayments, whereas structural models are solved by backward induction (McConnell and Buser, 2011). We follow standard industry practice and use a reduced-form prepayment model.

The exact details of practitioner models are not publicly available, but they vary in the choice of controls and weighting rules for historical data, and often make ad-hoc adjustments to incorporate likely effects of expected or announced policy changes affecting prepayments (for instance, the Home Affordable Refinance Program in 2009 or the introduction of additional agency fees on new mortgages since 2007). Therefore, in order to better capture market participants' expectations and be consistent with their pricing and spreads, we do not estimate our model on historical data, but instead extract prepayment model parameters from a survey of dealer models from Bloomberg LP. In these surveys, major MBS dealers provide their model forecasts of long-term prepayment speeds under different constant interest rate scenarios (with a range of +/- 300 bps relative to current rates).<sup>37</sup> Carlin et al. (2014) use these data to study the pricing effects of investors' disagreement measured from "raw" long-run prepayment projections. We, instead, extract model parameters of a monthly prepayment function by explicitly accounting for loan amortization, the path of interest rates, and changes in a pool's borrower composition.

Prepayment sensitivities to interest rates and other factors differ over time and across securities, and we thus estimate model parameters specific to each security and date. We model the date  $\tau$  single-month mortality rate (SMM), which is the fraction of a pool that prepays, of security j to match the average projected long-run survey speed for the different interest rate scenarios. These scenarios provide information on a pool's prepayment sensitivity to the incentive to refinance (INC<sup>*j*</sup><sub> $\tau$ </sub>).<sup>38</sup> The functional form of our prepayment model is:

$$\mathbf{s}_{\tau}^{j} = \chi_{\tau}^{j} \mathbf{s}_{1,\tau}^{j} + \left(1 - \chi_{\tau}^{j}\right) \mathbf{s}_{2,\tau}^{j} \text{ for } t < \tau \le t + \text{Maturity}_{j}$$

$$(4.1)$$

<sup>&</sup>lt;sup>37</sup>Until May 2003, dealers provided a single set of forecasts for each coupon (separately for Fannie Mae, Freddie Mac, and Ginnie Mae pass-through securities); since then, they provide separate forecasts for different vintages (for instance, a 5.5% coupon with average loan origination date in 2002 versus a 5.5% coupon with origination in 2005).

<sup>&</sup>lt;sup>38</sup>A notable detail is that in our model, we define INC as the end-of-month 10-year swap rate minus the pool's weighted average coupon (WAC). This is different from the "true" interest rate incentive faced by a borrower, which would be the mortgage rate minus WAC. However, our formulation has the major advantage that it does not require us to specify a model for the gap between mortgage rate and swap rate. The average gap between 30-year FRM rate and the 10-year swap rate over our sample period is about 1.2%.

where

$$s_{i,\tau}^{j} = b_{1}^{j} \min\left(Age_{\tau}^{j}/30, 1\right) + \kappa_{i} \cdot \frac{\exp\left(b_{2}^{j} + b_{3}^{j} \cdot INC_{\tau}^{j}\right)}{1 + \exp\left(b_{2}^{j} + b_{3}^{j} \cdot INC_{\tau}^{j}\right)} \text{ for } i = 1, 2.$$
(4.2)

This functional form allows us to capture a key feature of the time evolution of MBS prepayments: the so-called burnout effect, which is the result of within-pool heterogeneity in borrowers' sensitivity to the refinancing incentive. Because more sensitive borrowers are the first to exit the pool when rates decline, the pool's overall sensitivity to interest rates drops over time even if interest rates are unchanged.<sup>39</sup> To capture this effect, we assume the pool is composed of two types of borrowers: fast refinancers (group 1) and slow refinancers (group 2), with respective shares  $\chi_{\tau}$ and  $1 - \chi_{\tau}$  and shares  $s_{1,\tau}$  and  $s_{2,\tau}$ . This setup is a simplified version of the heterogeneous refinancing cost framework of Stanton (1995). As shown in equation (4.1), total pool prepayments are share-weighted averages of each group's prepayment speed. Each group's prepayment depends on two components. The first, which is identical to both groups, is governed by  $b_1$  and accounts for non-rate-driven prepayments, such as housing turnover. Because relocations are less likely to occur for new loans, we assume a seasoning of this effect using the industry-standard "PSA" assumption, which posits that prepayments increase for the first 30 months in the life of a security and are constant thereafter. The second component captures the rate-driven prepayments due to refinancing. This is modeled as a logistic function of the rate incentive (INC), with a sensitivity  $\kappa_i$ that differs across the two groups:  $\kappa_1 > \kappa_2$ . Since group 1 prepays faster,  $\chi_{\tau}$  declines over time in the pool. This changing composition, which we track in the estimation, captures the burnout effect. We provide more detail on the prepayment model and parameter estimation in Appendix E.

Figure 7 shows estimated prepayment functions for different loan pool compositions and using average parameters  $b_1$ ,  $b_2$ ,  $b_3$  across all securities in our sample. Prepayments (at an annual rate, known as the "constant prepayment rate," or CPR) display the standard S-shaped prepayment pattern of practitioner models. They are not very sensitive to changes in interest rates (and thus INC) for securities that are deeply ITM or OTM, but highly sensitive at intermediate moneyness ranges. The black (top) line shows that a pool with  $\chi = 1$  reaches a maximum predicted CPR of about 75% when it is deeply ITM, in contrast to only 35% when the share of fast refinancers is only 0.25 (red line). Thus, the changing borrower composition, even with a constant INC, implies a decline in prepayments over time because of the pool's burnout (decreasing  $\chi$ ).

<sup>&</sup>lt;sup>39</sup>In the extreme case, some borrowers never refinance even when their option is substantially in the money. Possible reasons for this non-exercise of the prepayment option include unemployment or other credit problems (Longstaff, 2005) or a lack of financial sophistication (sometimes called "woodhead" behavior; Deng and Quigley, 2012).

## 5 Model results

The pricing model produces the standard OAS measure as well as the  $OAS^{\mathbb{Q}}$ , which is adjusted for (or risk-neutral with respect to) not only interest rate risk but also prepayment risk. In this section we present the output of the model in terms of spreads in the cross section and time series. We then relate average  $OAS^{\mathbb{Q}}$  and prepayment risk premia to fixed-income and MBS-specific risk measures in order to help interpret model results and variation in MBS spreads. We finally discuss the response of MBS spreads to the Fed's first LSAP announcement in November 2008.

#### 5.1 OAS smile

The cross-sectional results are summarized in Figure 8. Similar to our findings for the TBA spreads (Figure 2), the OAS from our model exhibits a smile in moneyness (panel a): spreads are lowest for securities with moneyness near zero and increase for securities that are either OTM, or especially, ITM. As shown in panel (b), the  $OAS^Q$ , which strips prepayment risk from the OAS, does not appear to vary significantly with moneyness, suggesting that differences in liquidity do not contribute to the OAS smile. Instead, as shown in panel (c), the difference between the OAS and  $OAS^Q$  closely matches the smile pattern in the OAS; in other words, the differential exposure to prepayment risk explains the cross-sectional pattern in the OAS. Additionally, panel (d) displays the difference in implied long-run prepayment speeds between the risk-neutral (Q) and physical (IP) prepayment models. OTM securities tend to have slower risk-neutral speeds, while ITM securities tend to have faster risk-neutral speeds. Thus, in both cases the risk-neutral model tilts the prepayment speeds in the undesirable direction from the point of view of the investor. That is, market prices imply that prepayments are faster (slower) for securities that suffer (benefit) from faster prepayments, which is exactly what one would expect as market-implied prepayments include compensation for risk.

In Table 5, we use regressions to study if these cross-sectional patterns in the two components of OAS are robust to including month fixed effects (in order to focus on purely cross-sectional variation) and to ending the sample before the financial crisis period by sorting the different coupons in bins by moneyness, as in the earlier Table  $3.^{40}$  Panel (a) of Table 5 shows that there is little systematic pattern in  $OAS^Q$  across bins; results in columns (2) and (4) suggest in fact that ATM

 $<sup>^{40}</sup>$ We use fewer bins because our IO/PO strips have less coverage of very deeply OTM (moneyness < -1.75) or ITM (moneyness > 2.75) coupons.

coupons may have slightly *higher*  $OAS^Q$  than the surrounding coupons, but the differences are small. There is some evidence that the most deeply ITM coupons (moneyness  $\geq 2.25$ ) may command a positive premium, which could be driven by the reduced liquidity of these (generally very seasoned) coupons. Turning to the prepayment risk premium, panel (b) of Table 5 shows that the (slightly tilted) smile pattern shown in panel (c) of Figure 8 is robust to adding month fixed effects and excluding the financial crisis period. The coefficients suggest that the magnitude of the prepayment risk premium is economically meaningful: securities that are 1.25 percentage points or more ITM command a premium of 20 basis points (annual) or more relative to ATM securities.

In sum, while the prepayment risk premium in the cross section is strongly linked to the moneyness of the securities, we find little evidence that this is also the case for the remaining risk premium (*OAS*<sup>Q</sup>), suggesting that differential liquidity across coupons is likely not a major driver of cross-sectional variation in spreads (except perhaps for the most deeply ITM securities). Instead, the smile in prepayment risk premia suggests that both ITM and OTM securities earn positive compensation for prepayment risk, consistent with them not being hedges for one another. Going back to our discussion in Section 3.4, this would arise if either a single shock drives prepayments but with opposite effects on ITM and OTM securities (case 2), or, perhaps more realistically, if OTM and ITM securities were subject to distinct but independent shocks (case 3). For instance, OTM prepayments could be primarily driven by housing-relocation shocks, such as house prices, whereas variation in ITM prepayments could be due to shocks to refinancing activity.

Though Table 5 shows that the cross-sectional patterns in the two components of OAS are robust to including month fixed effects and to ending the sample in 2007, a potential concern is whether these results could be due to potential misspecification of the risk-neutral prepayment function as a multiple of the physical prepayment function. Note that the multiple  $\Lambda$  is security and date specific, so that our estimate of the Q prepayment function is semi-parametric and allows us to capture risk premia with the least restrictive prior assumptions. Nevertheless, in Appendix F we explore an alternative identification methodology that estimates the risk-neutral prepayment function. While this alternative approach does not allow the estimation of prepayment risk premia, it confirms this section's finding that the  $OAS^Q$  does not vary systematically with moneyness.

#### 5.2 Time-series variation

We now turn to the variation in the average OAS in the time series. As in Section 3, we construct a market value-weighted index of our model-implied OAS.<sup>41</sup> Comparing the OAS in Figure 9 to the corresponding series in panel (a) of Figure 2 confirms that our model output is close to its dealer counterparts. The level of the average OAS is generally close to zero, but increases in periods of market stress. Further, our pricing model finds the difference between OAS and  $OAS^Q$  to be small and the two series to tightly co-move, meaning that much of the OAS variation results from changes in  $OAS^Q$  (gray line). Thus, although it is an important determinant of the cross-sectional variation in spreads, prepayment risk does not appear to be the dominant driver of the OAS timeseries variation. Indeed as shown in Figure 1, the share of deeply OTM or ITM securities, which earn most compensation for prepayment risk, is limited; this arises because most securities are close to ATM when issued.<sup>42</sup> However, prepayment risk in the MBS universe can be significant when mortgage rates move sharply, as in early 1998, the summer of 2003, and in 2009 and 2010 as mortgage rates reached historic lows and the gap between the average OAS and  $OAS^Q$  widened.

We next investigate the determinants of the time-series variation in OAS, and in particular its two components: *OAS*<sup>Q</sup> and the prepayment risk premium. Table 6 shows results from monthly regressions of the OAS, and its components, on mortgage-specific risk factors, such as spreads on agency debt (or debentures) relative to swaps, agency MBS issuance (normalized by broker-dealer book equity, and subtracting Fed MBS purchases in 2009 and 2010), the average squared moneyness of the MBS universe, as well as dealer disagreement about future prepayment speeds, which Carlin et al. (2014) find to be significant predictors of MBS returns.<sup>43</sup> We construct our disagreement measure in the same way as Carlin et al. (2014), but based on the Bloomberg surveys on Fannie Mae prepayment speeds that we already use for the physical prepayment model (while Carlin et al. instead use forecasts of Ginnie Mae prepayments). We also include the credit spread, which was the main economically and statistically significant factor in the TBA analysis in Table 2.

We find that average  $OAS^{\mathbb{Q}}$  are related to spreads on (unsecured) agency debentures. As noted earlier, agency MBS are typically perceived as being free of credit risk, but since the government

<sup>&</sup>lt;sup>41</sup>We do this by first averaging spreads across trusts within a coupon (weighting each trust by its market value, given by its remaining principal balance times the sum of the prices of its IO and PO strips). Then, we average across coupons in a given month (weighting each coupon by its market value based on TBA prices, as in Section 3.3).

<sup>&</sup>lt;sup>42</sup>After issuance, the moneyness of securities fluctuates as a function of interest rates and the remaining balance declines with prepayments, lowering the importance of older issues on a value-weighted basis.

<sup>&</sup>lt;sup>43</sup>Agency debenture yields and swap yields come from Barclays; agency MBS issuance is from eMBS; Fed MBS purchases from the https://www.federalreserve.gov/regreform/reform-mbs.htm; and broker-dealer equity from Table L.130 in the Flow of Funds (interpolated linearly between quarter-ends).

guarantee on securities issued by Fannie Mae is only implicit, investors' perceptions of this guarantee (along with the perceived credit risk of agencies) may change over time and thus affect both spreads on agency debt and MBS. In particular, both *OAS*<sup>Q</sup> and agency debt spreads increased in the fall of 2008, when Fannie Mae and Freddie Mac were placed in conservatorship by the U.S. Treasury. The spreads on MBS and agency debt do, however, also co-move earlier in the sample, pointing to other common factors such as liquidity and funding costs of these securities.<sup>44</sup>

Credit spreads (Baa-Aaa) continue to be significantly related to OAS, mostly through  $OAS^Q$  rather than the prepayment risk component. The sensitivity of  $OAS^Q$  to credit spreads suggests common pricing factors in the MBS and credit markets, such as limited risk-bearing capacity of financial intermediaries (see, for example, Shleifer and Vishny, 1997; Duffie, 2010b; Gabaix et al., 2007; He and Krishnamurthy, 2013). In these models, financial intermediaries are marginal investors in risky assets; when their financial constraints bind, their effective risk aversion increases, raising risk premia in all markets. Thus, when the supply of risky assets relative to intermediaries' capital decreases, financial constraints are relaxed, lowering required risk compensation. In line with these predictions, we find that higher supply of MBS, measured by issuance relative to mark-to-market equity of brokers and dealers, also positively correlates with average  $OAS^Q$ .<sup>45</sup> We explore this channel further in the next section, where we study the effects of Fed MBS purchases, which absorb supply in the hands of investors, on the OAS and its components.

Disagreement about future prepayments (for given rates) is positively related to the prepayment risk premium, in line with the findings of Carlin et al. (2014); however, the economic magnitude of the coefficient is relatively small. Finally, as previously discussed, the OAS smile implies that spreads, and in particular their prepayment risk component, are largest for deeply OTM and ITM securities. This suggests that when the market-value weighted moneyness is either very positive or very negative, the average OAS and prepayment risk premium should be large. In line with this prediction, we find average squared moneyness to be positively related to the average prepayment risk premium.

