Bankruptcy Costs, Financial Constraints and the Business Cycle

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Abstract

This paper develops a business cycle model with heterogeneous debt constrained firms exposed to idiosyncratic production risk. An agency problem causes firms to trade off the benefit of debt (a reduction in the damage caused by malfeasant managers) and the cost of debt (an increase in the risk of costly bankruptcy). Firm debt capacity, the amount of debt a firm can take on without going bankrupt, is given by the liquidation value of the firm's capital, which is in turn determined by the market for old capital.

I show that a firm's production, investment and financial behavior depend on the firm's debt-capital ratio and the liquidation value of the firm's capital stock. The model predicts that a firm's debt-capital ratio affects the firm's behavior even when the debt capacity constraint is not momentarily binding, b) the model predicts a non-degenerate distribution of debt-capital ratios across firms, and c) debt capacity depends on the distribution of debt-capital ratios across firms.

Using a spectral approximation method, the paper shows how to calculate the general equilibrium dynamics of the model. Shocks to the distribution of firm debt-capital ratios provide a new source of business cycle impulses. Moreover, the perturbation of the distribution of firm debt-capital ratios by aggregate technology shocks (which occurs even in the presence of financial markets which are fully contingent on aggregate technology shocks) provides an amplification mechanism for business cycle disturbances. The time it takes for the distribution of debt-capital ratios to return its steady state provides a new propagation mechanism. The dependence of the liquidation value of capital on the distribution of debt-capital ratios across firms plays an important role in these effects.

The model nests the standard stochastic growth model and thus allows for an assessment of the quantitative importance of debt constraints, time-varying liquidation values and firm heterogeneity for business cycle dynamics.

[Preliminary]

1 Introduction

Macroeconomic models with a continuum of heterogeneous agents arise naturally in a wide range of contexts, for instance, when addressing the effects of irreversible investment (Bertola and Caballero 1994), adjustment costs in employment (Bentolila and Bertola 1990, Campbell and Fisher 1996, Caballero, Engel, and Haltiwanger 1997), precautionary saving by households (Carroll 1992, 1997, Huggett 1993, Rios-Rull 1995),

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vintage capital (Caballero and Hammour 1994, 1996, Campbell 1995, 1997), price adjustment with fixed costs (Caballero and Engel 1993, Tsiddon 1993), or financial intermediation (Bernanke and Gertler 1989, Greenwald and Stiglitz 1993, Phelan 1994). In all of these models, *ex ante* identical economic agents become *ex post* heterogeneous as they experience distinct histories of idiosyncratic shocks. Until recently, the general equilibrium analysis of these models was restricted to steady state and partial equilibrium settings, with the exception of a few papers which eliminated any interaction between agent heterogeneity and aggregate dynamics by making very strong functional form assumptions.[1] This lack of analysis of general equilibrium dynamics was due to the problems created by the combination of an infinite dimensional aggregate state variable with the rational expectations equilibrium concept: a rational expectation equilibrium requires that individual economic agents know the law of motion for the aggregate state variable when they formulate their optimal policies, while the law of motion for the aggregate state variable is in turn determined by the optimal policies of the individual economic agents. When traditional methods are used, this fixed point problem becomes increasingly difficult as the dimensionality of the aggregate state variable increases, making it impossible to compute the general equilibrium dynamics of models with a continuum of heterogeneous agents.

To address these concerns, this paper uses a new method to analyze the general equilibrium dynamics of a stochastic growth model with firm level uncertainty and partially reversible investment. The general idea behind this method is that the standard linearization techniques used in the analysis of homogeneous agent models can also be applied to models with a continuum of heterogeneous agents.[2] In the standard case of an economy with a finite number of agents, these linearization techniques solve for an economy's general equilibrium dynamics in two steps. The first step is to find the economy's linearized law of motion around the steady state, something which can be accomplished by linearizing the economy's feasibility conditions as well as the agents' first order conditions for optimization. This linear law of motion is finite dimensional due to the finite number of agents in the economy. The second step is then to determine the economy's jump variables as a function of the economy's predetermined variables. This second step can be accomplished by

[1]For instance, the papers cited above either analyze the steady state (Huggett, Bentolila and Bertola) or the partial equilibrium dynamics of an economy with exogenously given price processes (Caballero and Hammour) or policy process (Caballero and Engel, Tsiddon), or make functional form assumptions which rule out any interaction between agent heterogeneity and the transitional dynamics of the economy (for instance the exponential utility function in Phelan, the offsetting government policy in Rios-Rull, the fixed entry cost in Campbell and Fisher, or the short-lived agents in Bernanke and Gertler or Greenwald and Stiglitz).

