FISCAL DETERMINATION OF HYPERINFLATION (VERY PRELIMINARY)

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ABSTRACT. We develop and estimate a nonlinear general equilibrium model on hyperinflation. Our estimated results show that the recurrence of hyperinflation and the sustained end of it are determined by the amount of average seigniorage or shocks to seigniorage or both. A high level of average seigniorage is necessary but not sufficient to cause hyperinflation. Each country has its distinctive characteristic in its experience with hyperinflation. Unlike the existing literature on hyperinflation, we show that intricate interactions among beliefs, escapes, levels of average seigniorage, and sizes of shock variances explain different dynamic patterns of inflation we observe in different Latin American countries.

> Perhaps the simple rational expectations assumption is at fault here, for it is difficult to believe that economic agents in the hyperinflations understood the dynamic processes in which they were participating without undergoing some learning process that would be the equivalent of adaptive expectations.
>
> *Stanley Fischer*

> ... the only way to test theory and to analyze policy is by estimating the deep parameters of the model.
>
> *Zvi Eckstein*

I. INTRODUCTION

In the last three decades, a number of Latin American countries have experienced extraordinarily high inflation at the rates ranging from 100% to 400% per month, which we call hyperinflation. Why did hyperinflation occur repeatedly in these countries? How did such recurrence come to halt? Under what circumstance would these countries maintain their low inflation path in the long run? This paper answers these questions by estimating a

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theoretical model that allows both seigniorage and its shocks to change regime over time. We show that such a model is capable of explaining the time series of hyperinflation in several countries in Latin America.

The relationship between seigniorage and hyperinflation is one of the classic issues in monetary economics (Sargent and Wallace, 1981, 1987). The conventional view is that extremely high inflation is a result of high government expenditures financed by printing money (seigniorage). This view is a primary reasoning behind the role of the International Monetary Fund in promoting fiscal reforms and disciplines in countries that have experienced extremely high inflation rates.

The conventional view has been challenged on serval grounds. The correlation between seigniorage and hyperinflation is weak for many countries; the highest level of inflation does not necessarily occur at the same time when seigniorage reaches its highest level. In fact, as Cagan (1956) documented, inflation rates could go up well beyond the maximum revenues generated by money creation. In support of this finding, Sargent and Wallace (1973) provide empirical evidence that inflation Granger causes money, but not vice versa.

These empirical observations are supported by a rational expectations theory given by Sargent and Wallace (1987). In their model economy, the are two steady state stationary equilibria. They show that even with a low level of seigniorage, inflation dynamics converge to the high steady state equilibrium inflation rate. The high inflation equilibrium is stable and the low one is not – the so-called slippery side of the Laffer curve. These theoretical results led Bruno and Fischer (1990) to conclude that "the monetary anchor [of fixing the growth rate of money or nominal exchange rate] cannot be replaced by a fiscal anchor."[1]

The principal difficulty in resolving this controversy is that there is little successful empirical work on the cause of hyperinflation and on how hyperinflations can recur and then end suddenly. The goal of this paper is to put all the pieces of the puzzle together and provide careful empirical evidence for the fiscal determination of inflation in five Latin American countries: Argentina, Bolivia, Brazil, Chile, and Peru. Our model builds on Marcet and Nicolini (2003), discussed in more detail below, where the agents' beliefs about the next-period inflation rate evolve according to a constant-gain learning mechanism. Intricate interactions among these beliefs, a sequence of shocks to seigniorage, and regime switches between the low and high levels of average seigniorage generate the different dynamics of inflation we observe across the countries in our sample in the last three or four decades.

Following the approach of Sargent, Williams, and Zha (2004) and Sims, Waggoner, and Zha (2005), we estimate the key parameters of our model for each country. We show that in each country inflation dynamics all converge to a stable rate for each country and this inflation rate is close to the low rational-expectations-equilibrium (REE) inflation rate (if

[1]Even with a nominal variable being pegged, Sims (1994) and Woodford (1995) argue that other aspects of fiscal policy determines the initial price level and thus the path of the price level.

it exists) in the framework of Sargent and Wallace (1987). Our estimated model indicates that hyperinflation is caused by not only a high level of average seigniorage but also large shocks to seigniorage or rapidly changing beliefs. Contrary to Bruno and Fischer (1990), we show that responsible fiscal policy committed to low seigniorage can provide an anchor to the low inflation equilibrium and prevent hyperinflation from returning again.

II. Brief Literature Review

There has been some empirical work on testing the hyperinflation theory of Sargent and Wallace (1987). Sargent and Wallace (1987) set up an econometric framework for testing their theory. Imrohoroglu (1993) use this framework to estimate the Sargent and Wallace model using the German hyperinflation data in the early 1920s. The estimated model generates the single run-up of hyperinflation but is unable to explain the end of hyperinflation.

A recent paper by Marcet and Nicolini (2003) studies a learning model built on Sargent and Wallace (1987) and Marcet and Sargent (1989). Marcet and Nicolini propose a stopping rule to bring inflation temporarily back to the lower steady state REE inflation rate. They argue that this rule can be interpreted as being consistent with exchange-rate-based stabilization policies experienced by some hyperinflation countries in Latin America.

Marcet and Nicolini's stopping rule is an important feature in generating recurring hyperinflation, but their model is unable to explain why many Latin American countries have not had hyperinflation experienced one or two decades ago.[2] Marcet and Nicolini (in press) report empirical evidence on the relationships between money and prices by treating money supply as an exogenous process and calibrating the key structural parameters in the money demand. But endogeneity of money supply is an indispensable part of the theory of Sargent and Wallace (1987) in explaining hyperinflation. Further, the time series of hyperinflation is quite sensitive to the values of structural parameters, which have to be estimated in order to fit to the different patterns of time series of inflation in different countries. This sensitivity is one of the main reasons we introduce the time-varying parameters for seigniorage into our model, as will be shown in the next section.

III. The Theory of Hyperinflation

Our theoretical model builds on Marcet and Nicolini (2003), which is composed of the savings decisions of a representative private agent and the government budget constraint:

$$\frac{M_t}{P_t} = \frac{1}{\gamma} - \frac{\lambda}{\gamma}\frac{P^e_{t+1}}{P_t}, \tag{1}$$

[2]We are grateful to Albert Marcet for sharing their FORTRAN computer code in order for us to duplicate their results. Their learning mechanism is a combination of constant-gain learning and least-squares learning, but the phenomenon of recurring hyperinflation does not depend on such a combination. A constant-gain learning rule alone would generate a similar pattern of recurring hyperinflation.

$$M_t = \theta M_{t-1} + d_t(s_t)P_t, \tag{2}$$

$$d_t(s_t) = \bar{d}(s_t) + \eta_{dt}(s_t), \tag{3}$$

$$\Pr(s_{t+1} = i | s_t = j) = q_{ij},\ i, j = 1, ..., h, \tag{4}$$

where $0 < \lambda < 1$, $0 < \theta < 1$, $\gamma > 0$, $\bar{d}(s_t) > 0$, $s_t \in \{1, \ldots, h\}$ is a Hamilton (1989) type of regime state which is observable to the agent in the model but unobservable to an econometrician, P_t is the general price level at time t, M_t is per capita nominal balances at time t, P^e_{t+1} is the public's expectation of the price level at time $t+1$, and $\eta_{dt}(s_t)$ is an i.i.d. random shock. Each column of the transition probability matrix $Q = [q_{ij}]$ sums to 1 exactly.

Equation (1) is a rational-expectations version of Cagan's demand equation for high-powered money.[3] Equation (2) is Friedman (1948)'s version of the government budget constraint.[4] The coefficient $\bar{d}(s_t)$ measures the average amount of seigniorage financed by money creation and the exogenous assumption about seigniorage shocks is in the spirit of Sargent and Wallace (1981) and gives the fiscal authority a dominate role in its interactions with the monetary authority.

We follow Marcet and Sargent (1989) and Marcet and Nicolini (2003) to replace the rational expectation of inflation $\pi^e_{t+1} = E_t[P_{t+1}/P_t]$ by:

$$\pi^b_{t+1} = \beta_t$$

where the superscript b stands for belief. The belief β_t is updated with a constant-gain learning mechanism:

$$\beta_t = \beta_{t-1} + \varepsilon(\pi_{t-1} - \beta_{t-1}), \tag{5}$$

where $0 < \varepsilon << 1$ and π_t is the gross inflation rate at time t, defined as

$$\pi_t = P_t/P_{t-1}.$$

III.1. **Normalization.** It can be seen from the model (1)-(4) that inflation dynamics depend on $\gamma d_t(k)$, not the individual parameters γ and $d_t(k)$ separately. Hence we have the following proposition.

Proposition 1 (Normalization). The dynamics of π_t are unchanged if both $d_t(k)$ and $1/\gamma$ are normalized by the same scale.

Proof. Let $d_t(k)$ and $1/\gamma$ be multiplied by any real scalar κ. If we redefine P_t to be P_t/κ, the original system (1)-(4) remains the same. The redefinition of the price level simply means that the price index is re-based, which does not affect the dynamics of either M_t or π_t. □

[3] For the derivation of this saving decision in a general equilibrium setup, see Marimon and Sunder (1993), Marcet and Nicolini 2003), and various chapters in Ljungqvist and Sargent (2004).

[4] This equation has traditionally been used to measure the amount of seigniorage. See, for example, Fischer (1982) and Chang (2000).

This proposition has an important implication. In Marcet and Nicolini (2003), γ and $\bar{d}(k)$ are treated as separate parameters to be calibrated and the calibrated value of $\bar{d}(k)$ is interpreted as fiscal deficits as a share of GDP. But this interpretation is misleading, for these parameters cannot be identified separately and re-normalizing them in the manner of Proposition 1 gives the same equilibrium outcome.[5] For identification purposes, therefore, we normalize $\gamma = 1$ for the rest of the paper.

III.2. **Deterministic Steady State.** The deterministic version of model (1) - (4) can be obtained by fixing the state $s_t = k \in \{1,\ldots,h\}$ and setting $\eta_{dt}(k)$ to zero for all t .

Proposition 2. If

$$\bar{d}(k) < \frac{1+\theta\lambda - 2\sqrt{\theta\lambda}}{\gamma}, \tag{6}$$

then there exists two stationary equilibria for π_t:

$$\pi_1^*(k) = \frac{\left(1+\theta\lambda-\bar{d}(k)\right) - \sqrt{\left(1+\theta\lambda-\bar{d}(k)\right)^2 - 4\theta\lambda}}{2\lambda}, \tag{7}$$

$$\pi_2^*(k) = \frac{\left(1+\theta\lambda-\bar{d}(k)\right) + \sqrt{\left(1+\theta\lambda-\bar{d}(k)\right)^2 - 4\theta\lambda}}{2\lambda}. \tag{8}$$

Proof. It follows from Sargent and Wallace (1987) that

$$\pi_t = \left(\lambda^{-1} + \theta - \bar{d}(k)\lambda^{-1}\right) - \frac{\theta}{\lambda\,\pi_{t-1}}.$$

In stationary equilibrium, $\pi_t = \pi_{t-1}$. Substituting this into the above equation leads to (7) and (8). □

The condition represented by (6) will be imposed in our estimation. As shown in Marcet and Sargent (1989) and Marcet and Nicolini (2003), as long as the gain g is sufficiently small, π_t converges to π_1^* if the initial belief $\beta_0 < \pi_2^*(k)$. In contrast, π_t tends to converge to the high steady state inflation rate $\pi_2^*(k)$ under rational expectations (Sargent and Wallace, 1987). This steady state equilibrium, however, possesses a perverse static property such that inflation rises when seigniorage falls. The tendency of getting stuck in the high steady state equilibrium rate is a serious difficulty one faces when trying to explain recurring hyperinflation and its sharp decline.

[5]For a general discussion of normalization in econometrics, see Hamilton, Waggoner, and Zha (2004).

III.3. **Equilibrium and Escape.** We now consider the stochastic version of the model. Using (1)-(2)) and (5) we obtain the equilibrium path of inflation

$$\pi_t = \frac{\theta(1-\lambda\beta_{t-1})}{1-\lambda\beta_t - d_t(s_t)}. \tag{9}$$

The equilibrium holds only when both the numerator and denominator in (9) are positive. As will be shown in next section, moreover, the denominator must be bounded away from zero to ensure that the moments of inflation exist and for the inflation dynamics to converge. The equilibrium restrictions thus are

$$1-\lambda\beta_{t-1} > 0, \tag{10}$$

$$1-\lambda\beta_t - d_t(s_t) > \delta\theta(1-\lambda\beta_{t-1}), \tag{11}$$

where (11) bounds the denominator away for zero for some $\delta > 0$. It follows that inflation is bounded by $1/\delta$. Because the steady state REE inflation rate is bounded by $1/\lambda$ according to Proposition 2, we have $\lambda \geq \delta$.

When a run of seigniorage shocks η_{dt} push β_t over π_2^* and there is a tendency that β_t will escalate up and the equilibrium conditions (10) and (11) may be violated. When this violation occurs, we say that the inflation dynamics escape from the domain of attraction of the low REE inflation rate. An inflation escape is an undesirable outcome; once it happens, an equilibrium no longer exists. Marcet and Nicolini (2003) propose a stopping rule consistent with purchasing power parity and exchange rate interventions. The basic idea is that when the equilibrium conditions (10)-(11) are violated, π_t is reset to the low steady state REE rate $\pi_1^*(s_t)$. In reality we do not observe inflation exactly equal to $\pi_1^*(s_t)$. We modify this rule and let the inflation process after escape be

$$\pi_t = \pi_t^*(s_t) \equiv \pi_1^*(s_t) + \eta_{\pi t}(s_t), \tag{12}$$

where $\eta_{\pi t}(s_t)$ is an i.i.d. random shock such that

$$0 < \pi_t^*(s_t) < 1/\delta.$$

III.4. **Equilibria and Convergence.** In order to better understand the dynamics induced by learning, it is useful to determine whether the beliefs will converge to some equilibrium, and if so what are its characteristics. As in Sargent (1999), we call a limit of the learning rule a *self-confirming equilibrium* (SCE), as it has the property that agents' beliefs are consistent with their observations. A rational expectations equilibrium (REE) is a self-confirming equilibrium, but SCEs are not necessarily REEs. In particular, we have assumed that agents do not condition on the current regime when forecasting inflation, even though the regime is observed. Fully rational agents would incorporate this knowledge, and the REE will include forecasts which condition on the regime. Thus if beliefs converge, they will converge to an SCE which at best averages over the different regime-dependent forecasts.