<sup>&</sup>lt;sup>44</sup>The spread between Fannie Mae debentures and Treasury bonds of equal maturity fell following the conservatorship announcement, but then substantially increased through the end of 2008. Since there should have been essentially no difference in the strength of the debt guarantee between debentures and Treasuries at that point, and since the spread widening was stronger for shorter maturity bonds, Krishnamurthy (2010) argues that this reflects a flight to liquidity. In line with this interpretation, our *OAS*<sup>Q</sup> also reaches substantially higher levels in October compared to August 2008, despite the reduction in credit risk to investors.

<sup>&</sup>lt;sup>45</sup>As noted above, we net out monthly Fed purchases by settlement month over 2009-10 from new issuance to more properly measure fluctuations in net supply that is absorbed by investors. Relatedly, GSEs' conservatorship agreements have required them to divest their portfolios since 2010, a process that would affect net supply in the hands of other investors. However, during the first round of LSAPs, which we focus on, changes in GSE holdings of agency MBS were quite small relative to Fed purchases and issuance.

#### 5.3 Interpreting the OAS response to the Fed's LSAPs

As discussed above, MBS spreads are positively related to MBS supply, a finding that is consistent with intermediary asset pricing models with limited risk-bearing capacity. In this section we provide additional evidence on this channel by focusing on the Fed's large-scale asset purchase (LSAP) program. The program has entailed an unprecedented shift in the composition of the MBS investor base as the Fed now holds more than a quarter of the total agency MBS universe—up from nothing prior to the financial crisis. We decompose spreads using our pricing model and show how our model can explain the divergence in OAS across different coupons following the initial announcement of the program.

We focus on spread movements after November 25, 2008, when the Fed announced its first round of purchases of up to \$500 billion in agency MBS.<sup>46</sup> Based on the current coupon MBS, which is the focus of much of the research on this topic—with the important exception of Krishnamurthy and Vissing-Jorgensen (2013) which we discuss below—the announcement had a substantial effect on the MBS market (see, e.g., Gagnon et al. 2011 or Hancock and Passmore 2011; Stroebel and Taylor 2012 are more skeptical). According to different dealer models, the current coupon OAS, which had been at record levels of 75–100 basis points over October and November 2008, fell 30–40 basis points on the day of the announcement, and stayed around the lower level afterwards. Consistent with the decline in secondary MBS spreads and yields, headline 30-year FRM rates dropped nearly a full percentage point between mid-November and year-end 2008.

Spread movements on the current coupon MBS alone hide significant heterogeneity across the coupon stack, as evidenced by the series in Figure 10, which are median spreads across dealer models (the same as used in Section 3) for the four main coupons traded at that time. OAS that were all at similarly elevated levels in the fall of 2008 diverged following the announcement: spreads on low coupons (4.5 and 5) fell, while those on higher coupons were little changed and then even widened through year-end. Since high coupons represent the majority of outstanding MBS, this implies that, for specialized investors in this market, the recapitalization effect of monetary policy described in Brunnermeier and Sannikov (2012) was limited.

The earlier findings from our model suggest two potentially countervailing effects of Fed MBS purchases on OAS. On the one hand, Fed purchases reduce MBS supply to be absorbed by risk-

<sup>&</sup>lt;sup>46</sup>The Fed also announced that it would purchase up to \$100 billion in agency debt. The purchases began in early January 2009. The program was then extended in March 2009, when it was announced that an additional \$750bn in agency MBS, \$100bn in agency debt, and \$300bn in long-term Treasuries would be purchased over the following year.

sensitive investors, thereby reducing the required risk premium on all MBS (through  $OAS^Q$ ).<sup>47</sup> On the other hand, movements in mortgage rates associated with such purchases alter securities' moneynesses, shifting the OAS along the smile and changing the prepayment risk premium.

The results from our model are shown in the bottom panel of Figure 11. First, OAS movements (in black) for IO/PO pass-throughs are similar to the TBA ones.<sup>48</sup> In terms of the MBS supply effect, we discussed above how the  $OAS^Q$  component is flat across coupons and declines with a reduction in supply. Consistent with this, we find that the  $OAS^Q$  evolves similarly for the 4.5, 5, and 5.5 coupons. For the 6 coupon,  $OAS^Q$  increases in November and December, before converging toward the other coupons in January as actual LSAP purchases begin. The  $OAS^Q$  effect thus suggests that the LSAP program lowered non-prepayment risk premia across the coupon stack.<sup>49</sup>

The cross-coupon "homogeneous" *OAS*<sup>Q</sup> impact of the Fed's policy is, however, masked by changes in the prepayment risk premia that vary with MBS moneyness, shown in the top panel of Figure 11. The 4.5 starts out OTM and moves ATM as mortgage rates drop, while the 5.5 and 6, which are around ATM in October, move quite deeply ITM. Based on the OAS smile, the 4.5 should command a prepayment risk premium prior to November and the 5.5 and 6 coupons afterward. The bottom panel shows that this is indeed the case: the narrowing in the gap between the black and gray lines means that the decrease in the OAS of the 4.5 coupon is in part due to the decrease in its prepayment risk exposure following the drop in rates. In contrast, the prepayment risk premia on the 5.5 and 6 coupons are high from December onward as they move deeply ITM and are more sensitive to prepayment risk.

In sum, increases in the moneyness of high coupons following the November 2008 LSAP announcement led to an increase in their prepayment risk premium, which explains why their OAS did not fall, even though *OAS*<sup>Q</sup> declines across the coupon stack as the Fed started absorbing MBS supply. Differential OAS responses are not specific to the 2008 LSAP announcement. For example, Krishnamurthy and Vissing-Jorgensen (2013) discuss how during the "taper tantrum" around the June 19, 2013 FOMC meeting, the OAS increased substantially for low coupons, while the OAS on higher coupons stayed almost unchanged. These authors interpret this latter fact as evidence that capital constraints (or limited risk-bearing capacity) are unimportant at that time and argue

<sup>&</sup>lt;sup>47</sup>Over several months in early 2009, monthly Fed MBS purchases were absorbing essentially all new MBS issuance.

<sup>&</sup>lt;sup>48</sup>The strips we have available do not necessarily have the same characteristics as what the dealers assume to be cheapest-to-deliver in TBA trades; therefore, our OAS levels do not exactly line up with theirs for all coupons in all months. Nevertheless, patterns are very similar, especially in changes.

<sup>&</sup>lt;sup>49</sup>In addition to the supply effect, the Fed announcement may also have strengthened the perceived government backing of Fannie Mae and Freddie Mac and improved the liquidity of agency securities (Hancock and Passmore, 2011; Stroebel and Taylor, 2012).

for a "scarcity effect" for low coupons, which implies large price responses for coupons directly targeted by Fed purchases.<sup>50</sup>

Our model, which does not rely on cross-coupon segmentation, suggests that the increase in the quantity of securities that non-Fed investors had to hold because of the anticipated taper increases the required risk premium (through  $OAS^Q$ ) on all MBS. However, because rates increase at that point, the prepayment risk premium on high coupons (that were previously deeply ITM) falls as they become ATM, so that their overall OAS remains roughly constant. Because of the differential prepayment risk exposure across MBS, the stability in high-coupon OAS around this event is thus *not* evidence of a lack of capital constraints for MBS investors, implying that potential sales of high coupons might still increase OAS on lower coupons and increase mortgage rates.

## 6 Conclusions

Our pricing model has two main implications. In the cross section, risk premia associated with non-interest-rate prepayment factors explain the OAS smile, which is the fact that OAS tend to be lower for at-the-money MBS than for others. In the time series, the model implies that the average OAS is primarily driven by non-prepayment risk factors linked to credit spreads, MBS supply, and spreads on other agency debt. These results suggest that risk-bearing capacity of MBS investors, and the liquidity and default risk of agency securities, drive aggregate spread variation and are important determinants of homebuyers' funding costs.

From a broader perspective, this paper provides further evidence for intermediary asset pricing in fixed income markets. Recent literature (such as He and Krishnamurthy, 2013; Brunnermeier and Sannikov, 2014) has proposed that intermediaries' risk-bearing capacity impacts risk premia during periods of market stress. While much of our discussion has focused on the response of the *OAS*<sup>Q</sup> to Fed purchases in the wake of the crisis, the *OAS*<sup>Q</sup> reacts to changes in the supply of MBS and credit spreads even during normal market conditions, consistent with theories (e.g. Gromb and Vayanos, 2002; Brunnermeier and Pedersen, 2009; Adrian and Boyarchenko, 2012) that link risk premia to intermediary balance sheet constraints even in periods when intermediaries are well capitalized.

<sup>&</sup>lt;sup>50</sup>According to this channel, as demand for a specific coupon increases, the quality of pools delivered in the TBA contract (as measured by their prepayment characteristics) improves, so that the equilibrium price increases to elicit pool delivery. Because this scarcity channel works at the level of each coupon, it predicts that Fed purchases do not affect risk premia on non-targeted MBS, such as higher coupon TBAs or MBS not deliverable in the TBA market.

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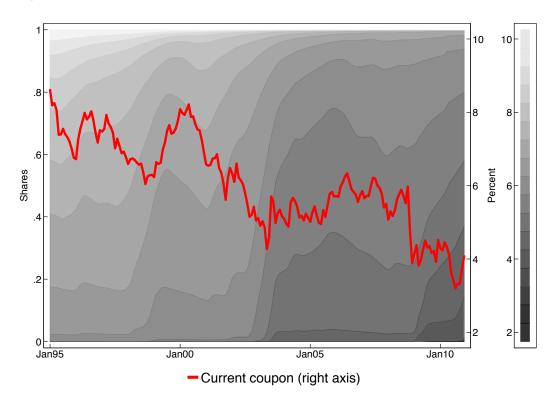
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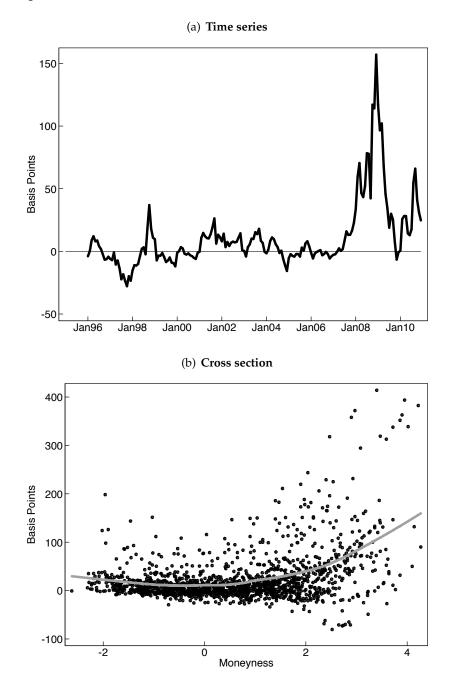
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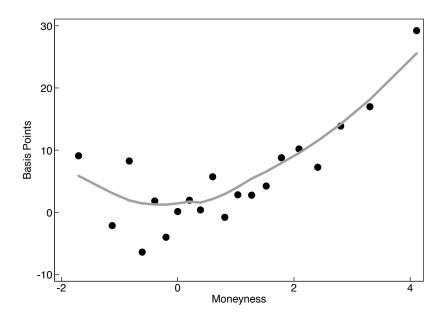
**Figure 1: Share of total MBS remaining balance by coupon against the current coupon rate.** The shaded gray areas are the share (left axis) of the total remaining principal balance (RPB) in each 30-year fixed-rate Fannie Mae MBS coupon relative to the total RPB of all coupons. RPBs are from eMBS. The red line is the current coupon (the interpolated coupon that trades at par) in the TBA market (right axis), obtained from dealer data.



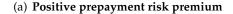
**Figure 2: Time-series and cross-sectional variation of the OAS based on dealer TBA data.** The top panel displays the time series at monthly frequency of the option-adjusted spread (to swaps) on a value-weighted index based on TBA quotes from six dealers. The bottom panel displays a scatterplot and a local smoother of the cross-sectional variation in the OAS across MBS coupons as a function of their moneyness. Moneyness is calculated as the coupon rate plus 50 basis points (to account for servicing and the guarantee fee) minus the 30-year fixed-rate mortgage rate obtained from Freddie Mac. The figure only includes coupons with remaining principal balance of at least 100 million in 2009 dollars. All data is as of month-end and covers the period 1996-2010. Further details on the construction of the value-weighted series is reported in Section 3. Appendix B.1 discusses the sample in more detail.

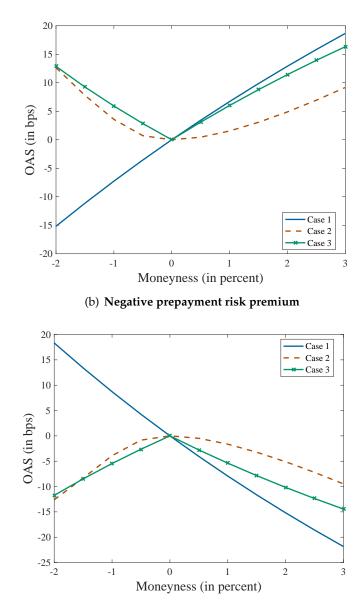


**Figure 3:** Cross-sectional variation in hedged returns. The figure shows monthly  $t \rightarrow t + 1$  hedged returns on MBS in the Bloomberg Barclays index by coupon against securities' moneyness as of the end of month *t*. Moneyness is calculated as the coupon rate plus 50 basis points (to account for servicing and the guarantee fee) minus the 30-year fixed-rate mortgage rate obtained from Freddie Mac. Each dot represents mean returns for one of twenty equally sized moneyness bins. The gray line represents a local smoother fitted to the underlying data, that is, each coupon's return. The sample period is January 1994 to June 2016.

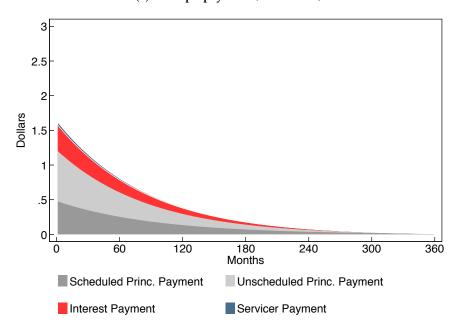


**Figure 4: OAS under different assumptions on the prepayment function.** The lines represent the OAS as function of moneyness for a hypothetical pass-through MBS with zero exposure to liquidity shocks ( $\alpha = 0$ ) and 29 years of maturity left. In Case 1 (blue solid line), prepayments are independent of the moneyness of the security ( $s_t = \gamma_1$ ). In Case 2 (dashed red line), prepayments are linear in the moneyness of the security ( $s_t = \bar{s} + \gamma_1 (c - r)$ ). In Case 3 (crossed green line), prepayments for OTM and ITM securities are driven by different shocks ( $s_t = \gamma_1 \mathbf{1}_{c < r} + \gamma_2 \mathbf{1}_{c \ge r}$ ). In all three cases, the parameters of the prepayment function are log-normally distributed,  $\gamma_i \sim \log \mathcal{N}(\mu_i, \sigma_i^2)$ , with ( $\mu_i, \sigma_i$ ) calibrated to historical prepayments. The top (bottom) panel shows OAS with positive (negative) prepayment risk premium. Calculations are based on the formulas in Section 3.2.