[2]These linearization techniques are familiar from the business cycle literature and are discussed in Blanchard and Kahn (1980), King, Plosser, Rebelo (1988) and Campbell (1994). Moreover, as argued by Gaspar and Judd (1997) and Judd and Guu (1997), these linearization techniques provide a good starting point for studying the nonlinear behavior of the economy.

decomposing the economy's linearized law of motion into the dynamics of a finite number of eigenvectors with their associated eigenvalues, and then picking the economy's jump variables so that the economy's state can always be written as a linear combination of the eigenvectors with stable eigenvalues.[3]

The method used in this paper proceeds in a similar fashion. As in the standard method described above, this paper's method represents the economy's aggregate state by a linear combination of the economy's stable eigenstates and then proceeds to analyze the economy's dynamics in terms of the dynamics of these eigenstates. The paper's innovation is to parameterize the dynamics of the distribution of heterogeneous agents using the eigenfunctions of this distribution when agents follow their steady state policies. The paper's method itself consists of five steps. The first step is to define the eigenstates (the equivalent of eigenvectors in the standard method) of the economy by their rate of convergence to the steady state. This step also delivers the effects of each eigenstate on aggregate consumption and wages. The second step is to find the eigenfunctions of the distribution of heterogeneous agents when agents follow their steady state policies – eigenfunctions which, like all eigenfunctions, are defined by their constant growth rate and invariant form. The third step is to find the optimal policies of the heterogeneous agents in the economic environment created by an eigenstate of the economy, and then to use these optimal policies to find the dynamics of the distribution of heterogeneous agents in this economic environment. The method's fourth step then uses the economy's aggregate resource constraint to determine which of the potential eigenstates found in step three are indeed actual eigenstates of the economy. This step also delivers an eigenstate's effect on aggregate output, investment and scrapping. The fifth and final step determines the effects of aggregate technology shocks on the economy's eigenstates by means of a projection procedure.

This way of computing the general equilibrium dynamics of the economy has several advantages. One advantage is that it does not rely on brute force iteration between the aggregate economy's law of motion and the optimal policies of the heterogeneous agents to arrive at the general equilibrium dynamics of the economy. Instead, the requirements of the rational expectations equilibrium concept are taken into account from the outset, and are solved for in one single step. A second advantage is that the method provides intuition as to why the economy behaves as it does by constructing the eigenstates of the economy, and then showing how these eigenstates are perturbed by aggregate shocks.

Recently, papers by Krusell and Smith (1994), Den Haan (1995), Campbell (1995), Veracierto (1996) and Cooley and Quadrini (1998) have also computed the general equilibrium dynamics of economies with a continuum of heterogeneous agents. Krusell and Smith study the dynamics of an economy with heterogeneous

[3] This second step also determines if there is a unique equilibrium.

households engaged in precautionary saving. Their solution method is to allow households to use the mean and variance of the distribution of household asset holdings as proxies for the information contained in the full distribution of household asset holdings. Using this restriction on household behavior, they solve for the general equilibrium dynamics of the economy by simulating the economy and then iterating between the economy's law of motion and the optimal policy of the households. In their parameterized economy, Krusell and Smith find that any one individual household would only gain minimally in utility terms from conditioning its actions on higher moments of the distribution of financial assets across households, which suggests that their approximation scheme is reasonable for the problem they are studying. However, they also find that household heterogeneity has very little influence on aggregate dynamics in their model. Given the near irrelevance of heterogeneity for the dynamics of this economy, it is therefore not surprising that their approximation method works well.

In contrast to Krusell and Smith, Den Haan, Campbell and Cooley and Quadrini find substantial effects of agent heterogeneity on the general equilibrium dynamics of the economies they study. Den Haan studies the asset pricing implications of precautionary saving, focusing on the effect of the cross-sectional dispersion of household assets on the interest rate. His solution method involves approximating the cross-sectional distribution of household asset holdings by means of a family of exponentials and then iterating between the optimal policy of the household and the economy's law of motion. This solution method seems to work well even in the presence of substantial effects of agent heterogeneity on the dynamics of the economy. However, as Den Haan notes, the approximation of the distribution of household asset holdings by a family of exponentials is not grounded in any prior knowledge about the dynamics of the distribution in general equilibrium, and it does not have an intuitive interpretation.

Cooley and Quadrini is closest this paper. They study the general equilibrium dynamics of debt constrained firms in a model which, in contrast to mine, has reversible capital investment, no hedging of aggregate shocks, and a limited participation monetary sector.

The literature on the macroeconomic effects of debt constrained firms reflects the focus on partial equilibrium models discussed in the introduction. There are a wide array of models of firm financial constraints (Gertler 1992, Hart 1995, Myers and Majluf 1984, Stiglitz and Weiss 1981, Townsend 1979), as well as empirical tests the importance of firm financial constraints for individual firm behavior (Bond and Meghir 1994, Fazzari, Hubbard and Petersen 1988, Gilchrist and Himmelberg 1995, Hubbard 1992, Kaplan and Zingales 1997, Lamont 1997, Whited 1992). The interest in firm financial constraints also extends to their importance for business cycles. There is a considerable amount of empirical work this issue (Bernanke, Gertler and Gilchrist 1996, Calomiris and Hubbard 1989, Carpenter, Fazzari and Petersen 1994, Gertler and Gilchrist 1994, Kashyap, Lamont and Stein 1994, Sharpe 1994). There are also a number of models concerned with the importance of financial constraints for business cycles. The most important of these are Bernanke and Gertler (1989, 1999), Greenwald and Stiglitz (1993) and Kiyotaki and Moore (1997). However, in least in part because they are trying to avoid dealing with firm heterogeneity, these papers lack interesting general equilibrium effects and do not nest the standard stochastic growth model.