III.4.1. *Self-Confirming Equilibria.* First, because it recurs so often we introduce some notation for the truncation event in (11). We truncate when

$$d_t \quad \geq \quad [1-\lambda\beta_t - \delta\theta(1-\lambda\beta_{t-1})]/\gamma \equiv \omega(\beta_t,\beta_{t-1}).$$

When we truncate, we draw π_t randomly as described above. Thus we have:

$$\pi_t = \mathscr{I}(d_t(s_t) < \omega(\beta_t,\beta_{t-1}))\frac{\theta(1-\lambda\beta_{t-1})}{1-\lambda\beta_t-\gamma d_t(s_t)} + \mathscr{I}(d_t(s_t) \geq \omega(\beta_t,\beta_{t-1}))\pi_t^*(s_t),$$

where $\mathscr{I}(A)$ is an indicator function for the event A. Hence we can write (5) as:

$$\beta_{t+1} = \beta_t + \varepsilon g(\beta_t,\beta_{t-1},d_t,\pi_t^*) \tag{13}$$

where

$$g(\beta_t,\beta_{t-1},d_t,\pi_t^*) = \mathscr{I}(d_t < \omega(\beta_t,\beta_{t-1}))\frac{\theta(1-\lambda\beta_{t-1})}{1-\lambda\beta_t-\gamma d_t} + \mathscr{I}(d_t \geq \omega(\beta_t,\beta_{t-1}))\pi_t^*(s_t) - \beta_t$$

We then further define $\tilde{g}(\beta_t,d_t,\pi_t^*) = g(\beta_t,\beta_t,d_t,\pi_t^*)$. Then assuming that Q has an invariant distribution $\bar{q}$ and $d \sim F(d|s)$ (which is log-normal conditional on s) we define:

$$\begin{aligned}
\bar{g}(\beta) &= E[\tilde{g}(\beta,d_t,\pi_t^*)] \\
&= \sum_{j=1}^{h}\left[\int_0^{(1-\delta\theta)\xi}\frac{\theta(1-\lambda\beta)}{1-\lambda\beta-\gamma d}dF(d|j) + [1-F((1-\delta\theta)\xi\,|\,j)]E(\pi^*|j)\right]\bar{q}_j - \beta \\
&= \sum_{j=1}^{h}\left\{\theta\xi\Psi_j(\beta,(1-\delta\theta)\xi) + \left[1-\Phi\left(\frac{\log((1-\delta\theta)\xi)-\log d(j)}{\sigma_d(j)}\right)\right]\pi^*(j)e^{\frac{\sigma_\pi^2}{2}}\right\}\bar{q}_j - \beta
\end{aligned}$$

where Φ is the standard normal cdf, we define $\xi = (1-\beta\lambda)/\gamma$ so that $\omega(\beta,\beta) = (1-\delta\theta)\xi$, and we use that $\log d|j \sim N(\log d(j),\sigma_d^2(j))$ and $\log\pi^*|j \sim N(\log\pi^*(j),\sigma_\pi^2)$. We collect the integral terms in Ψ which we define as:

$$\Psi_j(\beta,b) = \int_0^b \frac{1}{\xi(\beta)-d}dF(d|j).$$

As $b \to \xi(\beta)$ this integral diverges, but it is well behaved in our case for δ bounded away from zero.

Proposition 3. As $\varepsilon \to 0$ the beliefs $\{\beta_t\}$ from (13) converge weakly to the solution of the ODE

$$\dot{\beta} = \bar{g}(\beta) \tag{14}$$

for $\delta > 0$ and a broad class of probability distributions of $\eta_{dt}(s_t)$ and $\eta_{\pi t}(s_t)$.

Proof. Under our assumed distributions and truncation rule, this follows from **?**. □

Thus we see that self-confirming equilibria are zeros of the mean dynamics $\bar{g}$. In order for an SCE to be a limit of the learning rule, it must further be a stable equilibrium point of the ODE. Since we don't have an explicit expression for $\bar{g}$, we must be find the SCEs numerically. Thus we look for stationary points $\bar{\beta}$ such that $\bar{g}(\bar{\beta}) = 0$, and since the system is scalar the stability condition is simply $\bar{g}'(\bar{\beta}) < 0$. For comparison, we are also interested in the equilibria which would result if the economy were to remain in one regime for all time. We refer to these as "conditional equilibria," and they can be found by finding the stable equilibria of the conditional mean dynamics $\bar{g}(\beta, j)$ which are defined implicitly above and satisfy $\bar{g}\beta) = \sum_j \bar{g}(\beta, j)\bar{q}_j$. The stable conditional SCEs give the limit points if the regimes were fixed, while the stable SCE averages over the conditional equilibria.

It is important to note that we have convergence in a weak or distributional sense. As the regimes continue to change and the economy is hit by shocks, beliefs will continue to fluctuate. These fluctuations become proportionately smaller when the gain ε is smaller, but for any positive gain the beliefs will have a non-degenerate distribution. As the gain shrinks, this distribution collapses to a point mass on the solution of the ODE. But the results of Proposition 3 only describe the average behavior of beliefs for small gains. There may be extended periods in which beliefs are away from the SCE, particularly as some regimes may be experienced for extended periods.

III.4.2. *Rational Expectations Equilibria.* As a natural benchmark, we now consider the rational expectations equilibria of the model. We look for stationary Markov equilibria in which inflation and expected inflation are given by:

$$\begin{aligned}
\pi_t &= \pi(s_t, s_{t-1}, d_t) \\
E_t \pi_{t+1} &= E_t \pi(s_{t+1}, s_t, d(s_{t+1}) + \eta_{d,t+1}(s_{t+1})) \\
&= \sum_{j=1}^{h} \int \pi(s_j, s_t, d(s_j) + \eta) dF(\eta | j) q_{s_t,j} \equiv \pi^e(s_t).
\end{aligned}$$

Then going through calculations similar to those above we have:

$$\pi(s_t, s_{t-1}, d_t) = \frac{\theta(1 - \lambda \pi^e(s_{t-1}))}{1 - \lambda \pi^e(s_t) - \gamma d_t(s_t)}.$$

Again this only holds when the denominator is positive (which is the more stringent condition), so we truncate as above, giving:

$$\begin{aligned}
\pi(s_t, s_{t-1}, d_t) =& \mathscr{I}(d_t(s_t) < \omega(\pi^e(s_t), \pi^e(s_{t-1}))) \frac{\theta(1 - \lambda \pi^e(s_{t-1}))}{1 - \lambda \pi^e(s_t) - \gamma d_t(s_t)} \\
&+ \mathscr{I}(d_t(s_t) \geq \omega(\pi^e(s_t), \pi^e(s_{t-1}))) \pi_t^*(s_t)
\end{aligned}$$

Letting $\omega_{ij} = \omega(\pi^e(s_j), \pi^e(s_i))$ and taking expectations of both sides conditional on information at $t-1$ and setting $s_{t-1} = i$ yields:

$$\pi^e(i) = \sum_{j=1}^{h} \left\{ \theta \xi_i \Psi_j(\pi^e(j), \omega_{ij}) + \left[1 - \Phi\left(\frac{\log(\omega_{ij}) - \log d(j)}{\sigma_d(j)} \right) \right] \pi^*(j) e^{\frac{\sigma_\pi^2}{2}} \right\} q_{ij}, \quad (15)$$

where $\xi_i = (1 - \pi^e(i)\lambda)/\gamma$ and Ψ_j is as above. Thus we have h coupled equations determining $\pi^e(s_t)$. Substituting this solution into the expression for $\pi(\cdot)$ then gives the evolution of inflation under rational expectations. The equations are sufficiently complicated that an analytic solution is not available, and hence we must look for equilibria numerically. A simple iterative solution method for the equations consists of initializing the $\pi^e(j)$ on the right side of (15) and computing $\pi^e(i)$ on the left side and iterating until convergence. Alternatively, any other numerical nonlinear equation solver can be used.

III.4.3. *Multiplicity and Nonexistence.* Typically there will be multiple rational expectations equilibria of the model, but a unique stable SCE. As we've seen, in the deterministic counterpart of the model there are two REEs. With small enough shocks, we also find that there are two conditional SCEs in each regime. As discussed above, the true SCEs average across these conditional SCEs. Thus for example with two possible regimes and two conditional SCEs in each regime, there would typically be two SCEs, with one of them stable. REEs also average across the conditional SCEs, taking into account the probability of regime switches. Thus for example with two conditional SCEs in each regime, there are typically four REEs (switching between values close to the conditional SCEs in each regime). However, when the shocks to seignorage become large enough there may be only one conditional SCE in a regime, or a conditional SCE may fail to exist altogether. Depending on the weight these high-shock regimes have in the invariant distribution, the SCE may also fail to exist. Similarly, there may be fewer rational expectations equilibria or none at all.

As we see below, this is empirically relevant, as in some countries our estimates imply very large seignorage shocks in some regimes. In all cases we find that a stable SCE exists, even though there may not be a conditional SCE in the high shock regimes. This suggests that beliefs may tend to diverge in the regimes with high shocks, with agents expecting ever-growing inflation (up to the truncation point). But the regimes will usually not last long enough for this to actually happen, and the lower shock regimes will tend to bring beliefs back down. In one country, Peru, we find that the shocks are also large enough so that there is no REE. For the countries where a REE does exist, we focus on finding the stationary equilibrium with the lowest inflation rates and so is closest to the stable SCE, rather than exhaustively searching for all stationary equilibria (let alone nonstationary equilibria, sunspots, and so forth).

IV. LIKELIHOOD

We use the likelihood to measure the fit of our nonlinear model for each country. The likelihood is very complicated and its derivation requires several steps. We derive first a conditional likelihood and then the overall likelihood by integrating out the states.

Assume that the probability distribution of $\eta_{\pi t}(k)$ is truncated log-normal and the distribution of $\eta_{dt}(k)$ is log-normal for $k = 1, \ldots, h$. Specifically, the probability density functions are[6]

$$p_\pi(\eta_{\pi t}(k)) = \begin{cases} \dfrac{\exp\left\{-\frac{[\log(\pi_1^*(k)+\eta_{\pi t}(k)) - \log \pi_1^*(k)]^2}{2\sigma_\pi^2}\right\}}{\sqrt{2\pi}\sigma_\pi(\pi_1^*(k)+\eta_{\pi t}(k))\Phi\left((-\log(\delta)-\log(\pi_1^*(k))/\sigma_\pi\right)} & \\ \qquad\qquad \text{if } -\pi_1^*(k) < \eta_{\pi t}(k) < 1/\delta - \pi_1^*(k) & , \\ 0 \qquad\qquad \text{otherwise} & \end{cases} \tag{16}$$

$$p_d(\eta_{dt}(k)) = \begin{cases} \dfrac{\exp\left\{-\frac{[\log(\bar{d}(k)+\eta_{dt}(k)) - \log \bar{d}(k)]^2}{2\sigma_d^2}\right\}}{\sqrt{2\pi}\sigma_d(\bar{d}(k)+\eta_{dt}(k))} & \text{if } \eta_{dt}(k) > -\bar{d}(k) \\ 0 & \text{if } \eta_{dt}(k) \le -\bar{d}(k) \end{cases}, \tag{17}$$

where $\Phi(x)$ is the standard-normal cdf of x. Following the convention, we let $\log(0) = -\infty$ and $\Phi(-\infty) = 0$. Equation (17) implies that the geometric mean of $d_t(s_t)$ is $\bar{d}(s_t)$. Denote

$$\begin{aligned} S_t &= \{s_0, s_1, \ldots, s_t\}, \\ \Pi_t &= \{\pi_1, ..., \pi_t\}, \\ q &= \{q_{ij}\} \; \forall i, j = 1, \ldots, h, \\ \xi_d(s_t) &= 1/\sigma_d(s_t), \\ \xi_\pi &= 1/\sigma_\pi, \end{aligned}$$

and let ϕ be a collection of all structural parameters. As above, let $\mathscr{I}(\,)$ is an indicator function that returns 1 if the statement in parentheses is true and 0 otherwise. We use the superscript on $\tilde{\eta}_{dt}(s_t)$ to indicate that $\tilde{\eta}_{dt}(s_t)$ is a *random* variable that does not take any particular value, whereas $\eta_{dt}(s_t)$ is the realized value associated with π_t. With all these notations, we have the following proposition.

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[6] We have used a number of other distributions, including the truncated normal distribution used by Marcet and Nicolini (2003). None of these alternatives improved the fit of our model.

Proposition 4. Given the pdfs (16) and (17), the conditional likelihood is

$$
\begin{aligned}
p(\pi_t|\Pi_{t-1},S_T,\phi) &= p(\pi_t|\Pi_{t-1},s_t,\phi) \\
&= C_{1t}\,\frac{|\xi_\pi|\exp\left\{-\frac{\xi_\pi^2}{2}\left(\log\pi_t-\log\pi_1^*(s_t)\right)^2\right\}}{\sqrt{2\pi}\,\Phi\left(|\xi_\pi|(-\log(\delta)-\log(\pi_1^*(s_t)))\right)\pi_t} \\
&\quad + C_{2t}\left[\frac{\theta|\xi_d(s_t)|(1-\lambda\beta_{t-1})}{\sqrt{2\pi}\left[(1-\lambda\beta_t)\pi_t-\theta(1-\lambda\beta_{t-1})\right]\pi_t}\right. \\
&\quad \left.\exp\left\{-\frac{\xi_d^2(s_t)}{2}\left[\log[(1-\lambda\beta_t)\pi_t-\theta(1-\lambda\beta_{t-1})]-\log\pi_t-\log d(s_t)\right]^2\right\}\right],
\end{aligned}
\tag{18}
$$

where

$$
\begin{aligned}
C_{1t} &= \mathscr{I}(\beta_{t-1}\geq 1/\lambda)+\mathscr{I}(\beta_{t-1}<1/\lambda) \\
&\qquad \left(1-\Phi\left[|\xi_d(s_t)|\left(\log\left(\max[(1-\lambda\beta_t)-\delta\theta(1-\lambda\beta_{t-1}),0]\right)-\log d(s_t)\right)\right]\right), \\
C_{2t} &= \mathscr{I}(\beta_{t-1}<1/\lambda)\,\mathscr{I}\left(\frac{\theta\,(1-\lambda\,\beta_{t-1})}{\max\left(1-\lambda\,\beta_t,\,\delta\theta(1-\lambda\,\beta_{t-1})\right)}<\pi_t<\frac{1}{\delta}\right).
\end{aligned}
$$

Proof. We need to prove that

$$\int_0^{1/\delta} p(\pi_t|\Pi_{t-1},s_t,\phi)\,d\pi_t = 1.$$

With some algebraic work, one can show from (16) and (17) that Equation (18) is equivalent to the following expression

$$
\begin{aligned}
&\mathscr{I}(\beta_{t-1}\geq 1/\lambda)\,p_\pi(\pi_t-\pi_1^*(s_t))+\mathscr{I}(\beta_{t-1}<1/\lambda) \\
&\left\{\mathscr{I}\left(\frac{\theta\,(1-\lambda\beta_{t-1})}{\max\left(1-\lambda\beta_t,\,\delta(1-\lambda\beta_{t-1})\right)}<\pi_t<\frac{1}{\delta}\right)p_d(\eta_{d\,t}(s_t))\,\frac{d\,\eta_{d\,t}(s_t)}{d\,\pi_t}\right. \\
&\qquad \left.+\Pr\left[\tilde{\eta}_{d\,t}(s_t)\geq 1-\lambda\beta_t-\theta(1-\lambda\beta_{t-1})-\bar{d}(s_t)\right]p_\pi(\pi_t-\pi_1^*(s_t))\right\},
\end{aligned}
$$

where Pr() is the probability that the event in the parentheses occurs.