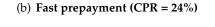




**Figure 5: MBS cash flows for different prepayment speeds.** The colored areas represent (undiscounted) monthly cash flows for a hypothetical MBS with original principal of \$100, loan rate of 4.5% and coupon of 4%. The 50 basis point difference between the loan and coupon rate is earned by the servicer and guaranteeing agency (blue area). Scheduled (amortization) and unscheduled (prepayment) principal payments are shown as gray areas. The sum of the two areas in each chart adds up to \$100. As a result PO strips benefit from faster prepayments (early repayment). The IO strip receives the monthly interest payment (coupon rate at monthly rate) on the principal balance outstanding. The top (bottom) panel shows a slow (fast) constant prepayment rate scenarios. With fast prepayments, interest payments (red area) are much smaller, thus IOs suffer from fast prepayments. Calculations are based on the formulas in Appendix E.4.



(a) Slow prepayment (CPR = 12%)



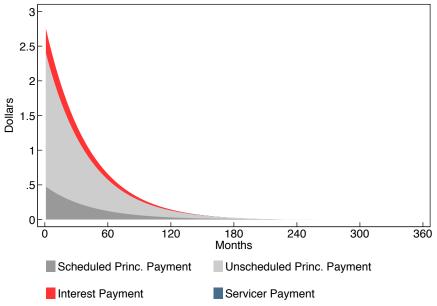
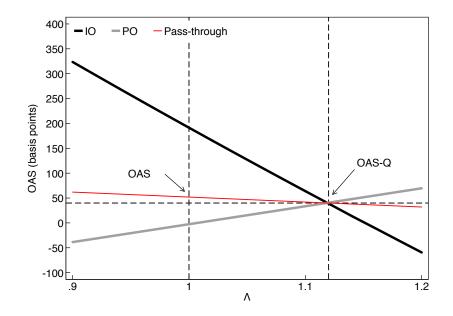
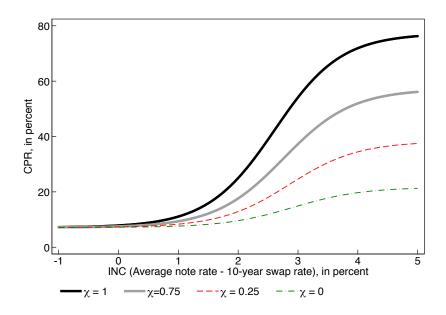


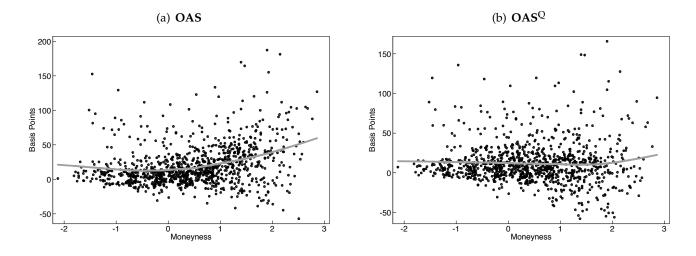
Figure 6: Visualization of the identification assumption for  $OAS^Q$ . The figure shows the OAS on a IO, PO and pass-through for the same underlying collateral as a function of the prepayment multiple ( $\Lambda$ ) on the physical prepayment speed. Higher  $\Lambda$  increases the market-implied prepayment speed relative to the physical one. The OAS on the IO (PO) declines (increases) in  $\Lambda$ . This follows from the relation between the value of each strip and prepayments shown in Figure 5. The OAS on the pass-through in this example also declines in  $\Lambda$  because the pass-through is a premium security ( $P^{IO} + P^{PO} >$ \$100). The three OAS differ at the physical speed ( $\Lambda = 1$ ) but are equalized at the risk-neutral speed. This value of  $\Lambda$  defines the market implied prepayment speed and the  $OAS^Q$ .



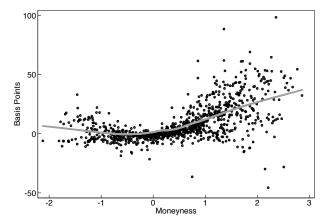
**Figure 7: Survey-implied average prepayment rates at different levels of burnout.** The figure shows the prepayment function in equation (4.2) with parameters  $b_1$ ,  $b_2$  and  $b_3$  set at their sample means. Differences in  $\chi$  parametrize the burnout effect. Fast refinancers (group 1) and slow refinancers (group 2) are present in the pool with shares  $\chi$  and  $1 - \chi$ . As the share of more sensitive borrowers  $\chi$  falls, the pool's overall sensitivity to interest rates is reduced. The vertical axis is the "conditional prepayment rate" (CPR, or annualized prepayment rate) and the horizontal axis is a measure of the incentive to prepay the mortgage.



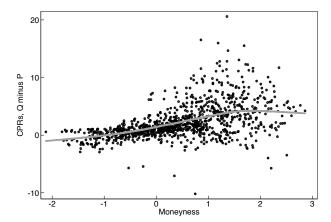
**Figure 8: Cross-sectional variation in model-implied OAS and prepayment speeds.** The panels show scatterplots and local smoothers of the cross-sectional variation in the model-implied OAS, OAS<sup>Q</sup>, the prepayment risk premium (OAS–OAS<sup>Q</sup>) and the difference between the market-implied and physical lifetime prepayment speed. All measures are for the recombined pass-throughs obtained as the sum of the value of the IO and PO components. Refer to Figure 6 for a summary of the relation between physical and market-implied spreads and prepayment speeds. The horizontal axis measures moneyness, which is calculated as the coupon rate plus 50 basis points (to account for servicing and the guarantee fee) minus the 30-year fixed-rate mortgage rate obtained from Freddie Mac. Additional details on the pricing model are given in Section 4.



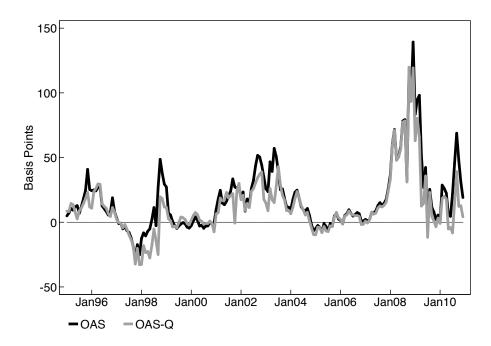
(c)  $OAS - OAS^{\mathbb{Q}}$ 



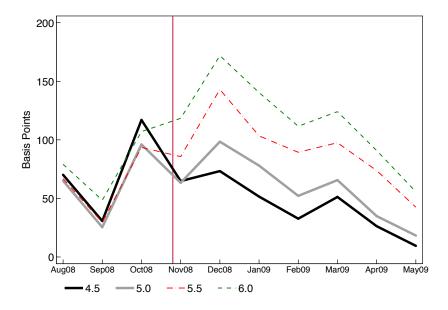
(d) **Prepayment speeds:**  $\mathbb{Q}$  vs.  $\mathbb{P}$ 



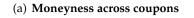
**Figure 9: Time series of the OAS on the pass-through index from IO/PO strips.** This figure shows time-series variation in OAS and  $OAS^Q$  on a value-weighted index computed from IO/PO prices. To construct the index, we first average spreads across trusts for each coupon using market weights. Then, for each month we average across coupons using each market value from TBA prices. For each trust, the value of the pass-through is the sum of the value of the IO and PO. Refer to Figure 6 for a summary of the relation between OAS and  $OAS^Q$  and to Section 4 for more details on the pricing model.

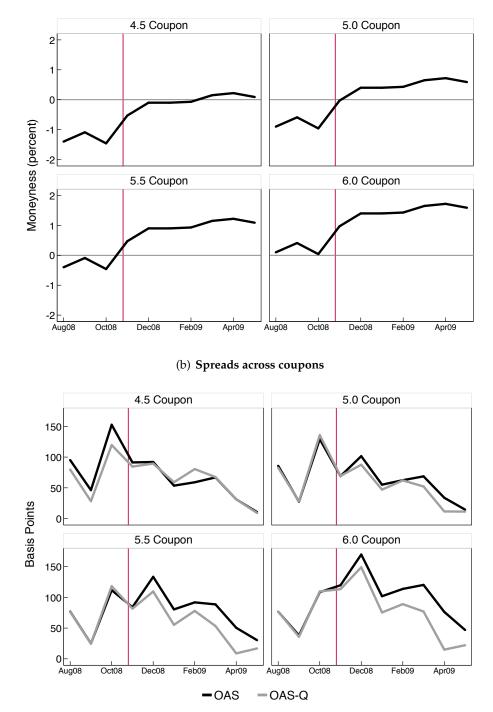


**Figure 10: Variation in OAS around the November 25, 2008 LSAP announcement by the Federal Reserve.** This figure shows the evolution of the OAS for Fannie Mae MBS with coupons 4.5, 5.0, 5.5, and 6.0%, which were the most heavily traded at that time. OAS data is based on median TBA quotes from six dealers and reported as of month-end. The vertical line indicates November 25, 2008, which is the announcement date.



**Figure 11: OAS decomposition around the November 25, 2008 LSAP announcement by the Federal Reserve.** This figure shows the MBS moneyness by coupon (upper panels) and movements in OAS and *OAS*<sup>Q</sup> (bottom panels) based on IO/PO prices and our pricing model. Moneyness is calculated as coupon plus 50 basis points minus the 30-year fixed-rate mortgage rate (from Freddie Mac). Data is as of month-end. Vertical lines indicate November 25, 2008, when the Federal Reserve announced its large-scale asset purchase program.





**Table 1: Equivalent probability measures of interest.** This table shows the three equivalent probability measures considered in this paper, the associated yield premium notation, and the necessary conditions for the discounted gains process of a security with cash flows:  $d\chi_t = \mu_{\chi t} dt + \sigma_{\chi r,t} dZ_{rt} + \sigma_{\chi \gamma,t} dZ_{\gamma t}$  under the physical measure that loses a fraction  $\alpha_{\chi}$  in a liquidity event to be a martingale under each measure.

Measure name	Measure notation	Yield premium	Necessary condition for martin- gale discounted gains process
Interest-rate-risk neutral	$Q_r$	$OAS_t$	$\mid \sigma_{\chi\gamma,t}=0,  lpha_{\chi}=0;  \forall \sigma_{\chi r,t}$
Interest-rate-and- prepayment-risk neutral	Qr,γ	$OAS_t^{\mathbb{Q}}$	$ \mid \alpha_{\chi} = 0; \forall \sigma_{\chi r, t}, \forall \sigma_{\chi \gamma, t}$
Risk-neutral	Q	0	$\mid \forall \sigma_{\chi r,t}, \forall \sigma_{\chi \gamma,t}, \forall \alpha_{\chi}$

**Table 2: Time-series regressions of OAS on the TBA index from dealer data.** Coefficient estimates from OLS regression of the TBA index OAS (shown in the upper panel of Figure 2) on the Aaa-Treasury spread, the Baa-Aaa spread, the slope of the Treasury yield curve (difference between the 10-year and 3-month Treasury yield) and the 2-year into 10-year swaption implied volatility. The Aaa and Baa corporate bond yields are from Moody's, while swaption volatility is from a dealer. Data is at monthly frequency and measured as of month-end. All regressors are standardized to have zero mean and unit standard deviation. Newey-West standard errors (6 lags) shown in brackets. Significance: \* p < 0.1, \*\* p < 0.05, \*\*\* p < 0.01.

	(1) OAS		(2) OAS				
Aaa - Treas	3.2**	[1.5]	2.3*	[1.2]			
Baa - Aaa	19.6***	[3.4]	9.7**	[3.8]			
Treas Slope	-0.5	[2.4]	2.0	[1.3]			
Swaption Vol.	2.7	[4.2]	-0.8	[2.4]			
Const	8.8***	[1.8]	3.6***	[1.3]			
Adj. R2	0.76		0.31				
Obs.	180		139				
Dates	199601.201012		199601.200707				

Table 3: Cross section of OAS on TBA securities from dealer data by moneyness. Coefficient estimates from OLS regressions of the OAS (annualized, in basis points) on different moneyness bin dummies, either including or excluding calendar month fixed effects. Moneyness of a coupon *j* at time *t* is defined as  $Moneyness_{j,t} = Coupon_j + 0.5 - FRMrate_t$ . The dummy for the moneyness bin surrounding zero, [-.25, .25), is the omitted category. Columns 1 and 2 in panel (a) show estimates for full sample 1996-2010; columns 3 and 4 exclude the period August 2007-December 2010 (thus excluding the financial crisis). In panel (b) the sample is split based on the moneyness of the MBS market: "Market ITM (OTM)" indicates that the balance-weighted average moneyness of all outstanding Fannie Mae MBS is > 0 (< 0). In columns 1 and 2 of panel (b), the first two bins are merged since there are only 7 observations with moneyness<-1.75 when the market is ITM; similarly in columns 3 and 4 the last two bins are merged since there are only 5 observations with moneyness≥2.75 when the market is OTM. Robust standard errors (clustered at the month level) shown in brackets. Significance: \* p < 0.1, \*\* p < 0.05, \*\*\* p < 0.01.

		(a) Full	l sample and j	pre-cris	sis sample			
		Full s	ample		Pre-crisis sample			
	(1)		(2)		(3)	(4)		
Moneyness bin:								
< -1.75	18.7***	[4.5]	18.0***	[2.3]	14.9***	[2.0]	14.1***	[2.4]
[-1.75, -1.25)	8.9***	[1.9]	$10.4^{***}$	[1.6]	7.7***	[1.2]	8.3***	[1.4]
[-1.25,75]	$4.0^{***}$	[1.0]	5.3***	[1.1]	4.3***	[0.9]	$4.8^{***}$	[0.9]
[75,25]	-0.0	[0.4]	-0.1	[0.4]	$0.8^{*}$	[0.4]	0.8**	[0.4]
[.25, .75)	1.5***	[0.5]	1.5***	[0.5]	-0.1	[0.4]	-0.1	[0.4]
[.75, 1.25)	6.4***	[1.2]	6.4***	[1.2]	2.6***	[0.9]	2.5***	[0.9]
[1.25, 1.75]	15.6***	[2.2]	15.0***	[2.1]	7.2***	[1.8]	7.2***	[1.7]
[1.75, 2.25]	27.2***	[3.7]	24.6***	[3.3]	11.9***	[3.0]	$11.4^{***}$	[3.0]
[2.25, 2.75]	41.3***	[6.4]	33.9***	[5.5]	26.0***	[5.3]	23.2***	[5.2]
$\geq 2.75$	93.4***	[13.3]	85.4***	[10.9]	50.5***	[7.2]	47.1***	[7.2]
Const	7.5***	[1.5]			0.5	[0.8]		
Month FEs?	No		Yes		No		Yes	
Adj. R2	0.22		0.61		0.24		0.38	
Adj. R2 (within)			0.31				0.25	
Obs.	1532		1532		1151		1151	
Dates	199601.201012		199601.201012		199601.200707		199601.200707	

(a) Full	sampl	le and	pre-crisis	sample
(**) - ****			P10 01010	

(b) Subsamples based on moneyness of MBS market

	Market ITM				Market OTM			
	(1)		(2)		(3)	(4)		
Moneyness bin:								
< -1.75					20.3***	[3.6]	18.5***	[2.5]
[-1.75, -1.25)	13.6***	[2.6]	18.1***	[2.4]	7.8***	[1.1]	7.8***	[1.1]
[-1.25,75]	5.7***	[1.5]	7.9***	[1.6]	1.2*	[0.7]	1.2*	[0.7]
[75,25]	0.3	[0.5]	0.2	[0.6]	-0.7	[0.5]	-0.7	[0.5]
[.25, .75)	0.7	[0.7]	0.7	[0.7]	3.4***	[0.8]	3.4***	[0.8]
[.75, 1.25)	4.3***	[1.6]	4.2***	[1.6]	11.3***	[1.4]	11.3***	[1.4]
[1.25, 1.75]	8.9***	[2.5]	8.6***	[2.4]	31.8***	[3.6]	30.8***	[3.2]
[1.75, 2.25]	17.4***	[3.8]	15.6***	[3.5]	55.4***	[7.9]	50.7***	[6.5]
2.25, 2.75)	44.0***	[7.6]	36.6***	[6.4]	27.1**	[11.8]	23.4**	[10.7]
$\geq 2.75$	98.5***	[13.4]	89.6***	[11.2]				
Const	8.5***	[1.6]			5.2	[3.3]		
Month FEs?	No		Yes		No		Yes	
Adj. R2	0.27		0.61		0.13		0.75	
Adj. R2 (within)			0.35				0.34	
Obs.	1039		1039		493		493	
Dates	199601.201012		199601.201012		199606.200810		199606.200810	

Table 4: Regressions of MBS returns on OAS and mortgage rates. Panels (a) and (b) show coefficient estimates from OLS regressions of 12-month ( $t \rightarrow t + 12$ , meaning from the end of month *t* to the end of month t + 12 and reported as "1-year") MBS returns and annualized 1-month ( $t \rightarrow t + 1$ , reported as "1-month") MBS returns from the Bloomberg Barclays index on the Barclays OAS to Treasuries as of the end of month *t*. In panel (a), returns are hedged, while in panel (b), returns are unhedged and net of the risk free rate. Panel (c) shows coefficient estimates from regressions of hedged returns in month t + 1 on changes in the Freddie Mac 30-year fixed mortgage rate, denoted "FRM", from the end of month t to t + 1. "ITM" is an indicator for an MBS being in-the-money (i.e. moneyness > 0) as of the end of month *t*. "Market ITM (OTM)" indicates that the balanceweighted average moneyness of all outstanding Fannie Mae MBS is > 0 (< 0). Moneyness is calculated as coupon plus 50 basis points minus the 30-year fixed-rate mortgage rate (from Freddie Mac). Newey-West standard errors (18 lags) in brackets for 1-year returns; robust standard errors (clustered at the month level) for 1-month returns. Significance: \* p < 0.1, \*\* p < 0.05, \*\*\* *p* < 0.01.