The three papers which are exceptions to this statement are Carlstrom and Fuerst (1997), Bernanke, Gertler and Glichrist (1998), and Cooley and Quadrini (1998). Carlstrom and Fuerst as well as Bernanke, Gertler and Gilchrist nest the standard stochastic growth model, but do so with models of the firm which do not allow for any relevant firm heterogeneity and which also do not allow firms to hedge aggregate risks. Cooley and Quadrini is the paper most similar to this paper. The main differences between their paper and this paper are that Cooley and Quadrini assume fully reversible investment while I assume partially reversible investment, that Cooley and Quadrini have a monetary sector while I do not, and finally that Cooley and Quadrini do not allow firms to hedge aggregate risks.

The business cycle model I develop in this paper is constructed using a debt constrained firms exposed to idiosyncratic production risk. I introduce an agency problem between the firm's equity owners and the firm's management to create a role for firm financial structure. Specifically, the possible arrival of malfeasant managers who can expropriate equity owners but not debt holders creates an incentive for the firm to issue debt, while costly liquidation when a firm hits its debt constrain creates an incentive for the firm not to issue too much debt.

I show that in this setting the firm's production, investment and financial behavior behavior depends on the firm's debt-capital ratio and the liquidation value of the firm's capital stock. In particular, the model predicts a) a non-degenerate distribution of firm debt-capital ratios ranging from a lower bound at which the firm is willing to invest and pay dividend to an upper bound at which the firm is liquidated, b) that a firm's debt capital ratio affect the firm's behavior even when the debt capacity constraint is not momentarily binding. Furthermore, I show that the, potentially time-varying, level of the firm's debt capacity, given by the firm's liquidation value, is an important determinant of firm behavior and that the determinant of the firm's liquidation plays an important role in business cycles.

The paper is organized as follows. Section 2 describes the firm's agency problem and contracting technology. Section 3 lays out the structure of the economy and states the conditions for general equilibrium. Section 4 characterizes the behavior of economic agents along the balanced growth path and solves for the economy's general equilibrium in the absence of aggregate shocks, while Section 5 characterizes the economy's general equilibrium dynamics in the presence of aggregate shocks. Both these sections also presents numerical results. Section 6 concludes.

2 The Firm's Financial Contracting Technology and Agency Problem

This section describes the firm's financial contracting technology and agency problem. The firm is run by a manager who uses debt and equity to finance the firm's operations. These two types of financial contracts are distinguished by their distinct cash flows, as well as by their distinct enforcement and issuing technologies. By assumption, debt pays out a cash flow (termed "interest") which is independent of the state of the firm, but which can depend on the state of the aggregate economy. Debt holders can enforce the payment of interest by making the firm pay out the face value of its debt, if necessary by liquidating the firm's capital. The firm can continuously issue and retire debt.

Again by assumption, equity pays out a cash flow (termed "dividends") at the discretion of the firm's manager. Dividends can, since they are paid at the discretion of the manager, depend on the state of the firm as well as on the state of the aggregate economy. With equity there is no enforcement mechanism which allows the firm to assure equity owners *ex ante* that any particular cash flow stream will be paid out. Finally, the firm cannot issue additional equity and dividends must be non-negative.[4]

The firm's agency problem arises from the assumption that there are two types of managers, good managers and bad managers. All firms are run by good managers. Good managers maximize the value of equity, which means that they maximize the value of future dividends to equity owners. The problem with good managers is that they turn into bad managers at the exogenously given rate ϵ. Bad managers do not maximize the value of equity. Instead, on arrival, a bad manager expropriates the firm's current equity owners and installs himself as the firm's new equity owner. As a result, the firm becomes worthless to the current equity owners. The bad manager who has taken control of the firm then hires a new good manager to run the firm for him. Thus, except for the replacement of the current equity owners, there are no effects on the firm from the arrival of a

[4] The assumption that the firm cannot issue equity is an extreme form of the assumption that it is costly for firms to issue equity, for instance, due to inspection costs needed to overcome adverse selection problems. Allowing for some types of costly equity issue would not change the qualitative aspects of the model.

bad manager.[5]

The crucial assumption about the firm's financial contracting technology is that, even though bad managers can expropriate equity owners, bad managers cannot expropriate debt holders. Instead, debt is always serviced in full regardless of the arrival of a bad manager. This technological assumption about the contracting technology of debt reflects the intuition that debt contracts use collateral and covenants to restrain managerial discretion in a way that equity contracts do not. We will see below that this inability of bad managers to default on debt, along with their ability to expropriate equity owners, motivates the use of debt in the firm's financial structure.