Consider the case where $\beta_{t-1}<1/\lambda$ (the other case is trivial). Denote

$$
\begin{aligned}
L_t &= \frac{\theta(1-\lambda\beta_{t-1})}{1-\lambda\beta_t}, \\
\varpi_t &= 1-\lambda\beta_{t-1}-\delta\theta(1-\lambda\beta_{t-1})-\bar{d}(s_t).
\end{aligned}
$$

It follows that

$$
\begin{aligned}
&\int_0^{\infty} p(\pi_t|\Pi_{t-1}, s_t, \phi) d\pi_t \\
&\quad = \int_{L_t}^{1/\delta} p_d(\eta_{dt}(s_t)) \frac{d\,\eta_{dt}(s_t)}{d\,\pi_t} d\,\pi_t \\
&\qquad + \Pr\left[\tilde{\eta}_{dt}(s_t) > \varpi_t\right] \int_0^{1/\delta} p_\pi(\pi_t - \pi_1^*(s_t)) \\
&\quad = \int_{-\bar{d}(s_t)}^{\varpi_t} p_d(\eta_{dt}(s_t)) d\,\eta_{dt}(s_t) + \Pr\left[\tilde{\eta}_{dt}(s_t) \geq \varpi_t\right] \\
&\quad = \Pr\left[\tilde{\eta}_{dt}(s_t) < \varpi_t\right] + \Pr\left[\tilde{\eta}_{dt}(s_t) \geq \varpi_t\right] \\
&\quad = 1.
\end{aligned}
$$

□

The overall likelihood by integrating out S_T is

$$
\begin{aligned}
p(\Pi_T|\phi) &= \prod_{t=1}^{T} p(\pi_t|\Pi_{t-1}, \phi) \\
&= \prod_{t=1}^{T} \left\{ \sum_{s_t=1}^{h} \left[p(\pi_t|\Pi_{t-1}, s_t, \phi) \Pr(s_t|\Pi_{t-1}, \phi) \right] \right\},
\end{aligned} \tag{19}
$$

where

$$
\Pr(s_t|\Pi_{t-1}, \phi) = \sum_{s_{t-1}=1}^{h} \left[\Pr(s_t|s_{t-1}, q) \Pr(s_{t-1}|\Pi_{t-1}, \phi) \right]. \tag{20}
$$

The probability $\Pr(s_{t-1}|\Pi_{t-1}, \phi)$ can be updated recursively. We begin by setting

$$
\Pr(s_0|\Pi_0, \phi) = 1/h.
$$

For $t = 1, \ldots, T$, the updating procedure involves the following computation:

$$
\Pr(s_t|\Pi_t, \phi) = \frac{p(\pi_t|\Pi_{t-1}, s_t, \phi) \Pr(s_t|\Pi_{t-1}, \phi)}{\sum_{s_t=1}^{h} \left[p(\pi_t|\Pi_{t-1}, s_t, \phi) \Pr(s_t|\Pi_{t-1}, \phi) \right]}. \tag{21}
$$

V. ESTIMATION

V.1. **Estimation Procedure.** In estimation we use the monthly CPI inflation for each country published in the International Financial Statistics. These data sets are relatively reliable and have samples long enough to cover the episodes of both hyperinflation and low inflation. The sample period is 1957:02–2005:04 for Argentina, Bolivia, Chile, and Peru and 1980:01–2005:04 for Brazil.

There are no reliable data on GDP, money, or the government deficit in many hyperinflation countries even on an annual basis because of "poorly developed statistical systems"

(Bruno and Fischer, 1990). The ingenious framework of Marcet and Nicolini (2003), however, enables us to estimate the structural parameters through the inflation likelihood derived in Section IV. On the other hand, we ask too much of the model to pin down all the parameters. Therefore we fix the values of the following three parameters as $\beta_0 = \pi_{-1}$, $\theta = 0.99$, and $\delta = 0.01$. The value of θ is consistent with economic growth and some cash taxes.[7] The value of δ implies that monthly inflation rates are bounded by $10,000\%$.[8]

The likelihood of inflation is quite complicated and have local peaks; its shape differs significantly from one country to another. As a result obtaining the maximum likelihood estimates (MLEs) proves to be an unusually challenging task. The optimization method we use combines the block-wise BFGS algorithm developed by Sims, Waggoner, and Zha (2005) and various constrained optimization routines contained in the commercial IMSL package. The block-wise BFGS algorithm, following the idea of Gibbs sampling and EM algorithm, breaks the set of model parameters into a few subsets and uses Sims's csminwel program to maximize the likelihood of one set of the model's parameters conditional on the other sets.[9] This maximization is iterated at each subset until it converges. Then the optimization iterates between the block-wise BFGS algorithm and the IMSL routines until it converges. The convergence criterion is the square root of machine epsilon. This optimization process applied to only one starting point. We begin with a grid of 300 starting points; after convergence, we perturbs each maximum point in both small and large steps to generate additional 200 new starting points and restart the optimization process again; the MLEs are obtained at the highest likelihood value.[10] The other converged points typically have much lower likelihood values by at least a magnitude of hundreds in log value.

V.2. **Estimation Results.** Since our theoretical model is highly restricted, one would not expect its fit to come close to be as good as an unrestricted AR model, needless to say about a time-varying AR model. Consequently, only certain moments or correlations are reported in the previous work. In this paper we take the fit of our model seriously and report it against the unrestricted statistical models. We compare not only various versions within our model but also our model with different types of AR models.

[7]One could impose a prior distribution of θ with values ranging from 0.96 to 1.0. This is one of few parameters we have a strong prior on. For the other structural parameters, however, it is difficult or impossible to have a prior distribution on the other structural parameters because the likelihood shape differs considerably across countries. If we center a tight prior around the location as odds with the likelihood peak, the model would be unduly penalized. It would be more informative to study the likelihood itself and let the data determine what the model estimates are for each country. One could interpret our likelihood approach as having a diffuse prior on the other structural parameters.

[8]Marcet and Nicolini (2003) set the bound at $5,000\%$.

[9]The csminwel program can be found on http://sims.princeton.edu/yftp/optimize/.

[10]For each country, the whole optimization process is completed in about 5 days on a cluster of 14 dual-processors, using the parallel and grid computing package called STAMPEDE provided to us by the Computing College of Georgia Institute of Technology.

For each country we have tried a variety of versions of our model, including the models with constant parameters, with 2 or 3 states for both $\bar{d}(s_t)$ and $\eta_{dt}(s_t)$, with 2 or 3 states for $\bar{d}(s_t)$ only, with 2 or 3 states for $\eta_{dt}(s_t)$ only, and with 2 states for $\bar{d}(s_t)$ and another independent set of 2 states for $\eta_{dt}(s_t)$ (we call it the 2×2 version). By the Schwarz criterion (SC) or Bayesian information criterion, the 2×2 version of the model is the best and some versions such as the constant-parameter case fits much worse.[11] We have also experimented with different distributions of $\eta_{dt}(s_t)$ and $\eta_{\pi t}(s_t)$ than (16) and (17) including the truncated normal distribution used by (Marcet and Nicolini, 2003), and the fit deteriorates in general. We only report the results for the 2×2 version of the model.

Table 1 reports the model fit for each country, compared with two unrestricted regime-switching AR(2) models as in Sims and Zha (in press).[12] Our model is used as a baseline for comparison. The notation "df" stands for degrees of freedom in relation to our baseline model. The 2-state AR(2) model allows both coefficients and shock variances to change regime at the same time. This model has 1 degree of freedom less than our model. The 2×2 AR(2) model allows the 2 states in coefficients to be independent of the 2 states in shock variances. It has 2 degrees of freedom more than our model. We have also estimated other types of AR models such as those with constant parameters and with regime changes only in coefficients or in shock variances. We do not report the results for those cases because the fit is substantially worse.

As shown in Table 1, our hyperinflation model is not as good as the 2×2 AR(2) but is quite competitive with the 2-state AR(2) model. This kind of fit has not been achieved in the existing literature on hyperinflation.

Tables 2-6 report the MLEs of structural parameters for Argentina, Bolivia, Brazil, Chile, and Peru. All the standard errors reported in the tables are computed from the inverse of the second derivatives matrix of the log likelihood formed as

$$\left[\sum_{t=1}^{T} \hat{g}_t \hat{g}_t'\right]^{-1}$$

where

$$\hat{g}_t = \frac{\partial \log p(\pi_t|\Pi_{t-1},\hat{\phi})}{\partial \hat{\phi}},$$

and $\hat{\phi}$ denotes the MLE of ϕ. It is clear from Tables 2–6 that most key parameters are sharply estimated.[13]

[11]See Sims (2001) for detailed discussions of how to use the SC for model comparison.

[12]Regime-switching AR models are simply a special case of regime-switching VAR models studied by Sims and Zha (in press).

[13]To obtain the standard errors, one could also use the numerical inverse Hessian of the log likelihood accumulated by the optimization routine as done in Sims (2001). We have verified that the results from

Each of the five countries under study has four regimes: high seigniorage with large variance in seigniorage shocks, high seigniorage with small shock variance, low seigniorage with large shock variance, and low seigniorage with small shock variance. The respective ergodic probabilities are 0.03, 0.39, 0.04, and 0.54 for Argentina, 0.01, 0.07, 0.13, and 0.79 for Bolivia, 0.0, 0.0, 0.36, and 0.64 for Brazil, 0.06, 0.54, 0.04, and 0.36 for Chile, 0.02, 0.32, 0.04, and 0.62 for Peru. For Chile, the amount of seigniorage in the high-seigniorage regime is already quite low, thus what really matters is the size of a shock variance. The ergodic probability for the regime associated with small shock variance is 0.90. This result confirms other evidence that Chile's fiscal situation is solid and better than all the other countries.

The regime with low seigniorage and small seigniorage shocks is the best in the sense that it guarantees low inflation with little probability of returning to hyperinflation. Our estimates indicate that this regime receives the most probability among all regimes. While the sample is not long enough for us to determine that this regime will be permanent, it is possible that it will be maintained in the future. Conditional on the best regime, we show below that mean dynamics of inflation for each of the five countries converge to a very low rate of inflation. This result is important because it implies that disciplinary fiscal policy is essential for maintaining a low inflation path.

V.3. **Equilibria Implied by Estimates.** We now examine what our estimates imply for the self-confirming and rational expectations equilibria discussed above. Table 7 lists the stable SCE and low-inflation REE in each country, as well as the conditional SCEs (which recall presume that the regime is always experienced). For comparison to the stable SCE, we also list the unconditional mean for the REE. In all cases we find that the mean dynamics converge when we allow regimes to switch according to our estimated transition probabilities (as a stable SCE exists), and the converged rate is often close to the mean REE rate if the REE rate exists.

The self-confirming equilibria are also illustrated by Figures 1-2. Each figure plots the mean dynamics $\bar{g}$, along with the conditional mean dynamics discussed above, for different values of the belief β. Equilibria occur where these lines cross zero, and stable equilibria are those with negative slopes. Although each of the countries has its own idiosyncratic character, there are some similarities which allow us to break the analysis into three groups.

The first group consists of Argentina and Bolivia, shown in Figure 1. In these countries the high shock regimes, which are columns 4 and 5 in Table 7 and whose conditional mean dynamics are shown with the two jagged lines at the top of the left side of the plots, are volatile but not destabilizing. Note that the proportionately larger shifts with the variances reflect the difference between our stochastic model and the deterministic version (parallel

---

this approximate approach are similar to what is reported in the tables, but they tend to vary considerably, depending on how many function iterations the optimization routine takes to converge.

to Marcet and Sargent (1989)) sketched above. This can also be seen in the table, where the inflation rates in both the REE and conditional SCEs tend to vary more with the shock regime than the mean seignorage regime. For Argentina, we see from the figure that the mean dynamics for the low shock variance regimes are very similar to the deterministic case, and in each case the lower stationary point is stable. For Bolivia, this is the case for the best regime (low mean and low shocks), but the regime associated with low average seignorage and high shocks has a very high conditional SCE which arises only due to truncation. In the high shock variance regimes, there is a single conditional SCE which is stable in both countries, although associated with higher inflation. The overall SCE balances among these regimes according to the invariant distribution of the Markov chain. Since this distribution doesn't put too much weight on the high variance states, the stable SCE overall is consistent with relatively low inflation. These results suggest that average inflation should be relatively low in these countries in the long run, although there may be significant fluctuations in the inflation rate due mainly to the seignorage shocks.

Brazil is a special case, with dynamics which differ rather substantially from the other countries. As seen in Table 4 average seignorage chain has an absorbing state. Thus after a transient period, the economy in Brazil will t have low average seignorage and switch between high and low seignorage shocks. This is illustrated in Figure 2 where we see that the regime-switching mean dynamics lie between (l,l) and (l,h) states, and in Table 7 where we see that the SCE and the mean rate in the REE are averages of the rates in those two states. However even high shocks regime is not nearly as volatile as in many of the other countries, resulting in a quite placid inflation experience in the long run. These results suggest that the outlook for Brazil is quite promising, as a transient period of relatively higher inflation would eventually give way to a sustained period of very low inflation.

In Chile and Peru, the last group, matters are rather different. These countries are somewhat similar to the first group of Argentina and Bolivia, but the high shock regimes are even more volatile and destabilizing. In Chile, this implies that there are no conditional SCEs in the high shock regimes, and that the overall SCE is associated with somewhat higher inflation. The conditional SCEs in the low shock regimes have relatively low inflation rates around 1-1.02, but there is enough weight on the unstable high shock regimes to pull up the SCE rate to 1.07. Note that in Chile however the overall range of variation is much lower than for many of the other countries, with even the high conditional SCEs occurring at rates of roughly 1.11-1.13, versus the high SCE rates of 1.4 or more for the other countries. The results for Peru are similar, in that again there is no conditional SCE in the high shock states. These states have a smaller weight in the invariant distribution however, so that the SCE inflation rate is relatively low. In both of these countries the destabilizing effects of the high shock states also affected the rational expectations equilibria. This is most extreme in Peru, where we were unable to find an REE. For Chile, we did find an REE which has relatively low mean inflation, but the forecasts in the high shock states are very close to the truncation point. Overall, the results for these countries are similar to what we described above. The high shock regimes are destabilizing and tend to lead to ever-increasing inflation expectations, but there is enough switching of regimes so that the lower shock regimes bring beliefs back down.

VI. EMPIRICAL RESULTS

Figures 4-18 are graphical representations of our empirical results. These results are the consequence of intricate interactions among beliefs, escapes, levels of average seigniorage, and sizes of shock variances. A high level of average seigniorage is a necessary condition for the cause of repeated hyperinflation, but it is not sufficient. Each country has its distinctive characteristic in its experience with hyperinflation. The figures show that these distinctive characteristics explain why hyperinflation happened repeatedly and why it appeared to end permanently across different countries.

For Argentina and Brazil, there is no single factor that explains the observed dynamics of hyperinflation; they are a result of interactions between a high level of average seigniorage and changing beliefs. The estimated gain in learning is large relative to the other countries. For Bolivia, the estimated gain is small but the level of average seigniorage during the hyperinflation period is much higher than that in the other countries (Table 3). Such an extremely high level of average seigniorage is a dominate source of hyperinflation dynamics. Chile, on the other hand, is a completely different case. Average seigniorage in both regimes are low and in fact lowest among all the five countries studied here (Table 5). Hyperinflation in the 1970s is mainly caused by large seigniorage shocks whose regime is short-lived. As for Peru, the burst of hyperinflation around 1990 is mainly driven by fast changing beliefs with the estimated gain larger than that for Bolivia and Chile.

There are a few new objects that are computed for Figures 4-18. The one-step median prediction is the median of π_t sampled from the joint distribution with the following density function

$$p(\pi_t, s_t | \Pi_{t-1}, \hat{\phi}),$$

where $\hat{\phi}$ is the MLE of ϕ. The probability of regime $s_t = k$ is computed as

$$\Pr(s_t = k | \Pi_T, \hat{\phi}).$$

In general, the probability $\Pr(s_t = k | \Pi_{t-1}, \hat{\phi})$ is close to the value of $\Pr(s_t = k | \Pi_T, \hat{\phi})$. On some occasions, however, these two values differ substantially. In these situations one needs be careful to use $\Pr(s_t = k | \Pi_T, \hat{\phi})$ to explain the probability band of a one-step prediction. The probability of escape at time t is computed as follows. If

$$1 - \lambda \beta_{t-1} \leq 0,$$

the probability is 1. Otherwise, the probability is equal to

$$\Pr\left(1 - \lambda \beta_t - d_t(s_t) \leq \delta \theta (1 - \lambda \beta_{t-1})\right).$$

VI.1. **Argentina.** The top chart of Figure 4 shows the actual path of inflation and one-step median forecasts for Argentina. The forecasts track the dynamic patterns not only qualitatively but also in magnitude, especially in the period of hyperinflation. The bottom chart reports the .90 error bands of one-step forecasts. Again, the bands track the actual path well and give high probability to hyperinflation around 1990.