	(a) Hedged returns and OAS								
	(1) 1-year	(2) 1-year	(3) 1-month	(4) 1-month	(5) 1-month	(6) 1-month			
OAS	1.36*** [0.21]	1.41*** [0.13]	1.75*** [0.63]	1.95*** [0.61]	0.50* [0.28]	1.15*** [0.27]			
ΔOAS					-2.37*** [0.30]	-2.33*** [0.25]			
Month FEs? Adj. R2	No 0.19 1990	Yes 0.70 1990	No 0.02 2076	Yes 0.53 2076	No 0.56 2064	Yes 0.78 2064			
Month FEs?					[0.30] No				

(a)	Hedge	d returns	and	OA	1S
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	(b) Unhedg	ed returns	s and OAS	
(1)	(2)	(3)	(4)	

	(1) 1-year	(2) 1-year			(5) 1-month	(6) 1-month
OAS	1.48*** [0.35]	0.80***	1.90** [0.76]	1.58* [0.87]	1.04 [0.74]	0.44
ΔOAS	[0.00]	[•]	[00]	[0.01]	-1.85*** [0.31]	-3.32*** [0.38]
Month FEs? Adj. R2 Obs.	No 0.02 1990	Yes 0.78 1990	No 0.01 2076	Yes 0.60 2076	No 0.11 2064	Yes 0.76 2064

(c) 1-month	hedged return	ns and mortgage rates	
(-) =			

	0		8.8		
	(1) Full	(2) Full	(3) Market ITM	(4) Market OTM	
ΔFRM	-0.58***		-0.74***	-0.39*	
	[0.18]		[0.26]	[0.21]	
$\Delta FRM \times ITM$	0.92***	$1.14^{***}$	0.91***	1.57***	
	[0.23]	[0.25]	[0.23]	[0.50]	
Month FEs?	No	Yes	No	No	
Adj. R2	0.03	0.55	0.02	0.16	
Obs.	2076	2076	1622	454	

Table 5: Cross sectional variation of OAS<sup>Q</sup> and prepayment risk premia (OAS – OAS<sup>Q</sup>) on pass-throughs from IO/PO strips. The table shows coefficient estimates from OLS regressions of the  $OAS^{Q}$  (panel a) and the prepayment-risk component in the OAS (panel b) on dummies for different moneyness levels either including or excluding time fixed effects. For each trust, the OAS is computed from the value of the pass-through obtained as the sum of the value of the IO and PO. Refer to Figure 6 for a summary of the relation between OAS and  $OAS^{Q}$  and to Section 4 for more details on the pricing model. Moneyness of a coupon *j* at time *t* is defined as Moneyness<sub>*j*,*t*</sub> = Coupon<sub>*j*</sub> + 0.5 – FRMrate<sub>*t*</sub>. Moneyness bin [-.25, .25) is the omitted category. Robust standard errors (clustered at the month level) in brackets. Significance: \* p < 0.1, \*\* p < 0.05, \*\*\* p < 0.01.

	(a) Cross section of OAS~									
	(1)		(2)		(3)		(4)			
< -1.25	1.5	[3.8]	-1.3	[1.5]	-1.8	[1.5]	-1.5	[1.7]		
[-1.25,75)	-0.2	[2.1]	-2.7**	[1.2]	-2.9**	[1.3]	-3.3**	[1.3]		
[75,25)	-1.4	[1.3]	-2.3**	[0.9]	-1.2	[1.0]	-2.5***	[0.9]		
[.25, .75)	-2.3***	[0.7]	-2.5***	[0.8]	-1.0	[0.8]	-1.5**	[0.8]		
[.75, 1.25)	-5.2***	[1.6]	-4.0***	[1.5]	-3.6**	[1.4]	-4.1***	[1.5]		
[1.25, 1.75]	-3.3	[2.5]	-1.4	[2.4]	-4.6*	[2.3]	-4.9*	[2.6]		
[1.75, 2.25]	-3.1	[4.6]	2.0	[3.8]	-9.7**	[4.4]	-4.4	[4.1]		
$\geq 2.25$	6.1	[6.9]	16.5***	[6.3]	9.1	[8.6]	$15.5^{*}$	[8.1]		
Const	13.8***	[1.6]	13.1***	[0.9]	7.9***	[1.1]	7.9***	[0.8]		
Month FEs?	No		Yes		No		Yes			
Adj. R2	-0.00		0.73		0.02		0.52			
Obs.	1005		1005		796		796			
Dates	199501.201012		199501.201012		199501.200707		199501.200707			

(a) Cross section	of $OAS^Q$
(a) CIUSS SECTION	01 OA3~

	(1)		(2)		(3)		(4)	
< -1.25	5.4***	[1.9]	6.8***	[2.2]	2.3	[1.7]	3.3*	[2.0]
[-1.25,75)	2.9***	[0.9]	5.2***	[1.2]	0.9	[0.8]	3.2***	[1.1]
[75,25)	-1.0***	[0.4]	0.7	[0.7]	-1.8***	[0.3]	-0.0	[0.7]
[.25,.75)	4.9***	[0.4]	4.1***	[0.5]	5.1***	[0.4]	4.5***	[0.5]
[.75, 1.25)	13.3***	[1.0]	12.4***	[1.1]	13.5***	[1.1]	12.8***	[1.3]
[1.25, 1.75]	22.1***	[1.5]	20.7***	[1.7]	22.3***	[1.7]	21.2***	[1.9]
[1.75, 2.25]	26.6***	[2.7]	23.1***	[2.7]	26.5***	[3.5]	23.3***	[3.5]
≥ 2.25	34.2***	[5.0]	27.5***	[4.3]	30.2***	[6.1]	25.6***	[5.1]
Const	-1.4***	[0.2]	-1.0	[0.7]	-1.5***	[0.3]	-1.2*	[0.7]
Month FEs?	No		Yes		No		Yes	
Adj. R2	0.42		0.59		0.42		0.58	
Obs.	1005		1005		796		796	
Dates	199501.201012		199501.201012		199501.200707		199501.200707	

Table 6: Time-series regressions of OAS,  $OAS^Q$  and prepayment risk premia on pass-throughs from IO/PO strips. The table shows coefficient estimates from OLS regressions of spreads reported in the top row of each panel on the Baa-Aaa corporate spread, MBS issuance to equity of brokers and dealers, average moneyness in the MBS universe squared, spreads on agency debt to swaps and dealer disagreement about prepayment speeds. For each trust, the OAS is computed from the value of the pass-through obtained as the sum of the value of the IO and PO. Refer to Figure 6 for a summary of the relation between OAS and OAS<sup>Q</sup> and to Section 4 for more details on the pricing and measured as of month-end. Columns (1) to (3) cover the entire sample period 1995-2010, Regressors are standardized to have zero mean and unit standard deviation. Newey-West standard model. See Section 5.2 for more detail on variable construction. Data is at monthly frequency while columns (4) to (6) exclude August 2007 to December 2010 (financial crisis and aftermath). errors (6 lags) in brackets. Significance: \* p < 0.1, \*\* p < 0.05, \*\*\* p < 0.01.

	(1) OAS		$^{(2)}_{OAS^{Q}}$		(3) OAS - OAS <sup>Q</sup>	5	(4) OAS		(5) OAS <sup>Q</sup>		(6) OAS - OAS <sup>Q</sup>	Q.
Baa - Aaa MBS Issuance / BD Equity	$15.5^{***}$ $5.8^{***}$	[2.8] [1.2]	$13.0^{***}$ $4.3^{***}$	[3.0] [1.0]	$2.5^{***}$ 1.5*	[0.6] [0.8]	$12.0^{***}$ $4.6^{***}$	[3.7] [1.5]	11.7** 2.3	[4.7] [2.0]	0.2 2.2	[2.5] [1.5]
Mean Moneyness <sup>2</sup>	2.5	[1.8]	-3.8**	[1.9]	6.3***	[1.1]	$4.1^{**}$	[1.7]	-1.1	[2.0]	$5.1^{***}$	[1.4]
Agency Debt Spreads	$5.9^{***}$	[2.1]	7.3***	[2.2]	-1.4	[1.0]	$6.2^{***}$	[2.1]	$8.0^{***}$	[2.4]	-1.7	[1.2]
Disagreement	-1.7	[2.0]	-3.1	[2.1]	$1.4^{**}$	[0.7]	$3.0^{***}$	[1.0]	0.8	[1.2]	2.2***	[0.7]
Const	$17.2^{***}$	[1.8]	$11.8^{***}$	[1.9]	$5.4^{***}$	[0.7]	$15.7^{***}$	[1.2]	$11.4^{***}$	[1.7]	4.3***	[1.1]
Adj. R2	0.69		0.62		0.59		0.67		0.47		0.50	
Obs.	192		192		192		151		151		151	
Dates	199501.201012		199501.201012	1	199501.201012		199501.200707		199501.200707		199501.200707	

## **Internet Appendix: Not for publication**

### A Proofs of Results in Section 3

**Evolution of principal balance.** Consider a mortgage pool with coupon rate *c*, remaining principal balance  $\theta_t$  at time *t* and maturity date *T*. Let  $\tilde{\theta}_t$  be the remaining loan balance absent unscheduled prepayment at date *t* and  $\tilde{c}$  is the fixed interest rate paid by the mortgagor (usually  $\tilde{c} = c + 50$ bps). Let the initial loan balance be  $\theta_0 = 1$ . In continuous time, the loan balance evolves as

$$\frac{\partial \tilde{\theta}_t}{\partial t} = \tilde{c}\tilde{\theta}_t - k_t$$

where *k* is the fixed payment from the borrower to the servicer. Solving for  $\tilde{\theta}_T = 0$ , we obtain that the fixed payment  $k = \tilde{c} (e^{\tilde{c}T} - 1)^{-1}$  and that the date *t* loan balance excluding unscheduled prepayment is

$$\tilde{\theta}_t = \frac{e^{\tilde{c}T} - e^{\tilde{c}t}}{e^{\tilde{c}T} - 1}.$$

Combining the scheduled and unscheduled prepayments, the remaining principal balance at date *t* is thus given by

$$\theta_t = e^{-\int_0^t s_u^* du} \frac{e^{\tilde{c}T} - e^{\tilde{c}t}}{e^{\tilde{c}T} - 1}.$$

Thus, the overall principal balance evolves as

$$d heta_t = \left(-s_t^* heta_t - ilde{c}e^{-\int_0^t s_u^* du} rac{e^{ ilde{c}t}}{e^{ ilde{c}T} - 1}
ight) dt \equiv -\mu_{ heta t} dt,$$

where  $\mu_{\theta t}$  is known at time *t*, and  $d\theta_t$  is locally deterministic.

**Proof of Result 3.1.** Consider the first-order Taylor series approximation to the price of the security around zero  $yp^{\mathbb{R}}$ :

$$P_t \approx P_t |_{yp_t^{\mathbb{R}}=0} + \left. \frac{\partial P_t}{\partial yp_t^{\mathbb{R}}} \right|_{yp_t^{\mathbb{R}}=0} yp_t^{\mathbb{R}}.$$

Using the definition of the yield premium in (3.7), we have

$$\frac{\partial P_t}{\partial y p_t^{\mathbb{R}}}\Big|_{yp_t^{\mathbb{R}}=0} = -\mathbb{E}_t^{\mathbb{R}}\left[\int_0^{T-t} \exp\left(-\int_0^s r_{t+u} du\right) s \left(1-\alpha\right)^{\int_0^s dJ_{t+u}} \mu_{X,t+s} ds\right].$$

Thus,

$$P_t \approx \mathbb{E}_t^{\mathbb{R}} \left[ \int_0^{T-t} \exp\left( -\int_0^s r_{t+u} du \right) \left( 1 - syp_t^{\mathbb{R}} \right) (1-\alpha)^{\int_0^s dJ_{t+u}} \mu_{X,t+s} ds \right].$$

Solving for the yield premium, we obtain

$$yp_t^{\mathbb{R}} \mathbb{E}_t^{\mathbb{R}} \left[ \int_0^{T-t} \exp\left(-\int_0^s r_{t+u} du\right) s \left(1-\alpha\right)^{\int_0^s dJ_{t+u}} \mu_{X,t+s} ds \right]$$
$$= \mathbb{E}_t^{\mathbb{R}} \left[ \int_0^{T-t} \exp\left(-\int_0^s r_{t+u} du\right) (1-\alpha)^{\int_0^s dJ_{t+u}} \mu_{X,t+s} ds \right] - P_t.$$

We can now use (3.6) to substitute for  $P_t$  and express the yield premium in terms of future returns

$$yp_t^{\mathbb{R}} = \mathbb{E}_t^{\mathbb{R}} \left[ \int_0^{T-t} \left( 1 - \exp\left( - \int_0^s dr x_{t+u}^{\mathbb{R}} \right) \right) w_{t,s} ds \right],$$

where the weights  $w_{t,s}$  are given by

$$w_{t,s} = \frac{\exp\left(-\int_{0}^{s} r_{t+u} du\right) (1-\alpha)^{\int_{0}^{s} dJ_{t+u}} \mu_{X,t+s}}{\mathbb{E}_{t}^{\mathbb{R}} \left[\int_{0}^{T-t} \exp\left(-\int_{0}^{s} r_{t+u} du\right) s (1-\alpha)^{\int_{0}^{s} dJ_{t+u}} \mu_{X,t+s} ds\right]},$$

decline in horizon *s*. That is, the yield premium under  $\mathbb{R}$  is a weighted average of expected excess returns over the lifetime of the security under the measure  $\mathbb{R}$ .