Finally, the amount of default-free debt the firm can issue is limited by the firm's debt capacity. Specifically, the liquidation value of the firm's capital stock determines the firm's debt capacity, with the firm's creditors liquidating the firm's capital stock when the firm's debt level reaches the liquidation value of the firm's capital stock. When the firm is liquidated, the firm's creditors receive the face value of their debt, and equity owners receive nothing. One can interpret the firm's debt capacity as a limit on the value of future cash flows the firm can commit to paying out with certainty.

Formally, the good manager's objective is to maximize the value of equity

$$V_{it} = \max_{\{D_{is}\}} E_t \; m_t^{-1} \int_t^{T_i} m_s e^{-(\rho+\epsilon)(s-t)} dD_{is}, \tag{1}$$

where V_{it} is the value of firm i's equity at time t, m_t is the equity owners' marginal utility of wealth at time t, ρ is the equity owners' subjective discount rate, ϵ is the hazard rate that a bad manager will appear and expropriate the equity owners, D_{it} is firm i's **cumulative** dividend flow, and T_i is the time at which firm i hits its debt capacity and is liquidated. The evolution of cumulative dividends D_{it} is governed by the firm's choices about its production, investment and financial policy, all of which are discussed in the next section.

This expression for the value of the firm displays the two types of financial contracting costs the firm faces when it makes decisions about its financial structure. The first type of financial contracting cost arises from the fact that current equity owners only have the probability of $e^{-\epsilon t}$ of actually receiving dividends at time t since they may be expropriated before then by a bad manager. This type of financial contracting cost is more severe for low debt firms since these firms have a higher value of equity. The second type of financial contracting cost arises from the fact that the firm faces costly liquidation if its outstanding debt level reaches its debt capacity. This type of financial contracting cost is more severe for high debt firms since they face a

[5] Bad managers could reduce the value of the firm to equity owners by less than 100% without changing the qualitative results of the paper.

higher likelihood that they will be liquidated in the future. The firm's optimal policy will lead it to minimize the net present value of these two types of financial contracting costs.

This paper follows the incomplete contracting literature by attributing the differences between debt and equity to the financial contracting technology instead of deriving them from a fully specified underlying problem. However, the properties of debt and equity are plausible in light of the technology of the legal system and the difficulty of renegotiating equity contracts. The legal system can prevent the expropriation of debt holders by bad managers by keeping managers from removing capital and breaching debt covenants. This assures debt holders that the firm will always be sufficiently valuable to service their claims. The legal system cannot, however, prevent the expropriation of equity holders by bad managers with equal effectiveness. There are two interconnected reasons for this inability of the legal system to effectively protect equity. First, with the exception of firms that are being liquidated, some fraction of the firm's equity value must always remain unprotected by collateralization due to the fact that the value of the firm is always higher than the liquidation value of the firm's capital. Secondly, when there is a cost to renegotiating the equity contract the fact that equity bears all risk in this model makes it costly to even partially protect equity by collateralization. The reason is that if equity is partially protected by collateralization then, since the amount of debt the firm has outstanding varies over time, the extent to which equity is protected by collateral has to vary over time. If this constant rewriting of the equity contract is sufficiently costly then even the partial protection of equity by collateralization will not occur. Debt, in contrast to equity, can be protected by collateralization and covenants, since there is never a need to renegotiate the collateralization of debt.

3 The Economy

The economy consists of three types of agents: a representative household, a continuum of financially constrained output producing firms, and a representative firm which converts scrapped capital into new investment. The representative household consumes output, provides labor, owns and trades debt, owns the continuum of production firms through equity contracts, and receives interest and dividend payments. The representative household also owns the scrapped capital converting firm. Each production firm produces a risky output stream, employs labor, owns capital, invests in new capital, issues and retires debt, and pays out dividends and interest payments. Furthermore, production firms go bankrupt when their debt level reaches their debt capacity and then sell their capital stock to the scraped capital converting firm. Finally, the representative household and the production firms can hedge aggregate shocks by contracting with each other. The rest of this section describes the details of the economy.

3.1 Aggregate Shocks and the Economy's Aggregate State Variable

All of the economy's aggregate dynamics are driven by an aggregate technology process which follows the geometric Brownian Motion

$$\frac{dA_t}{A_t} = g_A dt + \sigma_A dW_{At}, \tag{2}$$

where g_A is the expected growth rate of aggregate technology, σ_A is the instantaneous standard deviation of the log of aggregate technology, and W_{At} is a standard Brownian Motion. Aggregate technology shocks are the only source of aggregate uncertainty in the economy.