Figure 5 reports the probability of each regime. The high inflation period from 1975 to the early 1990s is associated with the regime with a high level of average seigniorage. The sizes of shock variances do not play an important role in generating repeated hyperinflation during that period. Rather, beliefs play an important role (Figure 6). As shown in the bottom chart of Figure 6, the patten of recurring hyperinflation during the period from 1975 to the early 1990s mimics that of beliefs in the period. In fact, the beliefs around 1990 is so high that the probability of escape is over 0.6 (the top chart of Figure 6).

Seigniorage shocks play an important role for periodic bursts of high inflation in the 1960s and in 2002.[14] But these shocks are temporary in nature and do not generate mega-inflation as in the period from 1975 to the early 1990s. Since 1991, the regime for high level of average seigniorage has completed vanished, which explains the sustained low inflation path we observe (Figure 5).

VI.2. **Bolivia.** The top chart of Figure 7 displays actual inflation and the one-step forecasts. Again, the median forecasts tracks well both the path of low inflation and the outbursts of hyperinflation. The error bands (the bottom chart) show a lot of uncertainty in the early period of the sample. This uncertainty is associated with the regime with large variances of shocks to seigniorage (the third chart of Figure 8). The error bands also show that the low inflation path since the late 19890s are attained except for a few isolated periods when the probability of the regime with large shock variances becomes one.

The outbursts of hyperinflation in the middle of 1980s are caused by an extremely high level of average seigniorage (Table 3 and the top two charts of Figure 8). When the seigniorage level is so high, even small shocks can generate hyperinflation according to our theory, which is confirmed by the second chart of Figure 8. The beliefs of future inflation are relatively high in the hyperinflation period, but unlike the cases for Argentina and Brazil, do not play a major role in producing hyperinflation (Figure 9. Indeed, the magnitude of rising beliefs is relatively small, due to the small gain in learning and the escape of probability remains low even in the hyperinflation period (see the top chart of Figure 9). Bolivia is a case that comes close to having high seigniorage becoming the dominate source of generating hyperinflation. Since the late 1980s, the regime of high seigniorage receives zero probability and the regime with low seigniorage and low shock variance persists except for a few isolated periods (Figure 8).

[14]The 2002 crisis is associated with the end of the exchange-rate stabilization policy adopted by the central bank of Argentina. As foreign reserves ran out, the central bank began to print money to finance the government's deficits.

VI.3. **Brazil.** The actual path of inflation and the one-step forecasts are reported in the top chart of Figure 10. The hyperinflation episodes around 1990 and 1994 are captured by the model's predictions, although the model predicts much higher rates. The error bands displayed in the bottom chart show that the model gives substantial probability to hyperinflation around 1990 and 1994 but not anywhere else. Indeed, the low inflation periods in the early 1980s and since 1995 are predicted well by the model.

The period before 1995 is the high-seigniorage regime as shown in Figure 11. As discussed before, such a regime is necessary but not sufficient for generating hyperinflation. We observe the relatively low inflation path in the early 1980s despite the high level of average seigniorage; small seigniorage shocks during the period play an important role in keeping inflation relative low. Large seigniorage shocks are associated with the two large run-up of hyperinflation around 1990 and in the late 1994. However large shock variances in the regime of high seigniorage do not generate as high inflation as these two run-ups (see the top two charts of 11). Beliefs play an important role in generating the two large run-ups of hyperinflation as shown in the bottom chart of Figure 12). The beliefs around 1990 and in the late 1994 are so high that the probability of escape spikes up to be 1 in these periods (see the top chart of Figure 12).

After 1995 the beliefs of future inflation comes down rather steeply, thanks to the relative large gain in learning. The regime of high seigniorage is vanished after 1995 as well. Our estimates give no probability that this regime will be returned. But the regimes of large and small shock variances switch frequently in the period after 1995 (the bottom two charts of Figure 11), signalling the possible unstable fiscal conditions in Brazil. In the long run, however, the estimated ergodic probability for the small-shock-variance regime dominates (with the .64 probability).

VI.4. **Chile.** Chile has been often viewed as a child prodigy in Latin America. Our estimated results confirm this view. Chile's fiscal situation is the best among all the five countries and the estimated levels of seigniorage in both regime are much lower than those in the other four countries (Table 5). Even so, Chile has experienced hyperinflation in the middle of 1970s and a short burst of high inflation in the middle of 1980s.

The top chart of Figure 13 depicts the path of actual inflation along with the one-step forecasts. Again, the forecasts track the observed series amazingly well. The error bands displayed in the bottom chart of Figure 13 show frequent occurrences of uncertainty in the pre-hyperinflation period, but the bands tighten considerably in the after-hyperinflation period except a few isolated periods. These tight bands are associated with the regime with small shock variances (see the second and fourth charts of 14), underscoring the importance of fiscal discipline even as the level of average seigniorage is already low.

The 3-month outbursts of hyperinflation from August 1972 to October 1972 are caused by a combination of a high level of seigniorage and large shocks to seigniorage (see the first chart of Figure 14). Large shocks play a crucial role in generating such hyperinflation as well as in causing the temporary burst of high inflation in the middle of 1980s. In the regime with high seigniorage and small shocks (the second chart of Figure 14), inflation fluctuates, which is reflected in the wide error bands displayed in Figure 13. Since the middle of 1990s, however, the regime with low seigniorage and small shock variances dominates and persists to today.

Chile is a case where beliefs play a minor role in the rise and fall of hyperinflation. The movements in beliefs, as shown in the bottom chart of Figure 12), are small relative to those in the other countries. The estimated gain is small as well, and so is the probability of escape throughout the entire sample.

VI.5. **Peru.** Our model tracks the inflation dynamics in Peru remarkably well, as shown in the top chart of Figure 16. The error bands give substantial probability to hyperinflation in the years around 1990 (the bottom chart). There are several isolated months in the pre-hyperinflation period that show considerable uncertainty around the low inflation path. These months are associated with the regime with low average seigniorage and small shocks to seigniorage (see the third chart of Figure 17). The regime with the low level of seigniorage has dominated since June 1994 and the ergodic probability is over 0.94. In particular, the regime with both low seigniorage and small shock variances receives the 0.62 ergodic probability. If such a regime is to be maintained, the low inflation path will continue in the future.

Peru is a clear case study in which a combination of high seigniorage and large shocks to seigniorage may not be sufficient (although necessary) to generate hyperinflation as shown in the first chart of Figure 17). The spike of hyperinflation in August 1990, for example, occurs in the regime with high seigniorage but small shock variances. The rapidly adapted beliefs play a crucial role for this hyperinflation to occur. As shown in Figure 18), the beliefs of future inflation are adjusted so fast that the probability of escape is one during the months around August 1990.

VII. CONCLUSION

Building on Sargent and Wallace (1987) and Marcet and Nicolini (2003), we develop a nonlinear general equilibrium model of hyperinflation. This model is fit to the data in Argentina, Bolivia, Brazil, Chile, and Peru. Unlike the previous literature, the time-series properties of this model are rigorously checked against the data in all these five countries. Our estimated model provides important insights that have not been explored in the existing literature. Our robust results show that a large amount of seigniorage is necessary for hyperinflation to occur repeatedly but the inflation dynamics depends crucially on a combination of many factors such as beliefs and fundamental shocks. On the other hand, low inflation can be achieved by disciplinary fiscal policy and will be sustained if such a policy is to be maintained.

TABLE 1. Log likelihood adjusted by the Schwarz criterion

	Hyperinflation Model	2-state AR(2) (df=-1)	2×2-state AR(2) (df=2)
Argentina	1232.5	1095.2	1490.1
Bolivia	1505.6	1483.7	1539.9
Brazil	750.34	782.97	838.02
Chile	1697.3	1605.4	1714.6
Peru	1651.0	1517.1	1652.3

TABLE 2. Argentina: MLEs for the 2×2-state model

λ : 0.667 (0.0059)	
$[\bar{d}(1)\ \bar{d}(2)]$: [0.0245 (0.0012) 0.0058 (0.0002)]	
$[\xi_d(1)\ \xi_d(2)]$: [0.081 (0.0464) 1.84 (0.0644)]	
ξ_π : 16.4 (1.1862)	
g : 0.10 (0.0021)	
Transition probability matrix for $\bar{d}(s_t)$:	
0.9867 (0.0118)	0.0098 (0.0047)
0.0133	0.9902
Transition probability matrix for $\eta_{dt}(s_t)$:	
0.3853 (0.1551)	0.0509 (0.0170)
0.6147	0.9491

The reported value in parentheses is standard error.

TABLE 3. Bolivia: MLEs for the 2×2-state model

λ : 0.562 (0.0483)	
$[\bar{d}(1)\ \bar{d}(2)]$: [0.0601 (0.0084) 0.0069 (0.0008)]	
$[\xi_d(1)\ \xi_d(2)]$: [0.158 (0.0314) 1.86 (0.0670)]	
ξ_π : 29.64 (2.9322)	
g : 0.027 (0.0083)	
Transition probability matrix for $\bar{d}(s_t)$:	
0.9456 (0.0281)	0.0050 (0.0033)
0.0544	0.9950
Transition probability matrix for $\eta_{dt}(s_t)$:	
0.4392 (0.0877)	0.0909 (0.0199)
0.5608	0.9091

The reported value in parentheses is standard error.

TABLE 4. Brazil: MLEs for the 2×2-state model

λ : 0.738 (0.0558)	
$[\bar{d}(1)\ \bar{d}(2)]$: [0.0169 (0.0083) 0.0041 (0.0029)]	
$[\xi_d(1)\ \xi_d(2)]$: [2.16 (2.4306) 5.70 (1.6658)]	
ξ_π : 3.76 (1.0738)	
g : 0.093 (0.0146)	
Transition probability matrix for $\bar{d}(s_t)$:	
0.9948 (0.1458)	0.0000 (0.0001)
0.0052	1.0000
Transition probability matrix for $\eta_{dt}(s_t)$:	
0.9356 (0.2383)	0.0364 (0.0257)
0.0644	0.9636

The reported value in parentheses is standard error.

TABLE 5. Chile: MLEs for the 2×2-state model

$\lambda: \; 0.876\,(0.0015)$	
$[\bar{d}(1)\; \bar{d}(2)]: \; [0.00032\,(0.000084) \;\; 0.00017\,(0.000046]$	
$[\xi_d(1)\; \xi_d(2)]: \; [0.323, (0.0501) \;\; 3.24\,(0.1063)]$	
$\xi_\pi: \; 14.00\,(4.7233)$	
$g: \; 0.025\,(0.0002)$	
Transition probability matrix for $\bar{d}(s_t)$:	
0.9713 (0.0098)	0.0441 (0.0206)
0.0287	0.9559
Transition probability matrix for $\eta_{dt}(s_t)$:	
0.7612 (0.0621)	0.0278 (0.0086)
0.2388	0.9721

The reported value in parentheses is standard error.

TABLE 6. Peru: MLEs for the 2×2-state model

$\lambda: \; 0.727\,(0.0044)$	
$[\bar{d}(1)\; \bar{d}(2)]: \; [0.0139\,(0.0006) \;\; 0.0043\,(0.0001)]$	
$[\xi_d(1)\; \xi_d(2)]: \; [0.375\,(0.0732) \;\; 2.733\,(0.0963)]$	
$\xi_\pi: \; 16.16\,(2.4356)$	
$g: \; 0.075\,(0.0012)$	
Transition probability matrix for $\bar{d}(s_t)$:	
0.9948 (0.0090)	0.0027 (0.0022)
0.0052	0.9973
Transition probability matrix for $\eta_{dt}(s_t)$:	
0.2447 (0.1356)	0.0464 (0.0133)
0.7553	0.9536

The reported value in parentheses is standard error.

TABLE 7. Equilibria for different countries. The third column gives the true stable SCE or the unconditional mean from the low-inflation REE. The last four columns give the conditional equilibria for the SCE or the forecasts conditional on the regime in the REE. For the regimes (h,l) denotes high average seignorage and low shock volatility and so forth.

Country	SCE/REE	All/Mean	Regime: (d,η_d)			
			(h,h)	(l,h)	(h,l)	(l,l)
Argentina	SCE	1.059	1.182	1.137	1.126	1.011
Argentina	REE	1.034	1.142	1.135	1.032	1.021
Bolivia	SCE	1.058	1.370	1.246	1.735	1.009
Bolivia	REE	1.069	1.251	1.230	1.079	1.039
Brazil	SCE	1.007	1.109	1.008	1.084	1.006
Brazil	REE	1.017	1.076	1.020	1.063	1.016
Chile	SCE	1.071	–	–	1.023	1.005
Chile	REE	1.023	1.142	1.142	1.011	1.007
Peru	SCE	1.042	–	–	1.060	1.008
Peru	REE	–	–	–	–	–

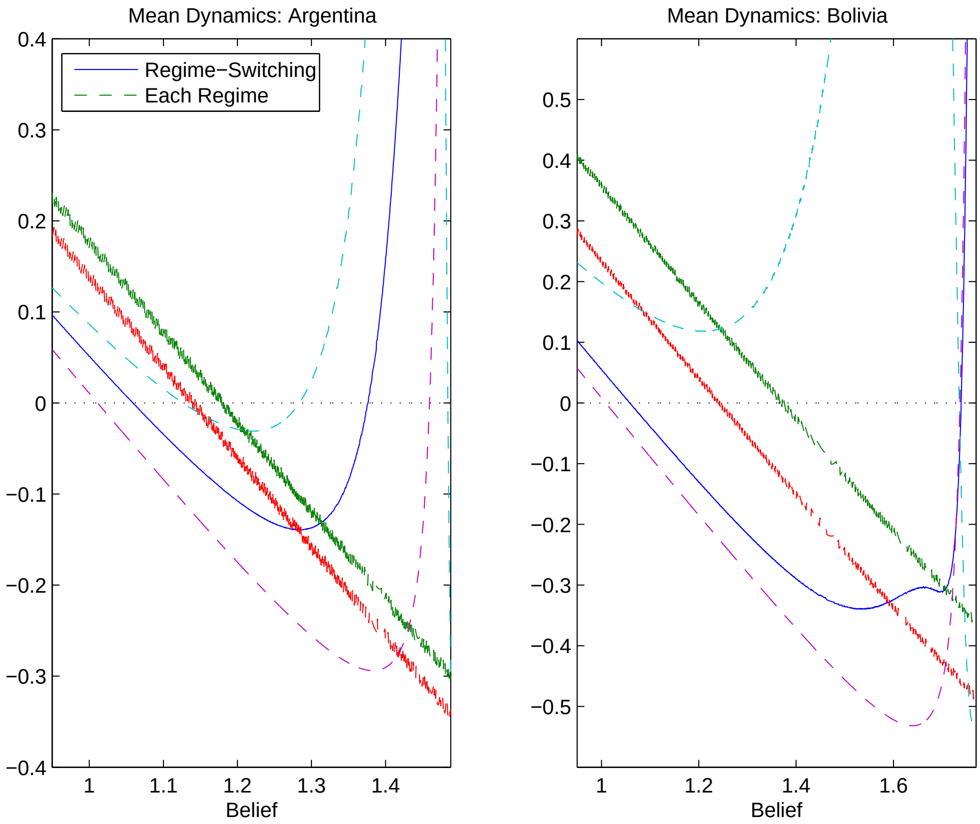


FIGURE 1. Mean dynamics (solid) and conditional mean dynamics (dashed) for Argentina and Bolivia.