**Proof of Result 3.2.** Notice first that, at time  $t + \Delta$ , the price of the pass-through security in terms of the time  $t + \Delta$  yield premium is given by

$$P_{t+\Delta} = \mathbb{E}_{t+\Delta}^{\mathbb{R}} \left[ \int_{0}^{T-t-\Delta} \exp\left( -\int_{0}^{s} \left( r_{t+\Delta+u} + yp_{t+\Delta}^{\mathbb{R}} \right) du \right) (1-\alpha)^{\int_{0}^{s} dJ_{t+\Delta+u}} \mu_{X,t+\Delta+s} ds \right]$$
  
=  $\mathbb{E}_{t+\Delta}^{\mathbb{R}} \left[ \int_{0}^{T-t-\Delta} \exp\left( -\int_{0}^{s} \left( r_{t+\Delta+u} + yp_{t}^{\mathbb{R}} + \Delta yp_{t}^{\mathbb{R}} \right) du \right) (1-\alpha)^{\int_{0}^{s} dJ_{t+\Delta+u}} \mu_{X,t+\Delta+s} ds \right],$ 

where  $\Delta y p_t^{\mathbb{R}} = y p_{t+\Delta}^{\mathbb{R}} - y p_t^{\mathbb{R}}$ . Expanding around  $\Delta y p_t^{\mathbb{R}} = 0$ , we can thus represent the time  $t + \Delta$  price of the pass-through security as

$$P_{t+\Delta} \approx \mathbb{E}_{t+\Delta}^{\mathbb{R}} \left[ \int_0^{T-t-\Delta} \exp\left( -\int_0^s \left( r_{t+\Delta+u} + yp_t^{\mathbb{R}} \right) du \right) (1-\alpha)^{\int_0^s dJ_{t+\Delta+u}} \mu_{X,t+\Delta+s} ds \right] - D_{t+\Delta} P_{t+\Delta} \Delta y p_t^{\mathbb{R}},$$

where  $D_{t+\Delta}$  is the modified duration of the security:

$$D_{t+\Delta} = -\frac{1}{P_{t+\Delta}} \frac{\partial P_{t+\Delta}}{\partial \Delta y p_t^{\mathbb{R}}}.$$

Thus,

$$drx_{t}^{\mathbb{R}} = \frac{dP_{t}^{\mathbb{R}} + dX_{t}^{\mathbb{R}}}{P_{t^{-}}} - r_{t}dt = \lim_{\Delta \to 0} \frac{P_{t+\Delta} - P_{t^{-}} + dX_{t}}{P_{t^{-}}} - r_{t}dt$$

$$\approx \lim_{\Delta \to 0} \frac{e^{\left(r_{t+\Delta} + yp_{t}^{\mathbb{R}}\right)\Delta} \left(1 - \alpha\right)^{-dJ_{t+\Delta}} \mathbb{E}_{t+\Delta}^{\mathbb{R}} \left[\int_{0}^{T - t - \Delta} \exp\left(-\int_{0}^{s} \left(r_{t+u} + yp_{t}^{\mathbb{R}}\right) du\right) \left(1 - \alpha\right)^{\int_{0}^{s} dJ_{t+u}} \mu_{X,t+s} ds\right]}{P_{t}}$$

$$- \frac{\mathbb{E}_{t}^{\mathbb{R}} \left[\int_{0}^{T - t} \exp\left(-\int_{0}^{s} \left(r_{t+u} + yp_{t}^{\mathbb{R}}\right) du\right) \left(1 - \alpha\right)^{\int_{0}^{s} dJ_{t+u}} \mu_{X,t+s} ds\right]}{P_{t}} - \lim_{\Delta \to 0} \frac{D_{t+\Delta} P_{t+\Delta} \Delta yp_{t}^{\mathbb{R}}}{P_{t}} - r_{t} dt$$

$$= yp_{t}^{\mathbb{R}} dt - D_{t} dyp_{t}^{\mathbb{R}},$$

where the last equality follows from evaluating the limit as  $\Delta$  approaches 0.

**Proof of Result 3.3.** Using the fund separation theorem (see e.g. Merton, 1972), we can represent the return on the pass-through security as

$$drx_t = \sum_{j=1}^{N_r} \beta_{rj,t} d\varphi_{jt} + d\widetilde{r} \widetilde{x}_t, \tag{A.1}$$

where  $\{d\varphi_{jt}\}_{j=1}^{N_r}$  are the instantaneous excess returns on a set of portfolios that spans the interest rate uncertainty in the economy, and  $d\tilde{rx}_t$  is the excess return on the pass-through that is earned as compensation for non-interest rate factors:

$$d\widetilde{rx}_t \equiv drx_t - \sum_{j=1}^{N_r} \beta_{rj,t} d\varphi_{jt}; \quad \mathbb{E}\left[d\widetilde{rx}_t d\rho_t\right] = 0.$$

That is, (A.1) decomposed the excess return on the pass-through into a beta-weighted sum of interest-rate-uncertainty-spanning portfolios and compensation for other sources of risk priced in the pass-through security.

Since  $\{d\varphi_{jt}\}_{j=1}^{N_r}$  span the interest rate uncertainty, we can also represent the pricing kernel  $M_t$  as

$$rac{dM_t}{M_t}=m_{0t}dt+\sum_{j=1}^{N_r}m_{jt}darphi_{jt}+rac{dM_t^{\mathbb{Q}_r}}{M_t^{\mathbb{Q}_r}},$$

with

$$r_t dt = -m_{0t} dt - \sum_{j=1}^{N_r} m_{jt} \mathbb{E}_t \left[ d\varphi_{jt} \right] - \mathbb{E}_t \left[ \frac{dM_t^{Q_r}}{M_t^{Q_r}} \right].$$

Thus,

$$\begin{split} \mathbb{E}_t \left[ d\widetilde{r}\widetilde{x}_t \right] &= -\mathbb{E}_t \left[ \frac{dM_t}{M_t} d\widetilde{r}\widetilde{x}_t \right] = -\mathbb{E}_t \left[ \left( \sum_{j=1}^{N_r} m_{jt} d\varphi_{jt} + \frac{dM_t^{\mathbb{Q}_r}}{M_t^{\mathbb{Q}_r}} \right) d\widetilde{r}\widetilde{x}_t \right] \\ &= -\mathbb{E}_t \left[ \frac{dM_t^{\mathbb{Q}_r}}{M_t^{\mathbb{Q}_r}} d\widetilde{r}\widetilde{x}_t \right] \\ &= -\mathbb{E}_t^{\mathbb{Q}_r} \left[ d\widetilde{r}\widetilde{x}_t \right], \end{split}$$

where the third equality follows from  $d\tilde{rx}_t$  being orthogonal to the interest-rate-risk-spanning portfolios  $\{d\varphi_{jt}\}_{i=1}^{N_r}$  and the fourth equality follows from the definition of the pricing kernel under  $\mathbb{Q}_r$ .

## **B** Data

#### **B.1** TBA sample and data-quality filters

The sample used for the analysis in Sections 3.3 and 3.4 spans 1996 to 2010, reflecting limited availability of data on MBS in the TBA market prior to 1996, and a limited liquidity in IO/PO strips, which we use later in the paper to decompose the OAS, post 2010. The OAS we use are the average (within each coupon/month) across six dealers from which we collect data. We do not necessarily have spreads for all dealers on the same coupons on each day. In addition, some of the dealers enter our data only after 1996. We clean each dealer's data to prevent spreads from being influenced by stale prices. To do so, we check whether a price for a coupon is unchanged relative to the previous day. If it is, and if the 10-year Treasury yield changed by 3 basis points or more on the same day (so we expect MBS prices to change), we drop the price and the corresponding spread. If the price is not constant, but had been constant more than twice in the same calendar month on days when the Treasury yield moves, we similarly drop it.

Descriptive statistics of the monthly coupon-level sample used for the analysis in Sections 3.3 and 3.4 are provided in Table A-1. The coupons in the sample change over time (as illustrated in Figure 1 in the main text), so that the data form an unbalanced panel.

#### **B.2** Stripped MBS data-quality filters

We start with daily price quotes from a large dealer for the period 1995 to 2010, and then clean these data using the following steps:

- 1. Remove/correct obvious outliers (such as prices of 0 or a few instances where IO and PO prices were inverted).
- 2. Remove prices that are stale (defined as a price that does not change from previous day despite a change in the 10-year yield of more than 3 basis points). In case of smaller yield changes, we check the previous 10 days and remove a price if there were more than two instances of stale prices on that security over that period.

**Table A-1: Descriptive statistics of TBA sample.** Based on Fannie Mae pass-through coupons from 1996-2010 for which we have OAS quotes from at least one dealer, subject to the data quality filters described in Section B.1. Data frequency is monthly; spreads and moneyness (= coupon plus 0.5 minus current FRM rate) are as of month-end. RPB stands for the total remaining principal balance in a coupon. Factor means current face value divided by issuance amount. Data on RPBs, factors, and weighted average loan ages are from eMBS.

	Mean	Std. Dev.	p5	p25	p50	p75	p95	Obs.
OAS (basis points, annual)	23.12	50.66	-16.45	-2.27	9.05	28.03	120.00	1532
Coupon (percent)	6.53	1.49	4.00	5.50	6.50	7.50	9.00	1532
Moneyness (percent)	0.56	1.40	-1.64	-0.58	0.51	1.60	2.93	1532
RPB (2009 USD, million)	126928.83	151009.18	590.43	9091.44	66862.53	194624.09	443893.38	1532
Factor of most recent vintage	0.89	0.21	0.26	0.91	0.98	0.99	1.00	1530
Weighted average loan age (months)	47.44	33.51	8.67	25.37	38.36	58.93	128.27	1532

- 3. For a subsample of trusts and months (starting in June 1999), we also have price quotes from two additional dealers. When available, we compare these prices (their average if both are available) to the price quoted by our dealer. When they are more than 5% apart, or if the overall range of price quotes is larger than 0.1 times the average price, we do not use our price quote in the analysis. This applies to about 10% of our price quotes.
- 4. Only retain trusts for which we have both the IO and PO strips, and which we can link to data on the underlying pool of mortgages (from eMBS). This restriction eliminates IO strips backed by excess servicing rights, for instance.
- 5. Only retain trusts for which the price on the recombined pass-through (=  $P_{PO} + P_{IO}$ ) is within \$2 of the TBA price of the corresponding coupon. (We also drop trusts if on that day we do not have a clean TBA price for the corresponding coupon.)
- 6. Only retain trusts with a factor (= current face value divided by issuance amount) of more than 5%.
- 7. Only retain trusts that we can match to a Bloomberg prepayment survey with the same coupon and absolute differences in WAC and WAM smaller than 0.3 percentage points and 60 months, respectively. (This affects almost exclusively observations before 2003, as we do not have individual vintages in the survey in the early years.)

Following these steps, the sample includes 3713 trust-month observations, or about 19 per month. The year with the lowest number of observations is 1999, where we have an average of 10 trusts per month, while after 2005 we have at least 20 trusts in all but one month.

# C OAS smile: Robustness

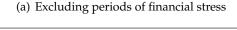
This Appendix provides additional evidence on the smile-shaped pattern of the OAS in the cross section of TBA coupons, which is a key empirical result as discussed in Section 3.4. We show

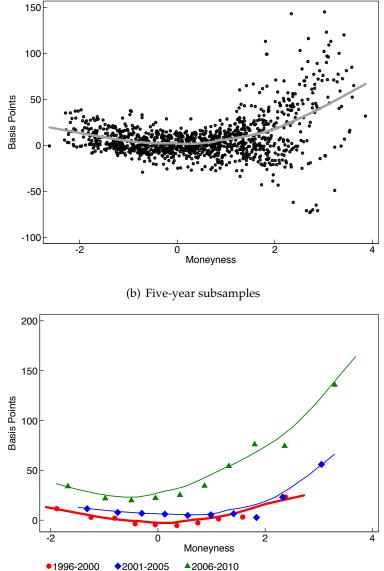
robustness to dropping periods of financial stress, stability across sub-periods, and regression estimates when excluding outlier observations, MBS coupons with relatively low balances, or coupons that may be "burned out" (as indicated by a low remaining factor).

One potential concern about the OAS pattern shown in panel (b) of Figure 2 is that it may be unduly influenced by periods of financial stress. Panel (a) of Figure A-1 replicates Figure 2 when excluding the post-July 2007 period, and the period from September 1998 to January 1999, when liquidity in many U.S. fixed income markets dried up following the failure of Long-Term Capital Management (LTCM). The figure shows that the distinctive smile pattern remains present even without these periods. Panel (b) of Figure A-1 provides evidence on sub-sample stability by providing "binned" scatter plots for three equally sized periods covering five years each. While the smile pattern is most distinctive during the last five years of the sample, it is also clearly visible in the earlier time periods.

We next present additional robustness checks of the regression evidence shown in Table 3 of the main text, focusing on the specifications that explore within-month variation in OAS across coupons. Table 3 showed that the smile pattern is robust to restricting the sample to certain time periods, either based on dates or based on the moneyness of the market as a whole. We now instead drop observations based on the characteristics of the underlying coupons, or if their OAS is an outlier. Column (1) of Table A-2 displays, for reference, the results of our baseline specification with month fixed effects (column 2 in Table 3(a)). In column (2), we drop from the sample all observations where the absolute level of moneyness exceeds 2, or where the OAS exceeds 100 basis points. The resulting coefficients confirm that the smile pattern is not driven by extreme observations. The final two columns instead restrict the sample based on characteristics of the pools underlying each coupon, to ensure that the smile pattern is not due just to small pools. In column (3), we restrict the included coupons to be those from the original sample with abovemedian remaining principal balance (in 2009 dollars); this cutoff is \$67 billion. We see that the coefficients on the most extreme bins are slightly reduced relative to column (1), but otherwise the smile pattern remains very similar. Finally, in column (4) we restrict the sample to include only those coupons where the most recently issued origination vintage (by year) has a remaining factor of at least 0.8 (meaning at most 20% of the original pool has prepaid). This excludes the coupons with the most "burned out" pools. This restriction has little effect on the estimated coefficients.

**Figure A-1: OAS smile: robustness and subsamples.** Panel (a) displays a scatterplot and a local smoother of the cross-sectional variation in OAS for MBS coupons with remaining principal balance (in 2009 dollars) of \$100 million or more, excluding from the sample the months September 1998 to January 1999 (LTCM turmoil) and August 2007 to December 2010 (financial crisis and aftermath). Panel (b) displays "binned" scatterplots for three five-year subsamples, where each dot represents mean OAS for one of ten equal-sized moneyness bins. The lines represent a local smoother fitted to the underlying data (i.e., each coupon's OAS, not just the moneyness bin averages).





**Table A-2: Cross section of OAS on TBA coupons: Robustness.** Coefficient estimates from OLS regression of the OAS on different moneyness level bins, with calendar month fixed effects. Robust standard errors (clustered at the month level) in brackets. Significance: \* p < 0.1, \*\* p < 0.05, \*\*\* p < 0.01.