The aggregate state of the economy can be jointly characterized by the economy's technology level A_t and the deviation of the economy from its balanced growth path y_t. For the moment y_t can simply be thought of as an infinite-dimensional state variable. In the absence of sunspot equilibria, the evolution of y_t is given by

$$dy_t = a(y_t)dt + b(y_t)\sigma_A dW_{At} \tag{3}$$

since aggregate technology shocks are the only aggregate shocks in this economy. For the moment $a(y_t)$ and $b(y_t)$ can be taken as parametrically given.

Together A_t and y_t determine the aggregate characteristics of the economy. In particular, A_t and y_t determine the equity owners' marginal utility of wealth

$$m_t = A_t^{-1} m(y_t),$$

the spot interest rate

$$r_t = r(y_t),$$

the wage rate

$$w_t = A_t w(y_t),$$

and the liquidation value of old capital

$$\Phi_t = \Phi(y_t).$$

The scaling of these prices with either A_t, A_t^0 or A_t^{-1} is verified below.

3.2 The Representative Household

The representative household has the utility function

$$E_t \int_t^{\infty} e^{-\rho(s-t)}[\log C_{Agg,s} - \nu L_{Agg,s}]ds, \tag{4}$$

where ρ is the representative household's subjective discount rate, $C_{Agg,t}$ is aggregate consumption, and $L_{Agg,t}$ is aggregate labor supply.

The representative household maximizes its utility subject to its budget constraint. An important feature of the model is that the representative household can enter into hedging contracts with production firms on shocks to the aggregate state variable y_t. When the representative household enters into such a hedging contract, it receives the certain cash flow $h_{Agg,t}p_{h,t}dt$ and in exchange pays out the risky cash flow $h_{Agg,t}\sigma_A dW_{At}$, where $h_{Agg,t}$ is the size and $p_{h,t}$ is the price of the hedge. The representative household's budget constraint also takes into account the household's interest income $r_t B_t$, dividend income $D_{Agg,t}$, labor income $w_t L_{Agg,t}$, cash flow from the scrapped capital converting firm Π_t and consumption expenditures C_t, and is given by

$$\begin{aligned} dB_{Agg,t} &= [r_t B_{Agg,t} + D_{Agg,t} + h_{Agg,t}p_{h,t} + w_t L_{Agg,t} + \Pi_t - C_t]\, dt \\ &\quad - h_{Agg,t}\sigma_A dW_{At} \end{aligned} \tag{5}$$

along with the transversality condition

$$\lim_{s\to\infty} E_t[e^{-(s-t)\rho} m_s B_{Agg,s}] = 0. \tag{6}$$

In the absence of sunspots, the existence of the hedging contract on the aggregate technology shock means that the household faces complete contingent markets for aggregate state variables.

3.3 Production Firms

3.3.1 Production Technology

The production side of the economy consists of a continuum of firms indexed by i on the unit interval $[0,1]$. Each firm has a constant returns to scale production technology which produces a risky **cumulative** output flow Y_{it}. This output flow depends on the aggregate technology level A_t, the firm's capital stock K_{it} and the firm's labor input L_{it}:

$$dY_{it} = (A_t L_{it})^{\alpha} K_{it}^{1-\alpha} dt + \sigma_Y (A_t L_{it})^{\alpha} K_{it}^{1-\alpha} dW_{it} \tag{7}$$

$$
\begin{aligned}
&= lkr_{it}^{\alpha} K_{it} dt + \sigma_Y lkr_{it}^{\alpha} K_{it} dW_{it} \\
&= \mu_Y(lkr_{it}) K_{it} dt + \sigma_Y(lkr_{it}) K_{it} dW_{it}.
\end{aligned}
$$

Here σ_Y is a parameter which describes the idiosyncratic risk associated with the firm's operating technology and W_{it} is a standard Wiener process specific to firm i. The firm can costlessly and instantaneously adjust its labor input L_{it}. Later on it will prove convenient to denote the firm's effective labor-capital ratio ALK^{-1} by lkr, as well as the drift of the firm's cumulative output per unit of capital by $\mu_Y(lkr)$ and the instantaneous standard deviation of the firm's cumulative output per unit of capital by $\sigma_Y(lkr)$.

The firm's cash flow from operations is given by the firm's revenue from output minus the firm's labor costs. The firm's cash flow from operations is therefore given by

$$
\begin{aligned}
dCF_{it} &= dY_{it} - w_t L_{it} dt \\
&= [\mu_Y(A_t L_{it} K_{it}^{-1})^{\alpha} - A_t w(y_t) L_{it} K_{it}^{-1}] K_{it} dt + \sigma_Y (A_t L_{it} K_{it}^{-1})^{\alpha} K_{it} dW_{it} \\
&= [lkr_{it}^{\alpha} - w(y_t) lkr_{it}] K_t dt + \sigma_Y lkr_{it}^{\alpha} K_{it} dW_{it} \\
&= \mu_{CF}(lkr_{it}, y_t) K_t dt + \sigma_{CF}(lkr_{it}) K_{it} dW_{it}.
\end{aligned} \tag{8}
$$