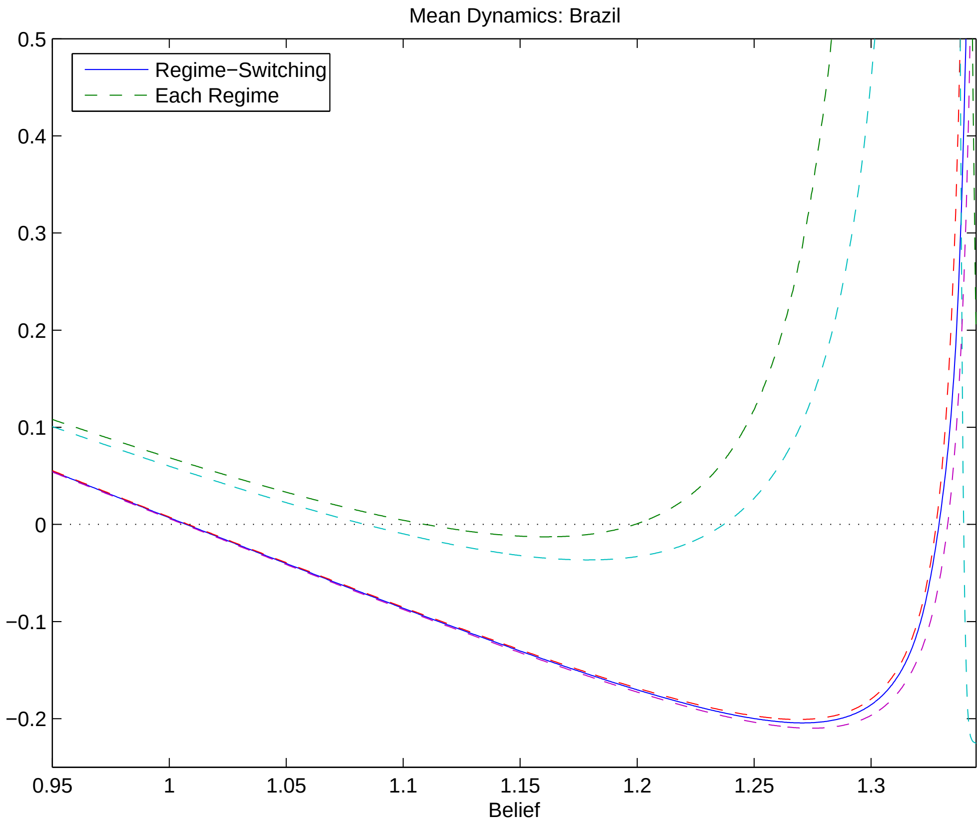


FIGURE 2. Mean dynamics (solid) and conditional mean dynamics (dashed) for Brazil.

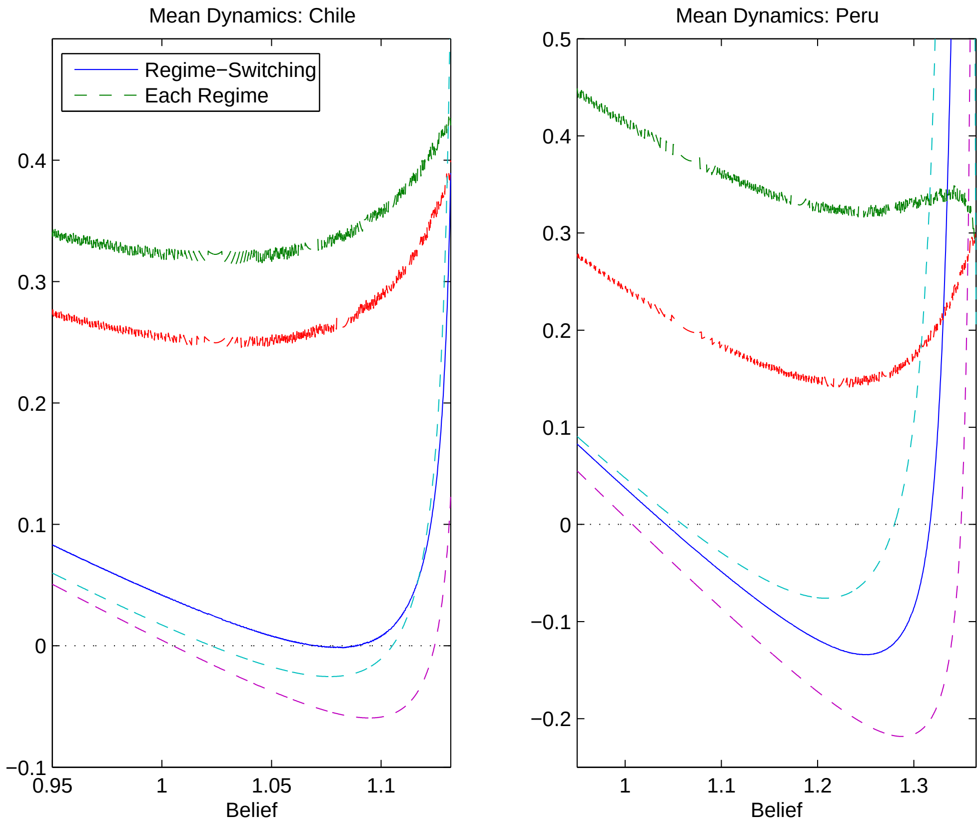
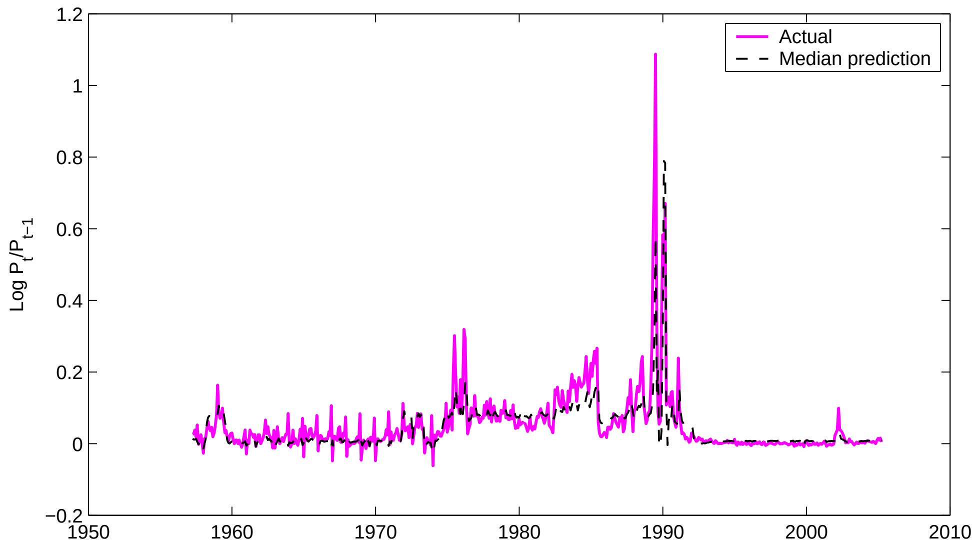


FIGURE 3. Mean dynamics (solid) and conditional mean dynamics (dashed) for Chile and Peru.



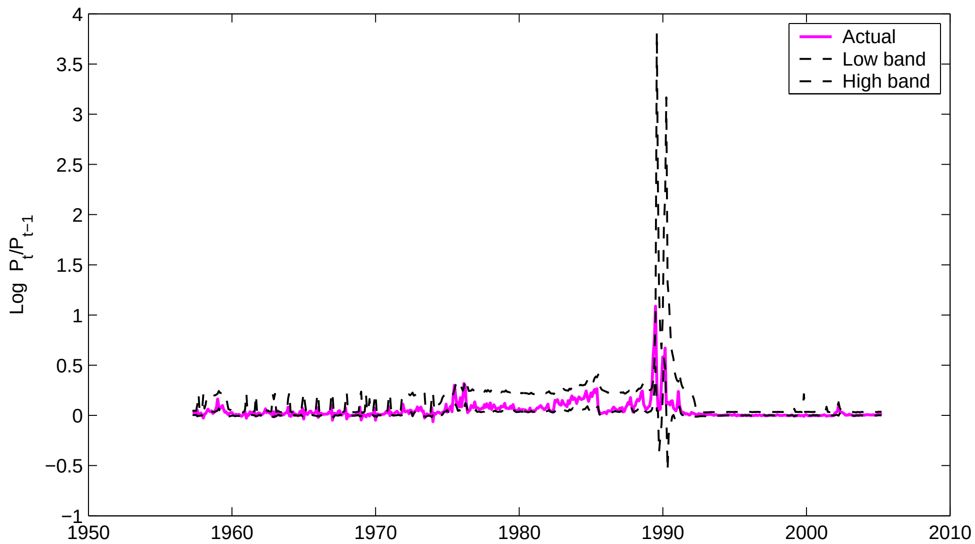


FIGURE 4. Argentinean inflation: actual versus one-step median forecast (top chart) and actual versus .90 probability bands of one-step prediction (bottom chart).

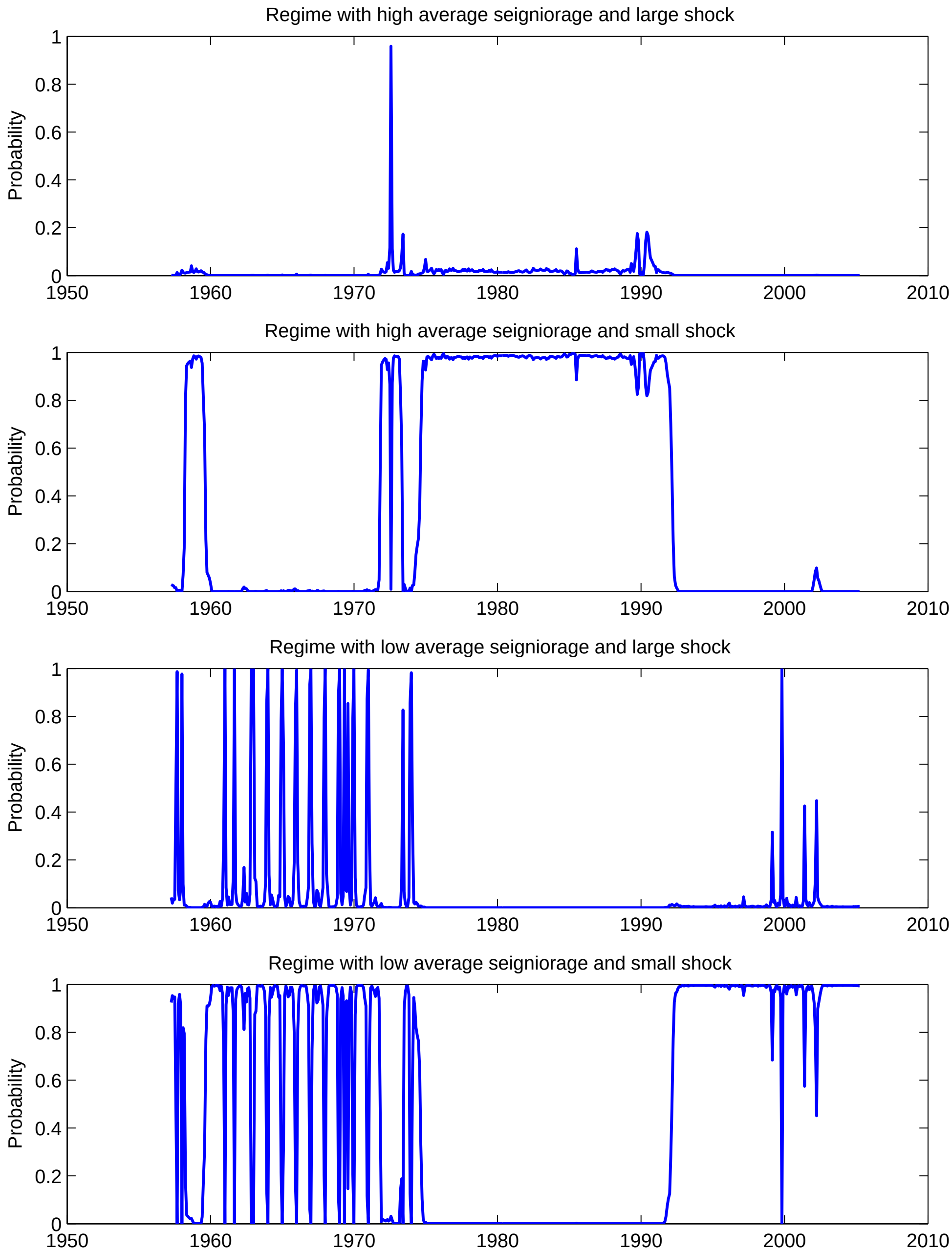
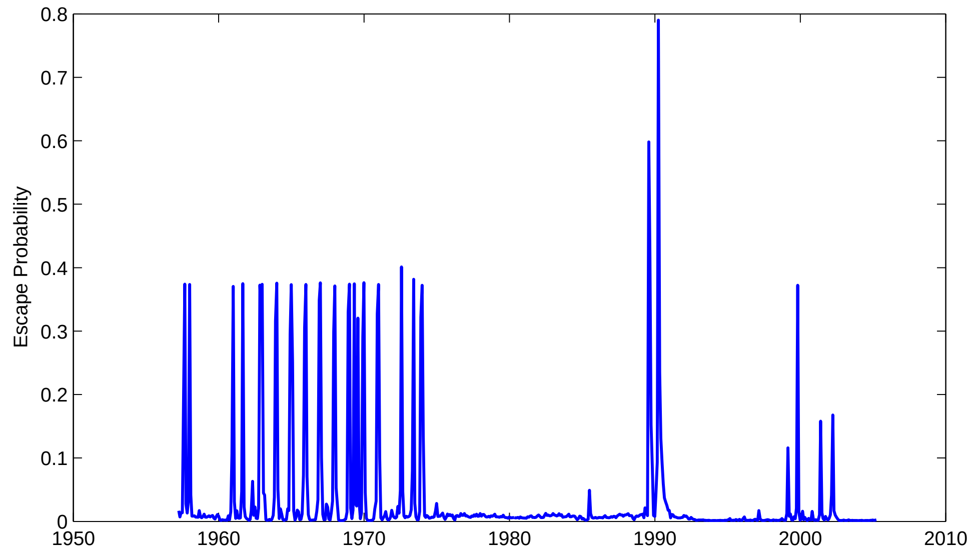


FIGURE 5. Argentina: probabilities of the four regimes conditional on the MLEs and the data.