	(1)		(2		(3)		(4	)
	Full		Money	m.  <2,	RPB>:	med.	Factor	> 0.8
			OAS<					
< -1.75	18.0***	[2.3]	16.5***	[2.6]	5.9***	[2.3]	17.9***	[2.2]
[-1.75, -1.25]	$10.4^{***}$	[1.6]	9.8***	[1.3]	3.7***	[1.1]	9.5***	[1.5]
[-1.25,75]	5.3***	[1.1]	4.9***	[0.8]	-0.4	[0.9]	5.8***	[1.0]
[75,25]	-0.1	[0.4]	0.1	[0.4]	-1.1	[0.7]	-0.1	[0.4]
[.25, .75)	$1.5^{***}$	[0.5]	1.2**	[0.5]	1.2*	[0.7]	$1.5^{***}$	[0.5]
[.75, 1.25)	$6.4^{***}$	[1.2]	5.1***	[1.0]	4.7***	[1.4]	6.3***	[1.2]
[1.25, 1.75)	15.0***	[2.1]	11.0***	[1.7]	12.8***	[2.8]	15.7***	[2.2]
[1.75, 2.25)	24.6***	[3.3]	14.6***	[3.2]	29.9***	[4.7]	26.2***	[4.1]
[2.25, 2.75]	33.9***	[5.5]			38.9***	[7.5]	$48.6^{***}$	[9.4]
$\geq 2.75$	85.4***	[10.9]			51.8***	[2.6]	91.6***	[15.2]
Month FEs?	Yes		Yes		Yes		Yes	
Adj. R2	0.61		0.68		0.88		0.76	
Adj. R2 (within)	0.31		0.15		0.31		0.32	
Obs.	1532		1215		766		1314	

## D Additional details on model

In this Appendix, we provide additional details and derivations for the simple framework described in the main text. We show that the simple intuitions for the sources of risk premia reflected in the OAS that we developed in Section 3 carry through under more general assumptions on the evolution of risk factors in the economy.

#### D.1 Cash flows

In this paper, we are interested in pricing three types of MBS securities: the interest-only (IO) strip, the principal-only (PO) strip and the pass-through security (the combination, denoted "PT," of IO and PO). For a pool with  $\theta_t$  remaining principal balance, the cash flows received by investors in an IO strip are  $dX_{t,IO} = c\theta_t dt$ , by investors in a PO strip are  $dX_{t,PO} = -d\theta_t$ , and by investors in the pass-through are  $dX_{t,PT} = c\theta_t dt - d\theta_t$ .

Recall that the remaining principal balance evolves as

$$\theta_t = e^{-\int_0^t s_u^* du} \frac{e^{\tilde{c}T} - e^{\tilde{c}t}}{e^{\tilde{c}T} - 1},$$

so that

$$d heta_t = \left(-s_t^* heta_t - ilde{c}e^{-\int_0^t s_u^*du} rac{e^{ ilde{c}t}}{e^{ ilde{c}T} - 1}
ight) dt.$$

Thus, the evolution of the remaining principal balance is locally deterministic, which implies that the cash flows to our three securities of interest are locally deterministic as well.

#### D.2 Pricing kernels and changes of measure

Under the assumption of no arbitrage, there exists a stochastic discount factor that prices all the assets in the economy, given by

$$rac{dM_t}{M_t} = -r_t dt - \pi'_{rt} dZ_{rt} - \pi'_{\gamma t} dZ_{\gamma t} - \log \pi_{lt} dN_t,$$

where  $r_t$  is the equilibrium risk-free rate,  $\pi_{rt}$  is the vector of the prices of risk associated with innovations affecting the term structure of interest rates in the economy (including  $\rho_t$ ),  $\pi_{\gamma t}$  is the vector of the prices of risk associated with the innovations to the non-interest rate factors,  $\pi_{lt}$  is the price of risk associated with the liquidity shocks  $dJ_t$ , and  $dN_t = dJ_t - \mu_t dt$  is the compensated liquidity jump process. Associated with the stochastic discount factor is the risk-neutral measure<sup>1</sup> Q. The risk-neutral measure Q is defined by its Radon-Nikodym derivative with respect to the

<sup>&</sup>lt;sup>1</sup>The risk-neutral measure is also sometimes called the equivalent martingale measure (EMM)

physical measure  $\mathbb{P}$ :

$$\frac{d\mathbb{Q}}{d\mathbb{P}} = \exp\left(-\frac{1}{2}\int_0^t \left(|\pi_{rs}|^2 + |\pi_{\gamma s}|^2\right)ds - \int_0^t \pi_{rs}' dZ_{rs} - \int_0^t \pi_{\gamma s}' dZ_{\gamma s} - \int_0^t \log \pi_{ls} dN_s\right)$$

As described in Section 3, in addition to the risk-neutral measure, we consider two intermediate changes of measure: one that is risk-neutral with respect to the interest rate shocks only,  $Q_r$ , and one that is risk-neutral with respect to all Brownian shocks,  $Q_{r,\gamma}$ . These two measures are also defined by their respective Radon-Nikodym derivatives with respect to the physical measure  $\mathbb{P}$ :

$$\frac{d\mathbb{Q}_r}{d\mathbb{P}} = \exp\left(-\frac{1}{2}\int_0^t |\pi_{rs}|^2 ds - \int_0^t \pi'_{rs} dZ_{rs}\right)$$
$$\frac{d\mathbb{Q}_{r,\gamma}}{d\mathbb{P}} = \exp\left(-\frac{1}{2}\int_0^t \left(|\pi_{rs}|^2 + |\pi_{\gamma s}|^2\right) ds - \int_0^t \pi'_{st} dZ_{rs} - \int_0^t \pi'_{\gamma s} dZ_{\gamma s}\right).$$

The pricing kernel under  $Q_r$  is thus given by

$$\frac{dM_t^{\mathbb{Q}_r}}{M_t^{\mathbb{Q}_r}} = -r_t dt - \pi_{\gamma t}' dZ_{\gamma t} - \log \pi_{lt} dN_t,$$

and under  $\mathbb{Q}_{r,\gamma}$  by

$$\frac{dM_t^{\mathbf{Q}_{r,\gamma}}}{M_t^{\mathbf{Q}_{r,\gamma}}} = -r_t dt - \log \pi_{lt} dN_t.$$

#### D.3 Expected excess returns

To simplify the exposition (and reduce repetition), let  $\mathbb{R}$  be a generic measure, defined through its Radon-Nikodym derivative with respect to the physical probability measure  $\mathbb{P}$ :

$$\frac{d\mathbb{R}}{d\mathbb{P}} = \exp\left(-\frac{1}{2}\int_0^t \left(|\psi_{rs}|^2 + |\psi_{\gamma s}|^2\right)ds - \int_0^t \psi_{rs}' dZ_{rs} - \int_0^t \psi_{\gamma s}' dZ_{\gamma s} - \int_0^t \log\psi_{ls}\left(dJ_s - \mu_s ds\right)\right).$$

Under  $\mathbb{R}$ , the vector of interest rate shocks  $\rho_t$  evolves as

$$d\rho_t^{\mathbb{R}} = (\mu_{rt} - \sigma_{rt}\psi_{rt}) dt + \sigma_{rt} dZ_{rt}^{\mathbb{R}},$$

and the vector of prepayment shocks  $\gamma_t$  evolves as

$$d\gamma_t^{\mathbb{R}} = (\mu_{\gamma t} - \sigma_{\gamma t} \psi_{\gamma t}) \, dt + \sigma_{\gamma t} dZ_{\gamma t}^{\mathbb{R}},$$

where  $dZ_{rt}^{\mathbb{R}}$  and  $dZ_{\gamma t}^{\mathbb{R}}$  are, respectively, an  $N_r$ - and an  $N_{\gamma}$ -dimensional Brownian motion under  $\mathbb{R}$ . The liquidity factor  $l_t$  follows a Poisson jump process with arrival rate  $\psi_{lt}\mu_t$  under  $\mathbb{R}$ . Let  $M_t^{\mathbb{R}}$  be the pricing kernel under  $\mathbb{R}$ , given by

$$\frac{dM_t^{\mathbb{R}}}{M_t^{\mathbb{R}}} = -r_t dt - (\pi_{rt} - \psi_{rt}) dZ_{rt}^{\mathbb{R}} - (\pi_{\gamma t} - \psi_{\gamma t}) dZ_{\gamma t}^{\mathbb{R}} - \log \frac{\pi_{lt}}{\psi_{lt}} dN_t^{\mathbb{R}},$$

where  $dN_t^{\mathbb{R}}$  is the compensated Poisson jump process under  $\mathbb{R}$ . With this notation, given an excess return process  $drx_t^{\mathbb{R}}$ , expected excess returns satisfy

$$\mathbb{E}_t^{\mathbb{R}}\left[drx_t^{\mathbb{R}}\right] = -\mathbb{E}_t^{\mathbb{R}}\left[\frac{dM_t^{\mathbb{R}}}{M_t^{\mathbb{R}}}drx_t^{\mathbb{R}}\right].$$

Next, recall that, the excess return process under  $\mathbb R$  is defined as

$$dr x_t^{\mathbb{R}} = \frac{dP_t^{\mathbb{R}}}{P_t} + \frac{dX_t}{P_t} - r_t dt$$

where  $dP_t^{\mathbb{R}}$  is the capital gains process on the security under  $\mathbb{R}$ . For our securities of interest, the cash flows  $dX_t$  are locally-deterministic. For securities with locally-deterministic cash flows, the expected excess return under  $\mathbb{R}$  thus satisfies

$$\mathbb{E}_{t}^{\mathbb{R}}\left[drx_{t}^{\mathbb{R}}\right] = -\mathbb{E}_{t}^{\mathbb{R}}\left[\frac{dM_{t}^{\mathbb{R}}}{M_{t}^{\mathbb{R}}}\frac{dP_{t}^{\mathbb{R}}}{P_{t}}\right].$$
(D.1)

Thus, to compute the expected excess return, we need to know the evolution of the gains process under  $\mathbb{R}$ .

Applying Ito's lemma (see e.g. Cont and Tankov, 2003, Proposition 8.14 for Ito's lemma for jump-diffusion processes), we can represent the capital gains process under  $\mathbb{R}$  as

$$\begin{split} dP_t^{\mathbb{R}} &= \frac{\partial P}{\partial t} dt + \frac{\partial P}{\partial \rho'_t} d\rho_t^{\mathbb{R}} + \frac{\partial P}{\partial \gamma'_t} d\gamma_t^{\mathbb{R}} + \frac{1}{2} tr \left( \frac{\partial^2 P}{\partial \rho'_t \partial \rho_t} d\rho_t^{\mathbb{R}} d\rho'_t^{\mathbb{R}} \right) + \frac{1}{2} tr \left( \frac{\partial^2 P}{\partial \gamma'_t \partial \gamma_t} d\gamma_t^{\mathbb{R}} d\gamma'_t^{\mathbb{R}} \right) + [P_t - P_{t-}] \\ &= \frac{\partial P}{\partial t} dt + \frac{\partial P}{\partial \rho'_t} \left( (\mu_{rt} - \sigma_{rt} \psi_{rt}) dt + \sigma_{rt} dZ_{rt}^{\mathbb{R}} \right) + \frac{\partial P}{\partial \gamma'_t} \left( (\mu_{\gamma t} - \sigma_{\gamma t} \psi_{\gamma t}) dt + \sigma_{\gamma t} dZ_{\gamma t}^{\mathbb{R}} \right) \\ &+ \frac{1}{2} tr \left( \frac{\partial^2 P}{\partial \rho'_t \partial \rho_t} \sigma_{rt} \sigma'_{rt} \right) dt + \frac{1}{2} tr \left( \frac{\partial^2 P}{\partial \gamma'_t \partial \gamma_t} \sigma_{\gamma t} \sigma'_{\gamma t} \right) dt + [P_t - P_{t-}] \,. \end{split}$$

Substituting into (D.1), we can thus express the expected excess return under  $\mathbb{R}$  as

$$\mathbb{E}_{t}^{\mathbb{R}}\left[drx_{t}^{\mathbb{R}}\right] = \frac{1}{P_{t^{-}}}\frac{\partial P_{t}}{\partial \rho_{t}'}\sigma_{rt}\left(\pi_{rt} - \psi_{rt}\right)dt + \frac{1}{P_{t^{-}}}\frac{\partial P_{t}}{\partial \gamma_{t}'}\sigma_{\gamma t}\left(\pi_{\gamma t} - \psi_{\gamma t}\right)dt + \alpha\mu_{t}\left(\frac{\pi_{lt}}{\psi_{lt}} - 1\right)dt,$$

so that the risk premium on the security under  $\mathbb{R}$ ,  $rp_t^{\mathbb{R}}$  is given by

$$rp_t^{\mathbb{R}} = \frac{1}{P_{t^-}} \frac{\partial P_t}{\partial \rho_t'} \sigma_{rt} \left( \pi_{rt} - \psi_{rt} \right) + \frac{1}{P_{t^-}} \frac{\partial P_t}{\partial \gamma_t'} \sigma_{\gamma t} \left( \pi_{\gamma t} - \psi_{\gamma t} \right) + \alpha \mu_t \left( \frac{\pi_{lt}}{\psi_{lt}} - 1 \right).$$
(D.2)

#### D.4 Identifying prepayment risk premia

**Proposition D.1.** Assume that the economy is as described in Section 3 and let  $\pi_{rt}$ ,  $\pi_{\gamma t}$  and  $\pi_{lt}$  be the prices of interest rate, prepayment and liquidity risk, respectively. Then:

*a)* Under the interest-and-prepayment-risk-neutral measure,  $Q_{r,\gamma}$ , the risk premia on the IO, the PO, and the pass-through are all equalized and are equal to

$$rp_{t,IO}^{\mathbb{Q}_{r,\gamma}} = rp_{t,PO}^{\mathbb{Q}_{r,\gamma}} = rp_{t,PT}^{\mathbb{Q}_{r,\gamma}} = \alpha \left(\pi_{lt} - 1\right) \mu_t,$$

where  $\alpha$  is the fraction of market value lost in case of the liquidity shock being realized.

b) Let  $\mathbb{R}$  be a measure defined through its Radon-Nikodym derivative with respect to the physical probability measure  $\mathbb{P}$  as

$$\frac{d\mathbb{R}}{d\mathbb{P}} = \exp\left(-\frac{1}{2}\int_0^t \left(|\psi_{rs}|^2 + |\psi_{\gamma s}|^2\right) ds - \int_0^t \psi_{rs}' dZ_{rs} - \int_0^t \psi_{\gamma s}' dZ_{\gamma s} - \int_0^t \log \psi_{ls} \left(dJ_s - \mu_s ds\right)\right).$$

Then, the expected excess returns on the IO, the PO and the pass-through under  $\mathbb{R}$  are equalized only if  $\mathbb{R}$  fully compensates for interest rate and prepayment risk:

$$\psi_{rt} = \pi_{rt}; \quad \psi_{\gamma t} = \pi_{\gamma t}.$$

*Proof.* Consider first part (a). In this case,  $\mathbb{R} = \mathbb{Q}_{r,\gamma}$  so that

$$\psi_{rt} = \pi_{rt}; \quad \psi_{\gamma t} = \pi_{\gamma t}; \quad \psi_{lt} = 1.$$

Substituting into (D.2), we thus obtain that, under the interest-rate-and-prepayment-risk-neutral measure  $Q_{r,\gamma}$ , the risk premium on the IO strip, the risk premium on the PO strip, and the risk premium on the pass-through are all given by

$$rp_{IO,t}^{\mathbb{Q}_{r,\gamma}} = rp_{PO,t}^{\mathbb{Q}_{r,\gamma}} = rp_{PT,t}^{\mathbb{Q}_{r,\gamma}} = \alpha\mu_t \left(\pi_{lt} - 1\right)$$

That is, under the interest-rate-and-prepayment-risk-neutral measure  $\mathbb{Q}_{r,\gamma}$ , holders of MBS securities earn an expected excess return as compensation for liquidity risk. Since every security written on the same pool are equally exposed to the liquidity shock, the risk premia under  $\mathbb{Q}_{r,\gamma}$  are equalized on all three securities.