Here CF_{it} is the firm's **cumulative** cash flow. Hence the drift of the firm's cumulative cash flow from operations per unit of capital is given by

$$
\mu_{CF}(lkr, y) = lkr^{\alpha} - w(y) lkr \tag{9}
$$

and the instantaneous standard deviation of the firm's cumulative cash flow from operations per unit of capital is given by

$$
\sigma_{CF}(lkr) = \sigma_Y lkr^{\alpha}. \tag{10}
$$

3.3.2 Investment

Each firm's capital stock K_{it} depreciates at the rate δ and is augmented by investment:

$$
dK_{it} = -\delta K_{it} dt + dI_{it}. \tag{11}
$$

Here K_{it} is the firm's capital stock, I_{it} is the firm's **cumulative** investment. So long the firm is not liquidated investment is irreversible and $dI_{it} \geq 0$.

3.3.3 Debt Capacity

The firm's debt capacity is given by the resale value of the firm's capital stock. With Φ_t denoting the liquidation price of old capital, this means that

$$B_{it} \leq \Phi_t K_{it}. \tag{12}$$

When the firm reaches its debt capacity it is liquidated to pay its creditors. In liquidation, equity owners receive nothing since the firm's debt level is exactly equal to the resale value of the firm's capital. Finally, as described below, the price of liquidated capital Φ is determined by the willingness of the scrapped capital converting firm to pay for liquidated capital.

3.3.4 Hedging Aggregate Shocks

Production firms can hedge their exposure to aggregate shocks. Just like the representative household, when a firm enters a hedging contract, it receives the certain cash flow $h_t p_{h,t} dt$ and in exchange pays out the risky cash flow $h_t \sigma_A dW_{At}$, where h_t is the size of the hedge and $p_{h,t}$ is the price of the hedge. By hedging, the firm can control the relationship between innovations in the economy's aggregate state variable and innovations in the firm's financial position.

One important effect of the firm's ability to hedge aggregate shocks is that it gives the firm control over the effective maturity structure of its debt: the firm can use the hedging contract to synthesize the interest rate risk of any longer term debt portfolio while only issuing short term debt.

3.3.5 Budget Constraint

The evolution of the firm's debt level is determined by the firm's interest payments on debt, its cash flow from operations, its dividend payments, its investment, and its income from hedging aggregate shocks. The firm's budget constraint is thus given by

$$\begin{aligned} dB_{it} &= [r_t B_{it} - \mu_{CF}(lkr_{it}, y_t) K_{it}]dt - \sigma_{CF}(lkr_{it}) K_{it} dW_{it} \\ &\quad + dD_t + dI_{it} \\ &\quad - h_{it} p_{ht} dt + h_{it} \sigma_A dW_{At}. \end{aligned} \tag{13}$$

along with the transversality condition

$$\lim_{s \to \infty} E_t[e^{-(s-t)\rho} m_s B_{is}] = 0. \tag{14}$$

3.4 The Scrapped Capital Converting Firm

The scrapped capital converting firm takes scrapped capital from liquidated production firms and converts it into new investment. This firm's production function is a matching function which uses both the stock of scrapped capital on-hand and the outflow of scrapped capital to new investment as inputs. Specifically,

$$I_{old,t} = A_{sc} I_{sc,t}^{\gamma} K_{sc,t}^{1-\gamma}, \tag{15}$$

where $I_{old,t}$ is the amount of the investment good created, $I_{sc,t}$ is the amount of scrapped capital used as an input, $K_{sc,t}$ is the amount of scrapped capital on-hand, and $A_s c$ and γ are parameters of the production function. The stock of scraped capital on-hand evolves according to

$$dK_{sc,t} = -\delta_{sc} K_{sc,t} dt + Liq_{Agg,t} - I_{sc,t}, \tag{16}$$

where $Liq_{Agg,t}$ is the inflow of liquidated capital from bankrupt production firms.

The firm is not financially constrained and operates as a price taker. Its cash flow is given by

$$\Pi_t = I_{old,t} - \Phi_t Liq_{Agg,t}. \tag{17}$$

and the firm maximizes the net present value of this cash flow to the representative household.

3.5 General Equilibrium

Definition 1 defines the general equilibrium of this economy in the usual manner, with all agents optimizing their objective functions given their technology and budget constraints, and with markets clearing.