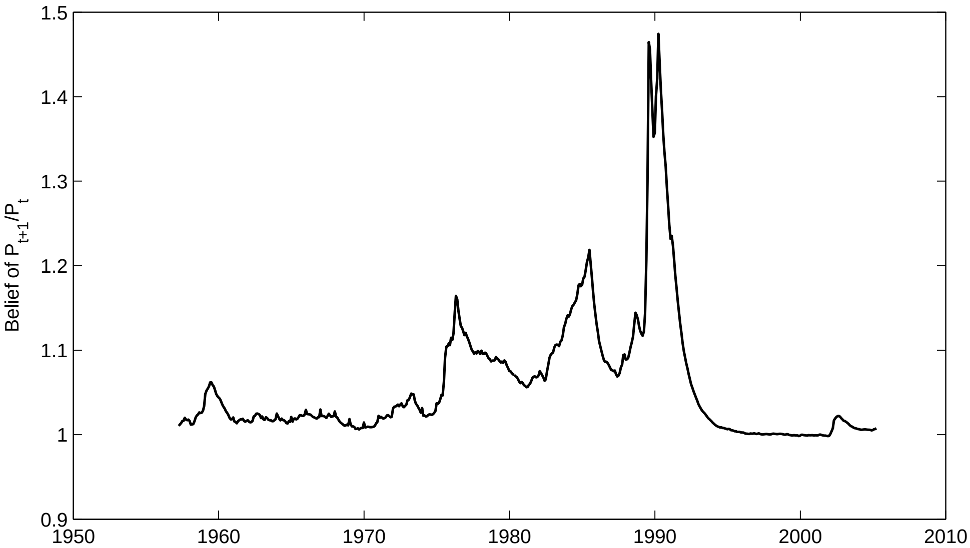
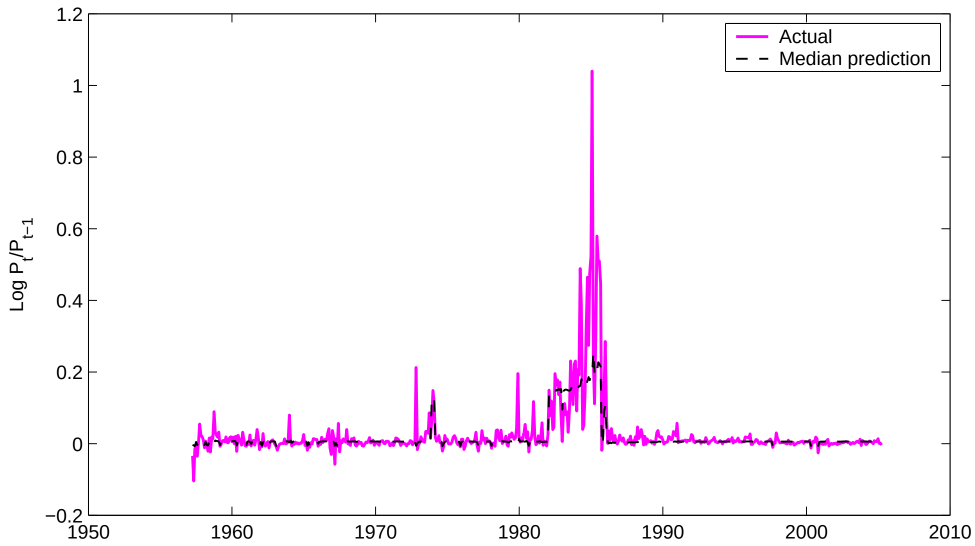


FIGURE 6. Argentina: probability of escape (top chart) and belief of next-period inflation β_t (bottom chart).



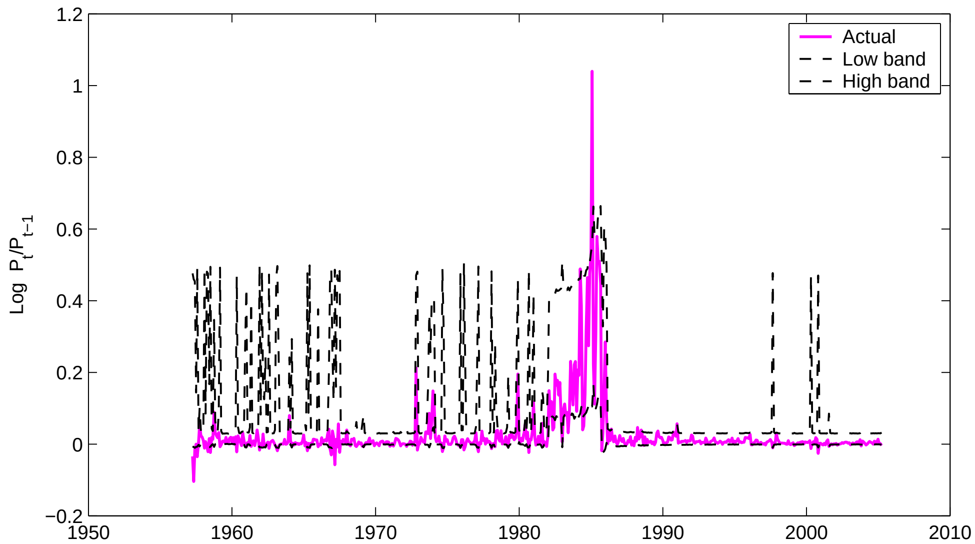


FIGURE 7. Bolivian inflation: actual versus one-step median forecast (top chart) and actual versus .90 probability bands of one-step prediction (bottom chart).

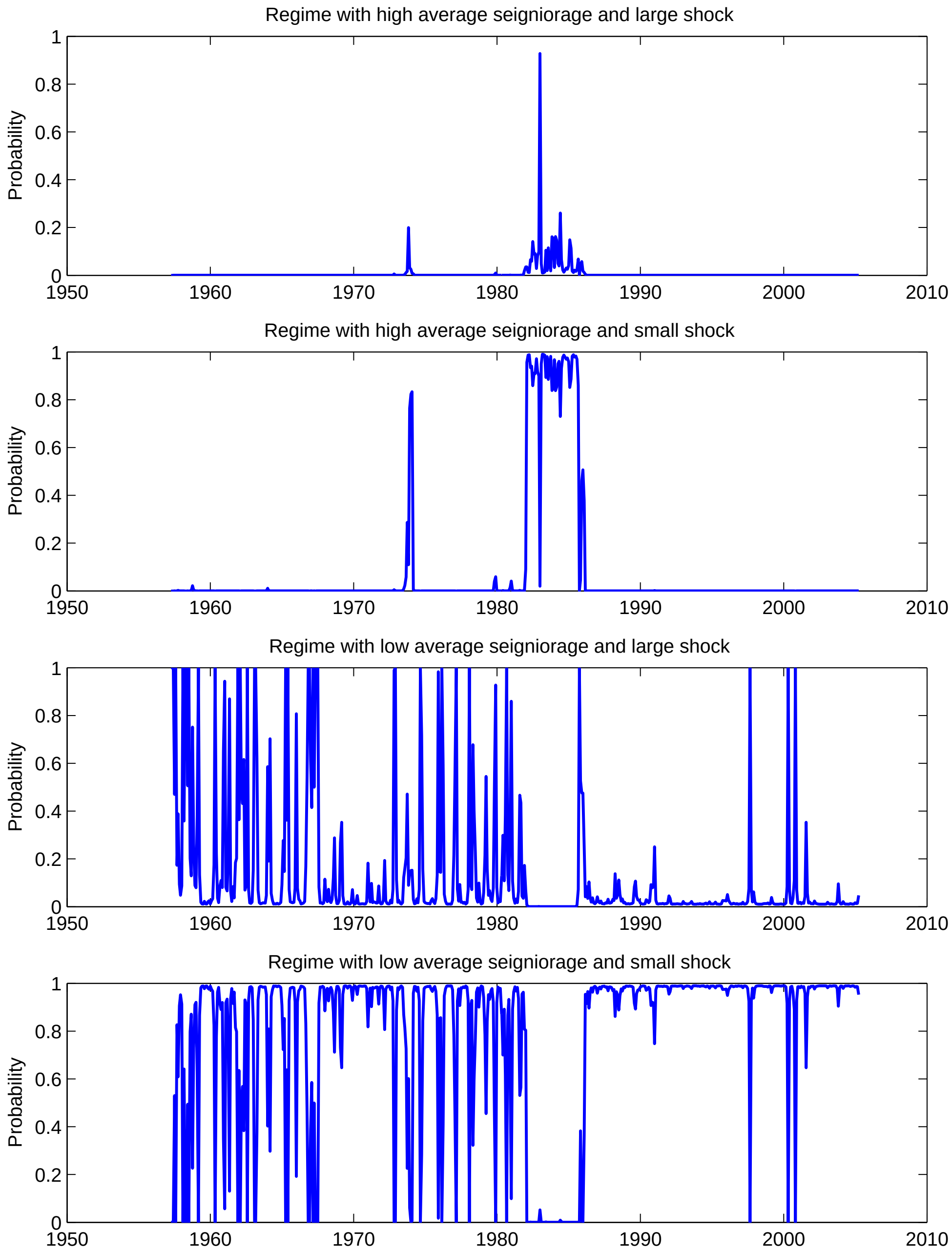
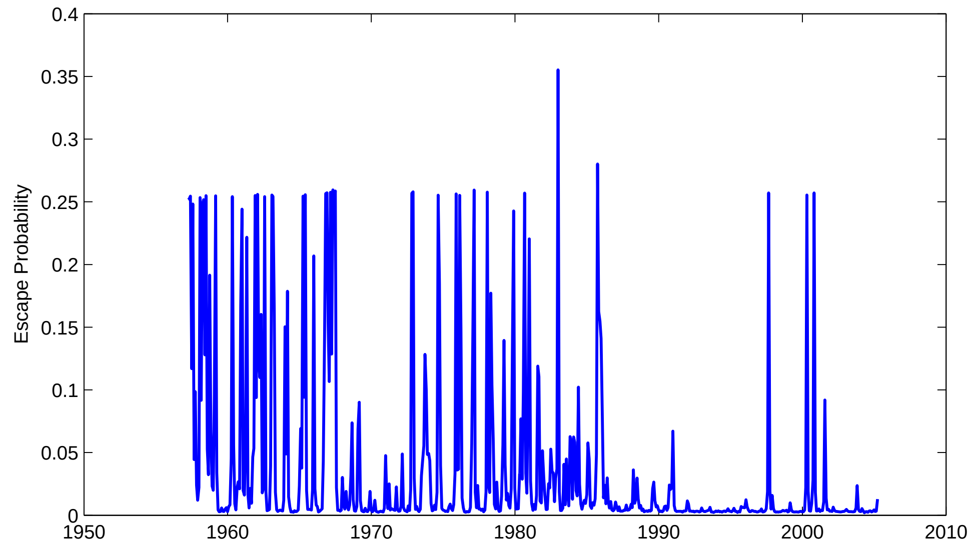


FIGURE 8. Bolivia: probabilities of the four regimes conditional on the MLEs and the data.



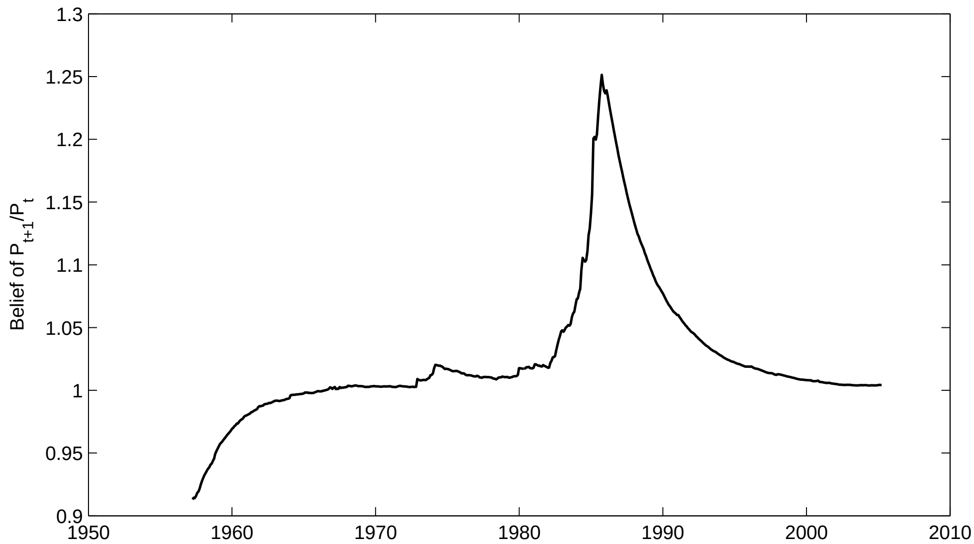
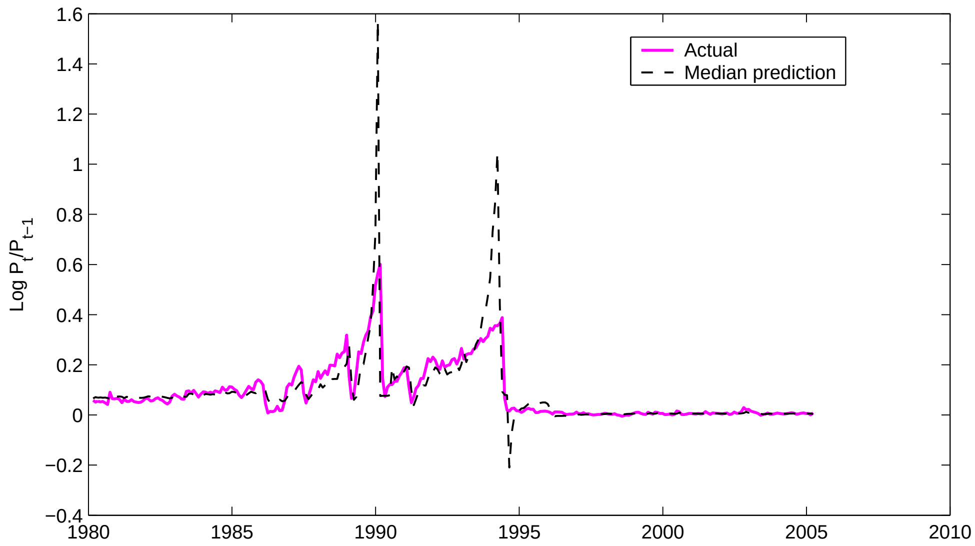


FIGURE 9. Bolivia: probability of escape (top chart) and belief of next-period inflation β_t (bottom chart).



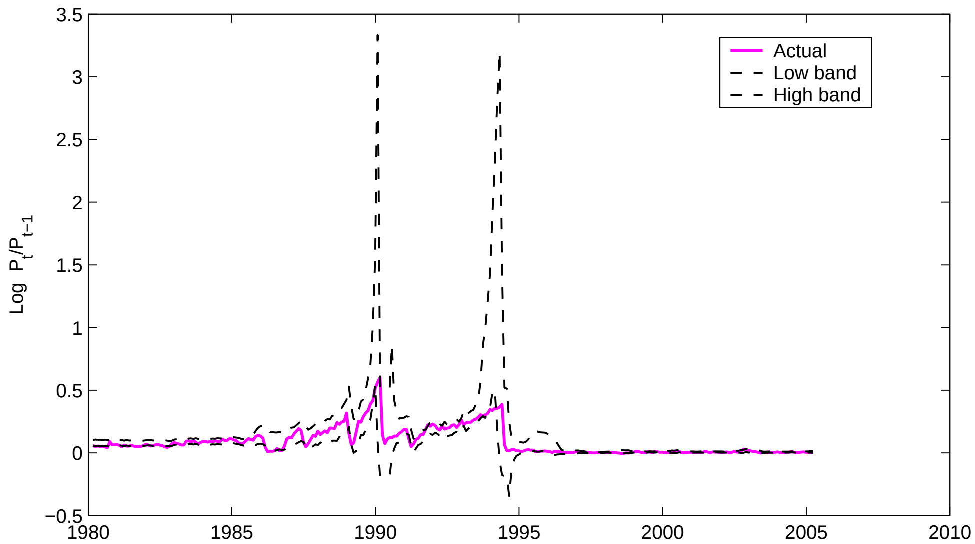


FIGURE 10. Brazilian inflation: actual versus one-step median forecast (top chart) and actual versus .90 probability bands of one-step prediction (bottom chart).