Turn now to part (b). From (D.2), the risk premium under  $\mathbb{R}$  on the IO strip is

$$rp_{IO,t}^{\mathbb{R}} = \frac{1}{P_{IO,t^{-}}} \frac{\partial P_{IO,t}}{\partial \rho'_{t}} \sigma_{rt} \left(\pi_{rt} - \psi_{rt}\right) + \frac{1}{P_{IO,t^{-}}} \frac{\partial P_{IO,t}}{\partial \gamma'_{t}} \sigma_{\gamma t} \left(\pi_{\gamma t} - \psi_{\gamma t}\right) + \alpha \mu_{t} \left(\frac{\pi_{lt}}{\psi_{lt}} - 1\right),$$

and the risk premium under  $\mathbb{R}$  on the PO strip is

$$rp_{PO,t}^{\mathbb{R}} = \frac{1}{P_{PO,t^{-}}} \frac{\partial P_{PO,t}}{\partial \rho'_{t}} \sigma_{rt} \left(\pi_{rt} - \psi_{rt}\right) + \frac{1}{P_{PO,t^{-}}} \frac{\partial P_{PO,t}}{\partial \gamma'_{t}} \sigma_{\gamma t} \left(\pi_{\gamma t} - \psi_{\gamma t}\right) + \alpha \mu_{t} \left(\frac{\pi_{lt}}{\psi_{lt}} - 1\right).$$

Since, in general, the IO and the PO strip have different exposures to interest rate and prepayment risk, so that

$$\frac{1}{P_{IO,t^{-}}}\frac{\partial P_{IO,t}}{\partial \rho'_{t}} \neq \frac{1}{P_{PO,t^{-}}}\frac{\partial P_{PO,t}}{\partial \rho'_{t}} \quad \text{and} \quad \frac{1}{P_{IO,t^{-}}}\frac{\partial P_{IO,t}}{\partial \gamma'_{t}} \neq \frac{1}{P_{PO,t^{-}}}\frac{\partial P_{PO,t}}{\partial \gamma'_{t}},$$

the risk premia on the IO and the PO are equalized only if

$$\pi_{rt} - \psi_{rt} = 0$$
 and  $\pi_{\gamma t} - \psi_{\gamma t} = 0$ .

Thus, the risk premia on the IO and the PO are equalized under measure  $\mathbb{R}$  if and only if  $\mathbb{R}$  is risk-neutral with respect to interest rate and prepayment risk.

## **E** Pricing model details

#### E.1 Interest rate model

We assume that swap rates follow a three-factor Heath, Jarrow, and Morton (1992) (HJM) model. Let f(t, T) denote the time t instantaneous forward interest rate for risk-free borrowing and lending at time T. We model the forward rate dynamics under the (interest rate) risk-neutral measure as

$$df(t,T) = \mu_{f}(t,T) dt + \sum_{i=1}^{3} \sigma_{f,i}(t,T) dW_{i,t}^{Q},$$

where  $W_{it}^{\mathbb{Q}}$  are independent standard Weiner processes under the risk-neutral measure  $\mathbb{Q}$ , and, under no arbitrage, the expected change in the forward rate is given by

$$\mu_f(t,T) = \sum_{i=1}^{3} \sigma_{f,i}(t,T) \int_t^T \sigma_{f,i}(t,u) \, du.$$

Thus, the risk-neutral dynamics of the instantaneous forward rate are completely determined by the initial forward rate curve and the forward rate volatility functions,  $\sigma_{f,i}(t, T)$ . Similarly to Trolle and Schwartz (2009), we assume that the volatility function of each factor  $\sigma_{f,i}(t, T)$  is

$$\sigma_{f,i}(t,T) = (\alpha_{0,i} + \alpha_{1,i}(T-t)) e^{-\gamma_i(T-t)}.$$
(E.1)

This specification has the advantage of allowing for a wide range of shocks to the forward rate curve while ensuring that the forward rate model above is Markovian.

Trolle and Schwartz (2009) show that, setting the volatility of the forward rates to be as in (E.1), the time *t* price of a zero-coupon bond maturing at time *T*, *P*(*t*, *T*), is given by

$$P(t,T) \equiv \exp\left\{-\int_{t}^{T} f(t,u) \, du\right\} = \frac{P(0,T)}{P(0,t)} \exp\left\{\sum_{i=1}^{3} B_{x_{i}}(T-t) \, x_{it} + \sum_{i=1}^{3} \sum_{j=1}^{6} B_{\phi_{ji}}(T-t) \, \phi_{ji,t}\right\},$$

where the state variables  $\{x_{it}, \phi_{ji,t}\}$  follow

$$dx_{it} = -\gamma_i x_{it} dt + dW_{it}^{Q}$$

$$d\phi_{1i,t} = (x_{it} - \gamma_i \phi_{1i,t}) dt$$

$$d\phi_{2i,t} = (1 - \gamma_i \phi_{2i,t}) dt$$

$$d\phi_{3i,t} = (1 - 2\gamma_i \phi_{3i,t}) dt$$

$$d\phi_{4i,t} = (\phi_{2i,t} - \gamma_i \phi_{4i,t}) dt$$

$$d\phi_{5i,t} = (\phi_{3i,t} - 2\gamma_i \phi_{5i,t}) dt$$

$$d\phi_{6i,t} = (2\phi_{5i,t} - 2\gamma_i \phi_{6i,t}) dt$$

The coefficients  $\{B_{x_i}, B_{\phi_{ji}}\}$  are functions of the parameters of the volatility function and the time to maturity  $\tau = T - t$ , and are given by

$$\begin{split} B_{x_{i}}\left(\tau\right) &= \frac{\alpha_{1i}}{\gamma_{i}} \left( \left(\frac{1}{\gamma_{i}} + \frac{\alpha_{0i}}{\alpha_{1i}}\right) \left(e^{-\gamma_{i}\tau} - 1\right) + \tau e^{-\gamma_{i}\tau} \right) \\ B_{\phi_{1i}}\left(\tau\right) &= \frac{\alpha_{1i}}{\gamma_{i}} \left(e^{-\gamma_{i}\tau} - 1\right) \\ B_{\phi_{2i}}\left(\tau\right) &= \left(\frac{\alpha_{1i}}{\gamma_{i}}\right)^{2} \left(\frac{1}{\gamma_{i}} + \frac{\alpha_{0i}}{\alpha_{1i}}\right) \left(\left(\frac{1}{\gamma_{i}} + \frac{\alpha_{0i}}{\alpha_{1i}}\right) \left(e^{-\gamma_{i}\tau} - 1\right) + \tau e^{-\gamma_{i}\tau} \right) \\ B_{\phi_{3i}}\left(\tau\right) &= -\frac{\alpha_{1i}}{\gamma_{i}^{2}} \left( \left(\frac{\alpha_{1i}}{2\gamma_{i}^{2}} + \frac{\alpha_{0i}}{\gamma_{i}} + \frac{\alpha_{0i}^{2}}{2\alpha_{1i}}\right) \left(e^{-2\gamma_{i}\tau} - 1\right) + \left(\frac{\alpha_{1i}}{\gamma_{i}} + \alpha_{0i}\right) \tau e^{-2\gamma_{i}\tau} + \frac{\alpha_{1i}}{2}\tau^{2}e^{-2\gamma_{i}\tau} \right) \\ B_{\phi_{4i}}\left(\tau\right) &= \left(\frac{\alpha_{1i}}{\gamma_{i}}\right)^{2} \left(\frac{1}{\gamma_{i}} + \frac{\alpha_{0i}}{\alpha_{1i}}\right) \left(e^{-\gamma_{i}\tau} - 1\right) \\ B_{\phi_{5i}}\left(\tau\right) &= -\frac{\alpha_{1i}}{\gamma_{i}^{2}} \left( \left(\frac{\alpha_{1i}}{\gamma_{i}} + \alpha_{0i}\right) \left(e^{-2\gamma_{i}\tau} - 1\right) + \alpha_{1i}\tau e^{-2\gamma_{i}\tau} \right) \\ B_{\phi_{6i}}\left(\tau\right) &= -\frac{1}{2} \left(\frac{\alpha_{1i}}{\gamma_{i}}\right)^{2} \left(e^{-2\gamma_{i}\tau} - 1\right). \end{split}$$

Consider now a period of length  $\nu$  and a set of dates  $T_j = t + \nu j$ , j = 1, ..., n. The time t swap rate for the period t to  $T_n$ , with fixed-leg payments at dates  $T_1, ..., T_n$  is given by

$$S(t, T_n) = \frac{1 - P(t, T_n)}{\nu \sum_{j=1}^{n} P(t, T_j)},$$
(E.2)

and the time *t* forward swap rate for the period  $T_m$  to  $T_n$ , and fixed-leg payments at dates  $T_{m+1}, \ldots, T_n$ 

by

$$S(t,T_n) = \frac{P(t,T_m) - P(t,T_n)}{\nu \sum_{j=m+1}^n P(t,T_j)}$$

Applying Ito's lemma to the time *u* forward swap rate between  $T_m$  and  $T_n$ , and switching to the forward measure  $\mathbb{Q}^{T_m,T_n}$  under which forward swap rates are martingales (see e.g. Jamshidian, 1997), we obtain

$$dS(u, T_m, T_n) = \sum_{i=1}^{3} \sum_{j=m}^{n} \zeta_j(u) B_{x_i}(T_j - u) dW_{iu}^{\mathbf{Q}^{T_m, T_n}},$$

where

$$\zeta_{j}(u) = \begin{cases} \frac{P(u,T_{m})}{\nu \sum_{j=m+1}^{n} P(u,T_{j})} & \text{if } j = m; \\ -\nu S(u,T_{m},T_{n}) \frac{P(u,T_{j})}{\nu \sum_{j=m+1}^{n} P(u,T_{j})} & \text{if } j = m+1, \dots n-1 \\ -(1+\nu S(u,T_{m},T_{n})) \frac{P(u,T_{n})}{\nu \sum_{j=m+1}^{n} P(u,T_{j})} & \text{if } j = n. \end{cases}$$

Notice that, since the  $\zeta_j(u)$  terms are stochastic, the forward swap rates are not normally distributed. We can, however, approximate  $\zeta_j(u)$  by their time *t* expected values, which are their time *t* values since these terms are martingales under the forward-swap measure. Thus, given date *t* information, the swap rate between dates  $T_m$  and  $T_n$  is (approximately) normally distributed

$$S(T_m,T_n) \sim \mathcal{N}\left(S(t,T_m,T_n),\sigma_N(t,T_m,T_n)\sqrt{T_m-t}\right),$$

where the volatility  $\sigma_N$  is given by

$$\sigma_N(t,T_m,T_n) = \left(\frac{1}{T_m-t}\int_t^{T_m}\sum_{i=1}^N\left(\sum_{j=m}^n\zeta_j(t)B_{x_i}(T_j-u)\right)^2du\right)^{\frac{1}{2}}.$$

#### E.2 Yield curve model

We closely follow the estimation of Gürkaynak et al. (2007) on Treasury yields using quotes on par swap yields with maturities between 1 and 40 years. We assume that instantaneous forward rates n-years hence are a function of six parameters:

$$f_t(n,0) = \beta_0 + \beta_1 \exp(-n/\tau_1) + \beta_2 (n/\tau_1) \exp(-n/\tau_1) + \beta_3 (n/\tau_2) \exp(-n/\tau_2).$$
(E.3)

We fit these parameters at month end by minimizing the sum of squared deviations between actual and predicted swap prices weighted by their inverse duration, which is approximately equal to minimizing the sum of squared yield deviations.

#### E.3 Prepayment model

As described in the main text, we begin by constructing a panel of monthly dealer prepayment forecasts by coupon-vintage using data from eMBS and Bloomberg LP. Specifically, we match pool characteristics from eMBS (WAC, WALA, WAM) to corresponding prepayment forecasts from Bloomberg. For each coupon until May 2003, and for each coupon-vintage from May 2003 onward, dealers report a prepayment forecast for each of the nine interest rate scenarios, as well as a WAC and WAM. To obtain additional pool characteristics, for the later sample, each survey is matched to its corresponding pool in eMBS. For the earlier sample, we match the survey to the vintage of the same coupon in eMBS with the minimum Mahalanobis distance based on WAC and WAM from the dealer's response. We only use securities that have a remaining principal balance in eMBS of more than \$1 million.

Dealers update their forecasts on different dates, so we use the most recent response as of the end of the month for each dealer (excluding dealers who did not update their response during that month), keeping only those securities in a month for which at least two dealers responded. Because we are interested in extracting prepayment model parameters that capture, for instance, the expectations of the rate-sensitivity of a security, we match each dealer's response to the swap rate of the day before that dealer's survey response was updated.

The prepayment forecasts in Bloomberg are reported in "PSA" terms, which can be translated into monthly CPRs using the following formula:

$$CPR_{\tau} = PSA/100 * \min(.2 * Age_{\tau}, 6) \text{ for } t \le \tau \le WAM$$
(E.4)

Thus, two securities with the same PSA forecast but of different ages (WALAs) will have different "average" CPRs if at least one of the securities is unseasoned. Because we would like to capture the prepayment speed forecast of the dealers with a single number for ease of estimation, we use the PSA forecast and the WALA<sup>2</sup> to compute the WAL (weighted average life), and thus the WAL-implied long-run CPR, defined as the constant monthly CPR that generates the WAL.

Specifically, we convert the monthly CPRs generated using equation E.4 to SMMs and compute the implied cash flows as in Section 3. The WAL is then defined as:

$$WAL_t = \frac{\sum_{j=t}^{\text{WAM}} j CF_j}{\sum_{j=t}^{\text{WAM}} CF_j}.$$
(E.5)

This gives us one long-run CPR forecast for each scenario per vintage per dealer. The nine different scenarios give us information about the expected rate sensitivity of the security. A common way to model this rate-sensitivity is through the use of an "S curve" as mentioned in the main text. Such a curve captures the observed behavior that prepayments are low for securities that are "out-of-the-money," i.e., the incentive to refinance is negative, and are mostly due to turnover and,

<sup>&</sup>lt;sup>2</sup>Since dealers don't actually report WALAs, we infer the WALA for a particular dealer's response by subtracting that dealer's surveyed WAM from the average sum of the WAM and WALA in eMBS.

to a lesser extent, cash-out refinancing or defaults. As a pool moves in-the-money (the refinancing incentive becomes positive) the refinancing component becomes a more important driver of prepayments, but at a declining rate: there is an incentive region in which prepayments are highly sensitive to changes in the interest rate (typically somewhere in the incentive region of 50-150 basis points) while beyond that, there is little sensitivity to further decreases in the available rate.

We convert our nine long-run CPRs into SMMs and fit the following S curve for each dealer for a vintage using nonlinear least squares:

$$SMM_i^{LR} = b_1 + b_4 \frac{\exp(b_2 + b_3 * INC_i)}{1 + \exp(b_2 + b_3 INC_i)} \text{ for } i = 1, 2, \dots, 9$$
(E.6)

where  $b_1, b_4 \in [0, 1]$  and  $b_1 + b_4 \le 1$  (these constraints ensure that the function is bounded by 0 and 1). Here, INC<sub>*i*</sub> is defined as the difference between the dealer's observed swap rate and WAC in scenario *i*.