Definition 1 General Equilibrium

The economy's general equilibrium is the set of state contingent sequences consisting of

- prices:

$\{m_t\}$, $\{r_t\}$, $\{p_{ht}\}$, $\{\Phi_t\}$, $\{w_t\}$

- quantities:

$\{Y_{Agg,t}\}$, $\{C_{Agg,t}\}$, $\{L_{Agg,t}\}$, $\{K_{Agg,t}\}$, $\{K_{sc,t}\}$, $\{I_{old,t}\}$, $\{I_{sc,t}\}$, $\{I_{Agg,t}\}$, $\{Liq_{Agg,t}\}$, $\{h_{Agg,t}\}$, $\{D_{Agg,t}\}$, $\{B_{Agg,t}\}$, $\{Y_{it}\}_{i=0}^{1}$, $\{L_{it}\}_{i=0}^{1}$, $\{K_{it}\}_{i=0}^{1}$, $\{I_{it}\}_{i=1}^{1}$, $\{h_{it}\}_{i=0}^{1}$, $\{D_{it}\}_{i=0}^{1}$, $\{B_{it}\}$

measurable with respect to $\{W_{At}\}$, $\{W_{it}\}_{i=0}^{1}$ *such that* *a) quantities aggregate correctly:*

- output

$$Y_{Agg,t} = \int dY_{it} di$$

- investment in capital

$$I_{Agg,t} = \int dI_{it} di$$

- capital

$$K_{Agg,t} = \int K_{it} di$$

- dividends

$$D_{Agg,t} = \int dD_{it} di$$

b) markets clear:

- the market for new goods

$$Y_{Agg,t} + I_{old,t} = I_{Agg,t} + C_{Agg,t}$$

- the market for liquidated capital

$$Liq_{Agg,t} dt = \int_{i \in \{B_{it}/K_{it} - \Phi_t = 0\}} K_{it} di$$

- the market for labor

$$L_{Agg,t} = \int L_{it} di$$

- the market for debt

$$B_{Agg,t} = \int B_{it} di$$

- the market for hedging contracts

$$h_{Agg,t} = -\int h_{it} di$$

c) the representative household maximizes its expected utility (4) subject to its budget constraint (5 - 6) by choosing $\{C_{Agg,t}, L_{Agg,t}, h_{Agg,t}\}$ *and the solution of this problem yields* $\{m_t\}$ *as the representative household's marginal utility of wealth*

d) production firms maximize their equity value (1) subject to their production technology (7), capital accumulation (11), debt capacity (12) and budget (13 - 14) constraints by choosing $\{L_{it}\}_{i=0}^{1}$, $\{I_{it}\}_{i=0}^{1}$, $\{K_{it}\}_{i=0}^{1}$ *and* $\{dD_{it}\}_{i=0}^{1}$

e) scrapped capital converting firms maximize the net present value of their cash flows (17) subject to their production technology (15)and capital accumulation constraint (11)

f) firms are liquidated when their debt level reaches their debt capacity (12).

4 Balanced Growth Path

Before proceeding to characterize the economy's general equilibrium dynamics in the presence of aggregate technology shocks, this section first studies the balanced growth path of the economy in the absence of aggregate uncertainty. Section 4.1 characterizes the optimal policy of the representative household, Section 4.2 characterizes the optimal policy of the production firm, Section 4.3 shows how to aggregate across production firms, Section 4.4 characterizes the optimal policy of the scrapped capital converting firm, and Section 4.5 finds the economy's balanced growth path. Finally, Section 4.6 illustrates the results using numerical examples.

4.1 The Household's Problem

The representative household faces the problem of maximizing its expected utility (4) subject to its budget constraint (5 - 6). The solution to this problem defines the household's marginal utility of wealth m_t, which equals the household's marginal utility of consumption:

$$m_t = C_t^{-1}. \tag{18}$$

Using this notation, the first order conditions for the household's problem are

$$w_t = \nu C_t \tag{19}$$

$$r_t = \rho - m_t^{-1} \frac{E_t[dm_t]}{dt}. \tag{20}$$

On the economy's balanced growth path and in the absence of aggregate technology shocks, all aggregate quantities grow at their balanced growth path rates, including aggregate consumption which grows at the rate $g_{C,BGP}$. The first order conditions for maximizing the representative household's expected utility (18 - 20) therefore imply that the wage rate, the household's marginal utility of wealth, and the spot interest rate evolve according to

$$w_{t,BGP} = w_0 e^{g_{C,BGP}\ t}$$

$$m_{t,BGP} = m_0 e^{-g_{C,BGP}\ t}$$

$$r_{t,BGP} = r_{BGP} = \rho + g_{C,BGP}.$$

4.2 The Production Firm's Problem

Each individual production firm maximizes the value of its equity to the representative household (1) subject to its budget constraint (13-14). To analyze the firm's problem it is useful to introduce the firm state variable

$$x = \Phi_{BGP} - B/K, \tag{21}$$

where Φ_{BGP} is the price of old capital along the economy's balanced growth path. The state variable x is the amount of additional debt the firm can take on per unit of capital when the price of old capital is Φ_{BGP}. If the economy is on its balanced growth path the firm hits its debt capacity when $x = 0$. x describes the firm's financial position and is a more convenient state variable to work with than B/K.