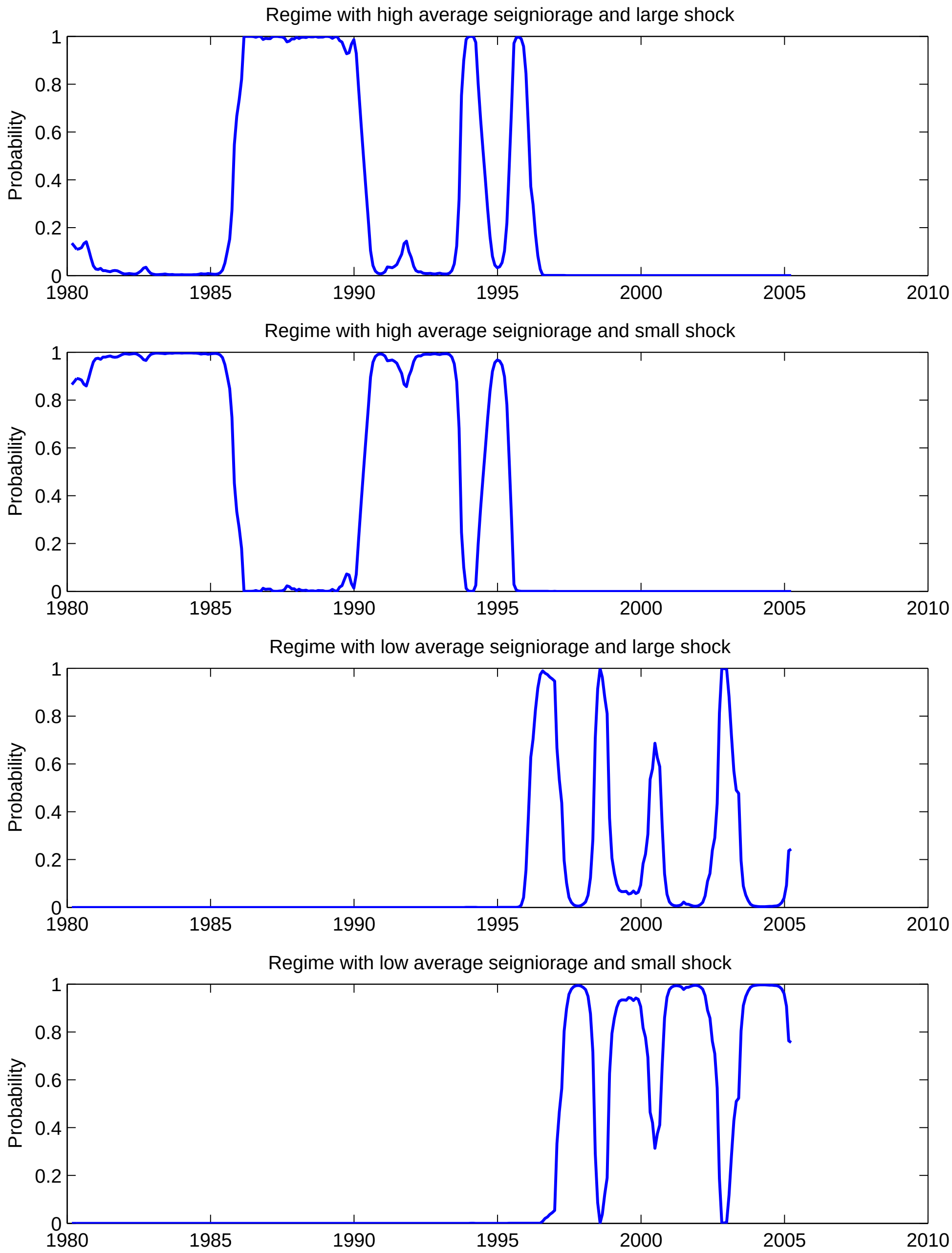


FIGURE 11. Brazil: probabilities of the four regimes conditional on the MLEs and the data.

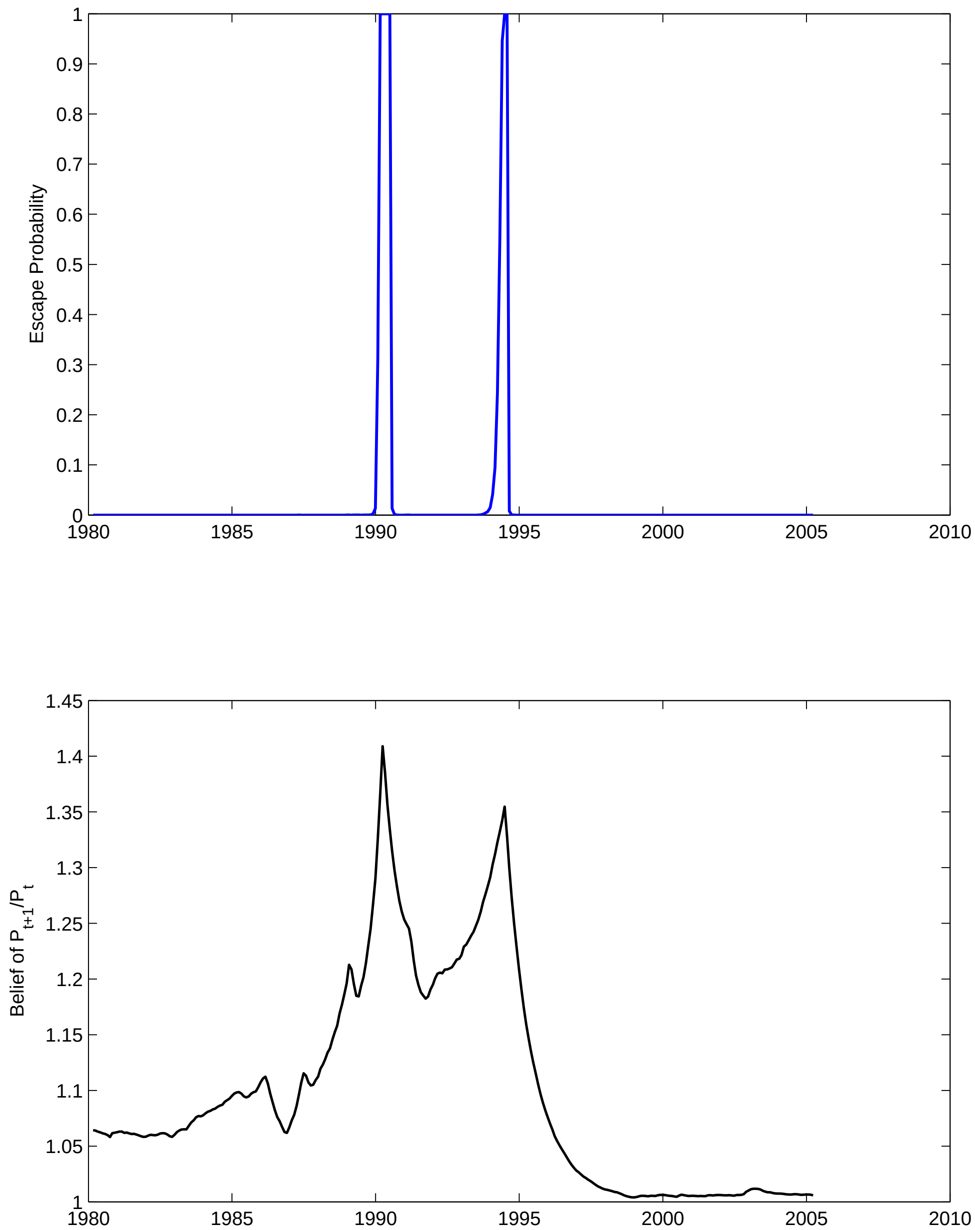


FIGURE 12. Brazil: probability of escape (top chart) and belief of next-period inflation β_t (bottom chart).

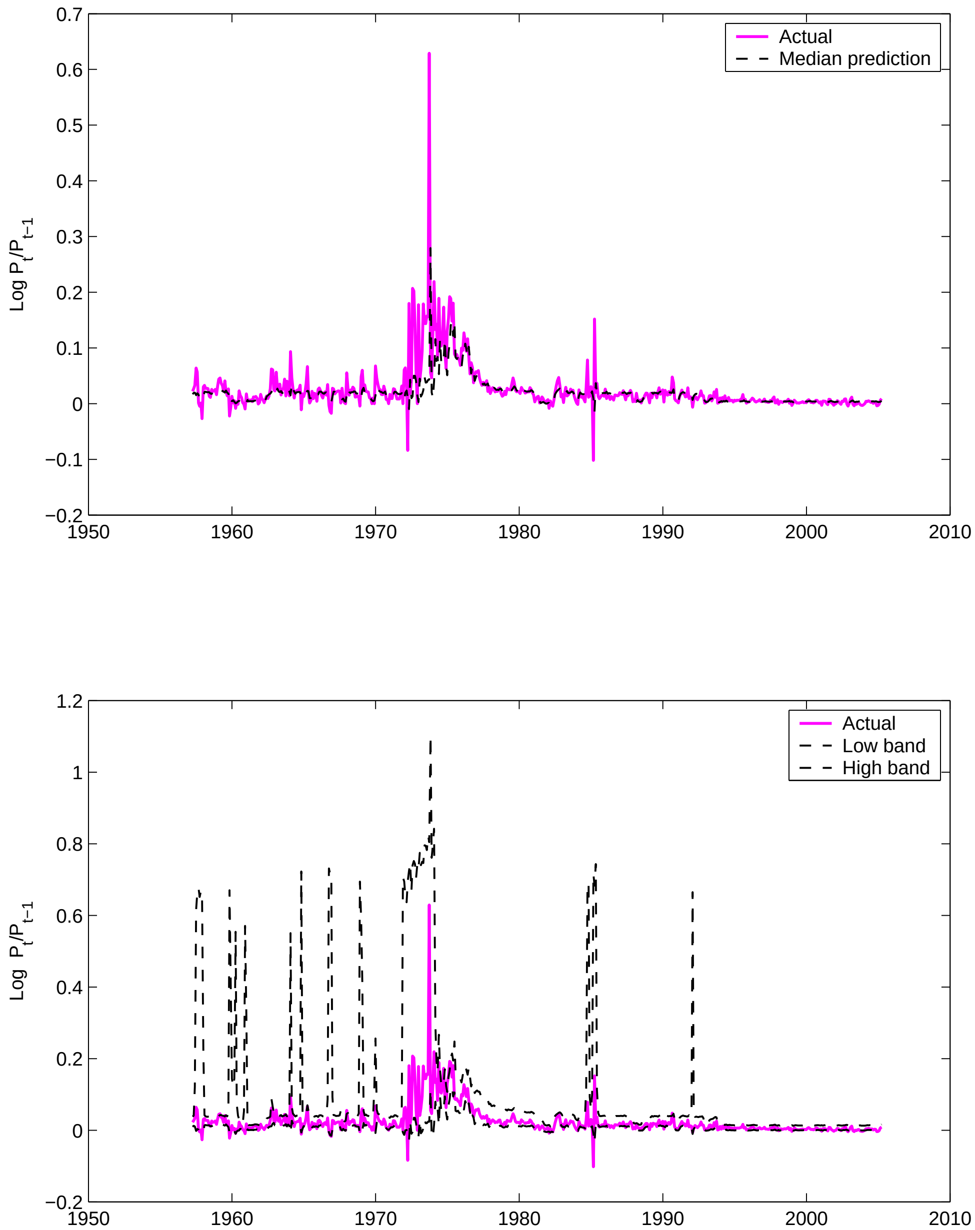


FIGURE 13. Chilean inflation: actual versus one-step median forecast (top chart) and actual versus .90 probability bands of one-step prediction (bottom chart).

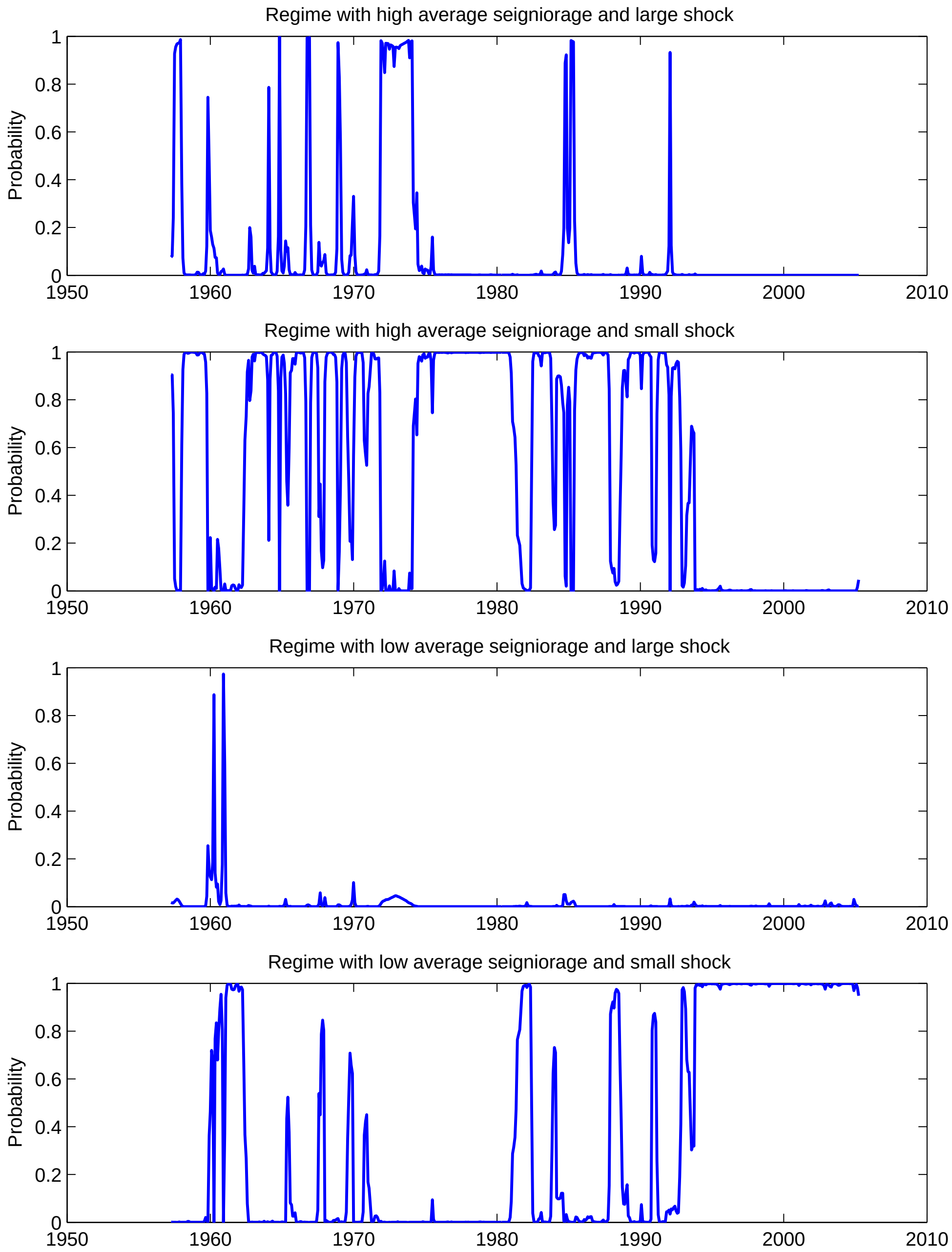


FIGURE 14. Chile: probabilities of the four regimes conditional on the MLEs and the data.