Estimating an S curve for each dealer allows us to "average" these dealer responses despite the fact that often the surveys were updated on different days and thus refer to slightly different interest rate scenarios. We take this average by averaging fitted dealer SMMs at 50 basis point intervals between -300 and 300 basis points, with the 0-scenario corresponding to the average 0-scenario across dealers.

Finally, because cash flows, and thus the OAS, depend on not just the average long-run prepayment rate, but also the time pattern of prepayments, we fit a series of monthly SMMs in the form of equations (4.1) and (4.2) to the dealer-averaged long-run CPR forecasts. As discussed in the main text, this functional form creates the "burnout effect" of prepayments. However, because the Bloomberg data provide no additional information as to the time pattern of prepayments, it is impossible to jointly identify  $\chi_t$ ,  $\kappa_1$ ,  $\kappa_2$  for each security. We therefore assume that  $\kappa_1$  and  $\kappa_2$  are universal parameters and let  $\chi_t$  vary across securities and time. To calibrate  $\kappa_1$  and  $\kappa_2$ , we exploit the fact that as INC  $\rightarrow \infty$ , SMM  $\rightarrow b_1 + \chi \kappa_1 + (1 - \chi) \kappa_2$  (for WALA> 30). Thus,  $b_1 + \kappa_1$  and  $b_1 + \kappa_2$  represent the speeds that a seasoned pool would prepay at if it were deeply in the money and composed of only fast or only slow borrowers, respectively. We therefore estimate  $\hat{\kappa}_1 = \kappa_1$  and  $\hat{\kappa}_2 = \kappa_2$  by taking the 99th and 1st%iles of survey SMMs (less an average  $b_1$ , which is negligible) for the -300 basis point interest rate scenario among seasoned ITM securities in our sample. This yields  $\hat{\kappa}_1 = 0.11$  and  $\hat{\kappa}_2 = 0.014$ . <sup>3</sup>

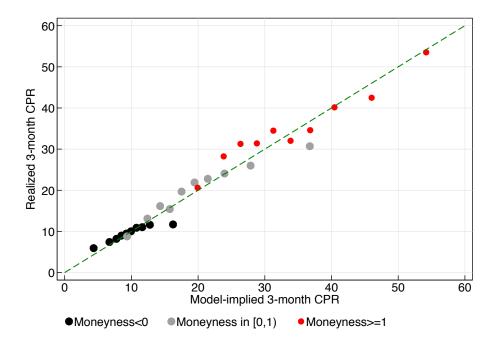
Given  $\kappa_1$  and  $\kappa_2$ , there are then four coefficients to be estimated for each security on each date:  $\chi_t$ ,  $b_1$ ,  $b_2$ , and  $b_3$ . We fit these four coefficients using nonlinear least squares with the thirteen dealeraveraged long-run fitted CPRs. Because of its flexibility, this model is able to fit the long-run CPRs quite well; the MAE across securities is less than 0.2.

Figure A-2 illustrates that the prepayment model that results from the previous procedure fits subsequently realized prepayment behavior well. We first divide the trusts in our sample into three groups based on their moneyness as of month *t*: OTM (moneyness<0; 1199 observations);

<sup>&</sup>lt;sup>3</sup>We have experimented with alternative calibrations, and obtained qualitatively similar results.

weakly ITM (moneyness  $\in [0, 1)$ ; 1573 obs.) and strongly ITM (moneyness>1; 941 obs.). Within each group, we then form deciles based on predicted three-month CPRs, and plot the average realized CPR (over months t + 1 to t + 3) against the average predicted CPR. The figure illustrates a few important points. First, as discussed in Section 4.3, moneyness (the refinance incentive) is the main determinant of predicted and realized CPRs. Second, however, it is not the only determinant: there is overlap between the three groups, meaning that predicted (and realized) CPRs are not always monotonic in moneyness. This reflects the factors emphasized in our specification in equations (4.1) and (4.2)—seasoning and burnout—as well as variation over time in turnover or refinancing sensitivity. Third, there is a lot less variation in the prepayment speeds of OTM securities than for ITM securities. Last, the model based on the dealer surveys fits actual prepayment behavior well: the dots are aligned along the 45 degree line.

**Figure A-2: Prepayments: Model projections vs. realized.** Figure shows binned scatterplot of realized three-month constant prepayment rate (CPR) over months t + 1 to t + 3 versus model-predicted three-month CPR at the end of month t. Within each of three moneyness groups (defined as of month t), ten deciles are formed by predicted CPR, and for each of the resulting 30 groups we plot average predicted and realized CPRs.



#### E.4 MBS cash flows

This section provides detail on the calculation of cash flows given a sequence of SMMs. Consider a fixed-rate MBS with an original balance of \$1, and let  $\theta_t$  be the "factor," or remaining balance relative to origination, at date *t*. In level-payment fixed-rate mortgages, the principal is repaid gradually rather than with a bullet payment at maturity and the borrower makes fixed payments every month. Denote the loan maturity measured in months by *T* (at the pool level, this is referred to as weighted average maturity, or WAM). Let *k* be the monthly installment from the borrower to the servicer, *w* the interest rate on the loan (or weighted average coupon, WAC, at the pool level), and *c* the coupon paid to investors. The difference between the loan and coupon rates is earned by servicers or by the guaranteeing agency. To compute the fixed payment *k* note that, net of this payment, the loan balance absent any prepayment, denoted  $\tilde{\theta}_t$  grows at rate (1 + w), or:

$$\tilde{\theta}_t = (1+w)\,\tilde{\theta}_{t-1} - k. \tag{E.7}$$

Solving for  $\tilde{\theta}_T = 0$ , it then follows that  $k = \left(\frac{w(1+w)^T}{(1+w)^T-1}\right)$ . The evolution of the loan balance  $\theta_t$  allowing for early prepayment generalizes equation (E.7). After accounting for loan amortization and unscheduled principal payments, the factor evolves according to:

$$\theta_t = (1 - \mathrm{SMM}_{t-1})(1+w)\theta_{t-1} - k\,\hat{\theta}_t,\tag{E.8}$$

where w is the interest rate on the loan (or weighted average coupon, WAC) and k is the constant monthly payment composed of the scheduled principal and interest payments. SMM<sub>t</sub> is the "single month mortality," or the fraction of the remaining balance that was prepaid in month t due to unscheduled principal payments, and  $\hat{\theta}_t$  is the cumulated fraction of unit principal that has not prepaid since the inception of the mortgage, which is also known as the *survival* factor,  $\hat{\theta}_t = \prod_{s=0}^{t-1} (1 - \text{SMM}_s)$ . It then follows that  $\theta_t = \hat{\theta}_t \times \tilde{\theta}_t$ .<sup>4</sup> Given prepayment rates, cash flows passed through to investors per unit of principal are:

$$X_t = (\theta_{t-1} - \theta_t) + c \,\theta_{t-1},\tag{E.9}$$

where the principal payment is equal to the decline in principal  $(\theta_{t-1} - \theta_t)$  and the coupon payment from the borrower to the investor net of the servicing and agency guarantee fees is  $c \theta_{t-1}$ .

#### E.5 Monte Carlo simulations

As discussed in Section 3, computing the OAS requires Monte Carlo simulations of swaps and discount rates. Along each simulation, we use the prepayment model to compute MBS cash flows. We take the OAS to be the constant spread to swaps that sets the average discounted value of cash flows along these paths equal to the market price. To construct these paths, we first simulate 1,000 paths of the three factors of the interest rate model using draws of the state variables described in Appendix E.1. We use antithetic variables as a variance reduction technique, giving us 2,000 paths in total.

<sup>&</sup>lt;sup>4</sup>The prepayment speed is often reported in annualized terms, known as the "conditional prepayment rate" or  $CPR_t = 1 - (1 - SMM_t)^{12}$ .

## **F** Identifying *OAS*<sup>Q</sup> from the cross-section

In this Appendix, we relax the assumption that the risk-neutral prepayment function is a multiple of the physical prepayment function and explore identification of  $OAS^Q$  from the cross section of MBS. This alternative identification strategy relies on the cross section of stripped securities being priced fairly relative to each other at any given point in time, without requiring us to make assumptions about the physical prepayment function.

In particular, we model the date  $\tau$  risk-neutral single-monthly mortality (Q-SMM) rate of security *j* as

$$s_{j,\tau}^{\mathbf{Q}} = b_1^{\mathbf{Q}} \min\left(\mathrm{Age}_{\tau}^j/30, 1\right) + \kappa^{\mathbf{Q}} \cdot f_{\tau}^j \cdot \frac{\exp\left(b_2^{\mathbf{Q}} + b_3^{\mathbf{Q}} \cdot \mathrm{INC}_{\tau}^j\right)}{1 + \exp\left(b_2^{\mathbf{Q}} + b_3^{\mathbf{Q}} \cdot \mathrm{INC}_{\tau}^j\right)},$$

where  $\text{INC}_{\tau}^{j}$  is the interest rate incentive to refinance and  $f_{\tau}^{j}$  is the "factor" (a pool's remaining balance relative to origination), at date  $\tau$ . In this prepayment function we use the factor as a proxy for the "burnout" effect; this is a simplified version of the specification in the main text. Mortgage rates have mostly trended lower in our sample and, absent the large volume of data from prepayment surveys, one cannot separately identify empirically the effect of "burnout" (remaining borrowers are largely rate insensitive) from the effect of a lower sensitivity of all borrowers to rates (that would lead to the upper-flatness in the "S-shaped" prepayment function). In the simplified version we assume that securities that exhibited more prepayments in the past (low  $f_{\tau}^{j}$ ) tend to prepay more slowly, implying fewer parameters to be estimated than in the main text ( $\chi$  and  $\kappa$  in equations (4.1) and (4.2) in Section 4.3).

For each pool  $j \in J_{\tau}$  that trades at date  $\tau$ , we compute the *OAS* implied by a set of parameters  $(b_1^Q, b_2^Q, b_3^Q, \kappa^Q)$  for the IO and the PO strip on that pool, denoted, respectively, by  $OAS_{j,IO}(b_1^Q, b_2^Q, b_3^Q, \kappa^Q)$  and  $OAS_{j,PO}(b_1^Q, b_2^Q, b_3^Q, \kappa^Q)$ . At each date  $\tau$ , we then minimize the remaining-principal-balance-weighted sum of the squared difference between OAS on the IO and the PO strips written on the same pool:

$$\arg\min\sum_{j\in J_{\tau}}\frac{\theta_{j\tau}}{\sum_{j\in J_{\tau}}\theta_{j\tau}}\left(OAS_{j,IO}\left(b_{1}^{Q},b_{2}^{Q},b_{3}^{Q},\kappa^{Q}\right)-OAS_{j,PO}\left(b_{1}^{Q},b_{2}^{Q},b_{3}^{Q},\kappa^{Q}\right)\right)^{2},\tag{F.1}$$

where  $\theta_{j\tau}$  is the remaining principal balance in pool j at date  $\tau$ . The risk-neutral *OAS* on the pass-through security on pool j (the combination, denoted "PT," of IO and PO) at date  $\tau$  is then computed as the *OAS* implied by the set of parameters  $(b_1^Q, b_2^Q, b_3^Q, \kappa^Q)^*$  that solves equation (F.1) at date  $\tau$ :

$$OAS_{j}^{\mathbb{Q}} \equiv OAS_{j,PT} \left( \left( b_{1}^{\mathbb{Q}}, b_{2}^{\mathbb{Q}}, b_{3}^{\mathbb{Q}}, \kappa^{\mathbb{Q}} \right)^{*} \right).$$

This identification strategy relies on the assumption that the cross section of securities at a given

date are priced fairly relative to each other, thus relaxing both the assumption that the risk-neutral SMM rate is a (pool- and date-specific) multiple of the physical SMM rate and that the IO and the PO on a given pool are equally exposed to non-interest and non-prepayment shocks. In addition, this strategy does not rely on any P-measure prepayment function.

Figure A-3 shows the cross-sectional scatterplot of the resulting  $OAS^Q$  on the passthrough security, together with the local smoother. Though the level of the  $OAS^Q$  that is identified from the cross-section is somewhat higher than in the estimation that relies on both pricing and physical prepayment information, the overall conclusion remains the same: the  $OAS^Q$  does not vary significantly with moneyness, suggesting that differences in pool-level liquidity do not drive the OAS smile. This is confirmed in Table A-3, which shows the equivalent regressions to those in Table 5 in the main paper. As was the case there, the only bin that exhibits significantly higher  $OAS^Q$  than for ATM pools is the deeply ITM bin (moneyness $\geq$ 2.25). The coefficient on that bin becomes larger if we add month fixed effects or if we end the sample in July 2007. However, the magnitude remains quite similar to the one in Table 5.

Despite these similarities, this identification strategy has a number of limitations relative to the one used in the main text. The method in this appendix only recovers the risk-neutral prepayment speed but does not allow the estimation of the prepayment risk premium (OAS-OAS<sup>Q</sup>), which is central to our analysis. One can only back out the prepayment risk premium from physical prepayment speeds. In our main analysis, we are fitting a parsimonious functional form to market participants' forecasts of future prepayment speeds under different rate scenarios; this helps against misspecification of the prepayment function relative to the real-time  $\mathbb{P}$ -expectations of market participants. The fact that the "standard" OAS (under  $\mathbb{P}$ -prepayments) look similar as in the dealer data further assuages fears that misspecification is driving our findings about the OAS smile.

Table A-3: Cross section of OAS <sup>Q</sup> on pass-throughs, based on alternative identification from
prices only. Coefficient estimates from OLS regression of the OAS <sup>Q</sup> resulting from the cross section
of IO and PO prices each month on different moneyness level bins either including or excluding
time fixed effects. Robust standard errors (clustered at the month level) in brackets. Significance: *
p < 0.1, ** p < 0.05, *** p < 0.01.

	(1)		(2)		(3)		(4)	
< -1.25	5.0	[3.7]	-0.1	[2.0]	2.5	[1.9]	1.8	[1.8]
[-1.25,75)	2.2	[2.2]	-2.0	[1.8]	0.8	[1.7]	-0.7	[1.8]
[75,25]	0.8	[1.5]	-0.3	[1.4]	2.6**	[1.2]	0.9	[1.3]
[.25, .75)	-4.6***	[1.2]	-4.9***	[1.3]	-4.4***	[1.3]	-5.4***	[1.2]
[.75, 1.25)	-7.8***	[2.2]	-6.3***	[2.2]	-6.6***	[2.0]	-7.6***	[2.1]
[1.25, 1.75]	-5.6*	[3.2]	-3.5	[3.3]	-5.4*	[3.1]	-6.6*	[3.5]
[1.75, 2.25]	-0.5	[5.1]	4.0	[4.2]	-0.5	[4.4]	2.8	[4.7]
≥ 2.25	6.3	[8.9]	$16.2^{*}$	[9.2]	21.1**	[10.2]	20.3*	[11.2]
Const	11.1***	[2.1]	10.7***	[1.2]	4.2***	[1.6]	4.9***	[1.2]
Month FEs?	No		Yes		No		Yes	
Adj. R2	0.01		0.66		0.04		0.44	
Obs.	1005		1005		796		796	
Dates	199501.201012		199501.201012		199501.200707		199501.200707	

**Figure A-3: Cross-sectional variation in OAS**<sup>Q</sup>**, based on alternative identification from prices only.** This figure shows the scatterplot and local smoother of the cross-sectional variation in OAS<sup>Q</sup> on pass-through securities, estimated using the cross section of IO and PO prices each month.