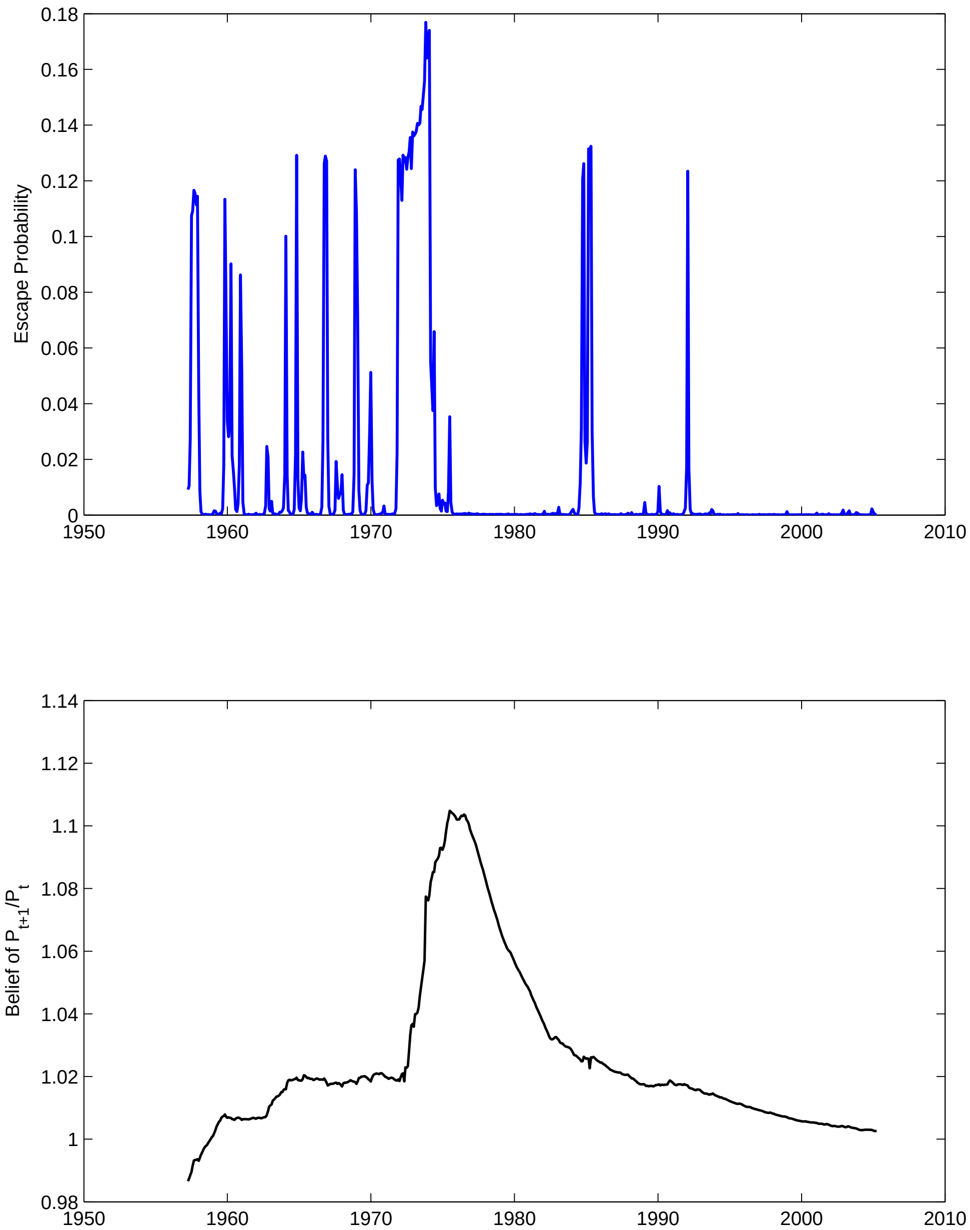


FIGURE 15. Chile: probability of escape (top chart) and belief of next-period inflation β_t (bottom chart).

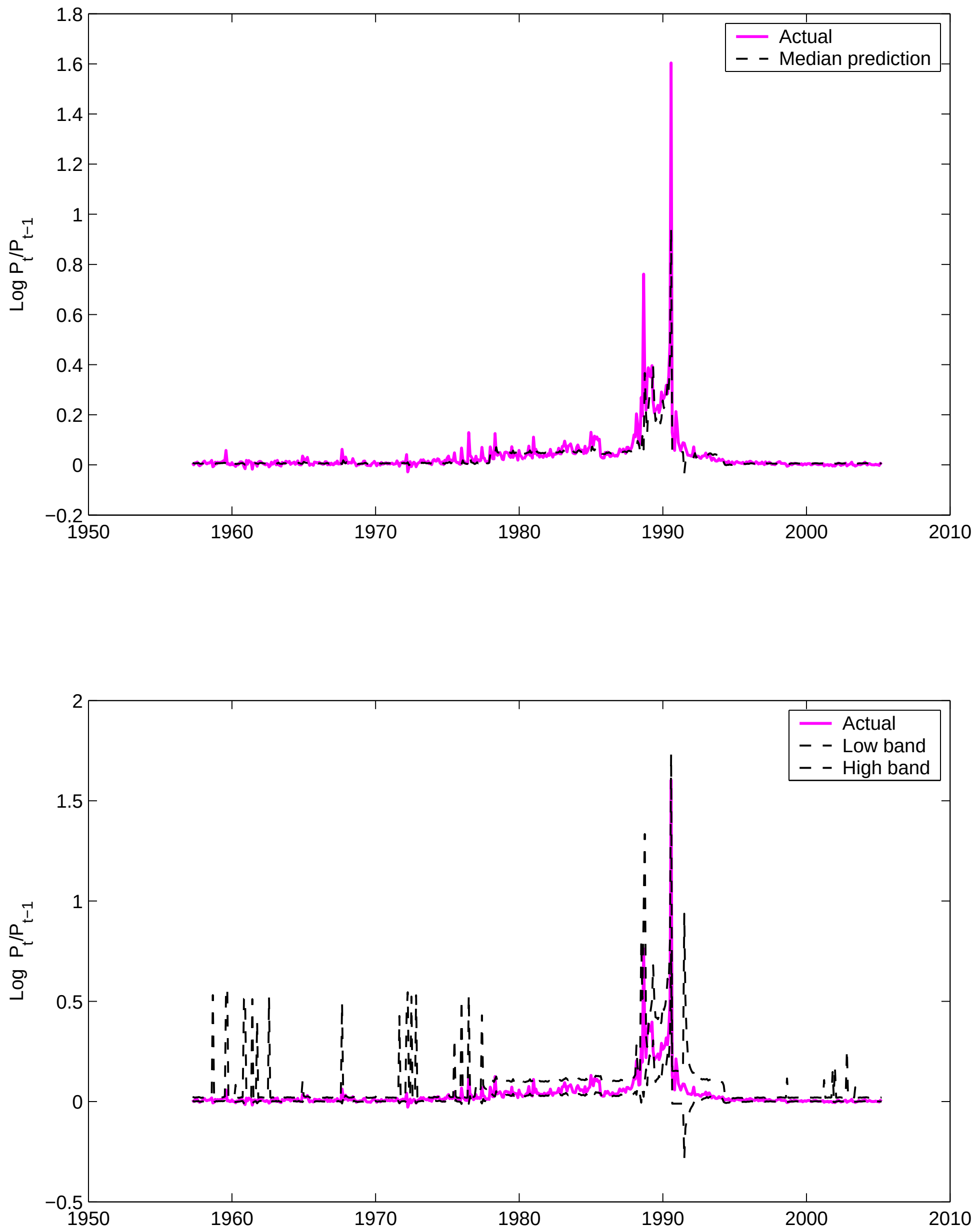


FIGURE 16. Peruvian inflation: actual versus one-step median forecast (top chart) and actual versus .90 probability bands of one-step prediction (bottom chart).

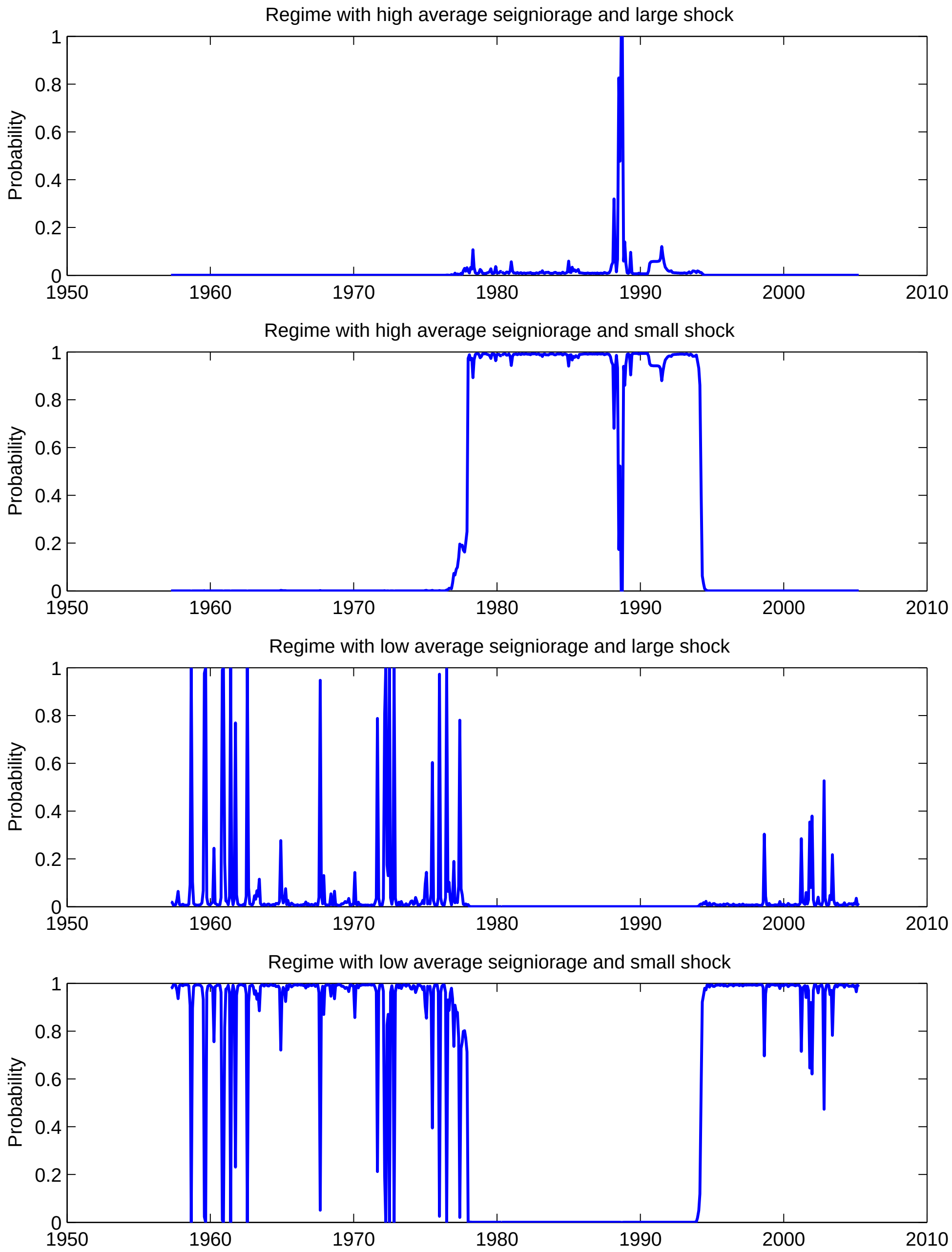


FIGURE 17. Peru: probabilities of the four regimes conditional on the MLEs and the data.

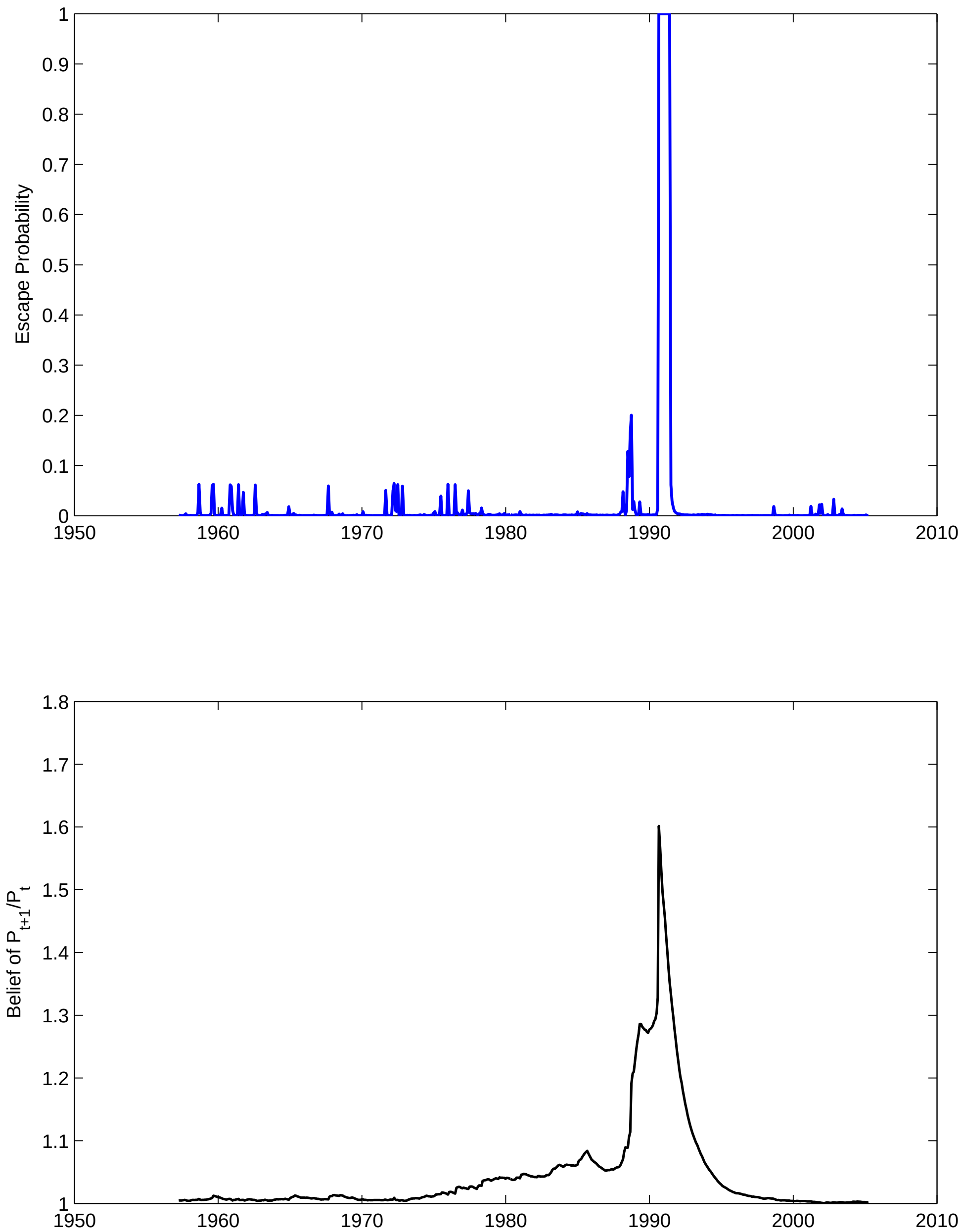


FIGURE 18. Peru: probability of escape (top chart) and belief of next-period inflation β_t (bottom chart).

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