Proposition 1 characterizes the firm's value function. The proposition shows that the firm chooses an effective labor-capital ratio lkr based on the state variable x. The proposition does this by relating the required expected appreciation of the firm's equity value to the drift and variance of the firm's cash flow using Ito's Lemma and then using this relationship to find the firm's optimal choice of its effective labor-capital ratio. Three boundary conditions tie down the unique solution for the value function. The first boundary condition, at the boundary where the firm hits its debt capacity, i.e. when $x = 0$, stems from the fact that the firm's equity value is zero in this situation. The second boundary condition occurs at the boundary where the firm pays out dividends (denoted by $x = p_{BGP}$ for "payment") and requires that cash inside the firm and cash outside the firm be equally valuable to the representative household owning the firm. This boundary condition is know in the literature as a "value matching condition". The third boundary condition, know as a "smooth pasting condition", assures that the location of the boundary at which the firm pays dividends maximizes the value of the firm.

Proposition 1 The Production Firm's Value Function and Choice of Labor Input
In the economic environment created by the economy's balanced growth path the firm's value function is given by

$$V(K, B) = Kv(x),$$

where $x = \Phi_{BGP} - B/K$. Here v(x) is characterized by

a) the ordinary differential equation (ODE):

$$(r_{BGP} + \delta + \epsilon)v(x) = [\mu_{CF}(lkr(x), 0) - (r_{BGP} + \delta)(\Phi_{BGP} - x)]v_x(x)$$

$$+\frac{1}{2}\sigma^2_{CF}(lkr(x))v_{xx}(x),$$

where

$$\begin{aligned} lkr(x) &= \arg\max\{[\mu_{CF}(lkr(x),0)-(r_{BGP}+\delta)(\Phi_{BGP}-x)]v_x(x) \\ &\quad +\frac{1}{2}\sigma^2_{CF}(lkr)v_{xx}(x)\}. \end{aligned}$$

Hence the first order condition for the optimal choice of lkr *is*

$$\frac{\partial\mu_{CF}(lkr,0)/\partial lkr}{\partial\sigma^2_{CF}(lkr)/\partial lkr}=-\frac{1}{2}\frac{v_{xx}(x)}{v_x(x)}.$$

b) the boundary conditions, for a positive and finite p_{BGP}*,*

$v(0)=0,$

$v_x(p_{BGP})=1,$

$v_{xx}(p_{BGP})=0.$

Finally, $v(x)$ *has the properties, for* $0\le x<p_{BGP}$*:*

i) $v(x)$ *is twice differentiable,*

ii) $v_x(x)>1,$

iii) $v_{xx}(x)>0.$

Proof. *See Appendix H.*

Proposition 2 reiterates the fact that the firm pays dividends when the firm state variable x equals the trigger level p_{BGP} and that the firm does not pay dividends when the firm state variable x is below this trigger level p_{BGP}. Proposition 2 also shows that the firm's investment policy is similar to its dividend policy, with the firm investing only when its state variable x reaches the trigger level I_{BGP}.

Proposition 2 The Production Firm's Dividend and Investment Policy

The firm's optimal dividend policy in the balanced growth path economic environment is to pay dividends when $x=p_{BGP}$*.* $x=p_{BGP}$ *is a reflective barrier for the state variable* x*, and the firm pays dividends to ensure that* $x\le p_{BGP}$*.*

Similarly, the optimal investment policy of the firm is to invest in capital when $x=I_{BGP}$*, where* I_{BGP} *solves the equation*

$$1=\frac{v(I_{BGP})}{v_x(I_{BGP})}+\Phi_{BGP}-I_{BGP}.$$

$x = I_{BGP}$ is a reflective barrier for the state variable x, and the firm invests to ensure that $x \leq I_{BGP}$.

Proof. *See Appendix H.*

In a partial equilibrium setting there is no reason why a firm should be willing both to pay dividends and to invest, and hence there is no reason why $p_{BGP} = I_{BGP}$ should hold in partial equilibrium. In general equilibrium, however, firms have to be willing both to pay dividends and to invest. Otherwise, we either have that $p_{BGP} > I_{BGP}$ and equity never pays any dividends along the economy's balanced growth path, which would not maximize the value of equity when $r_{BGP} > g_{Y,BGP}$, or we would have that $p_{BGP} < I_{BGP}$, which would imply that no investment would ever occur and that the economy's capital stock does not grow at the rate required for balanced growth. For this reason, $p_{BGP} = I_{BGP}$ must hold along the economy's balanced growth path in general equilibrium. We will see in Section 4.5 below that the wage adjusts to assure that firms are willing both to pay dividends and to invest in the balanced growth path economy.

Proposition 3 relates the firm's risk aversion towards idiosyncratic risk to the firm state variable x. The proposition states that the firm's risk aversion towards idiosyncratic risk declines monotonically with the firm state variable x, with the firm becoming indifferent to idiosyncratic risk when $x = 0$. In other words, high debt firms are more risk averse towards idiosyncratic risk than low debt firms, with the lowest debt firm being indifferent towards idiosyncratic risk. Furthermore, proposition 3 shows that, as a consequence of the firm's changing risk aversion towards idiosyncratic risk, the firm chooses higher labor-capital ratios as its debt-capital ratio falls.

Proposition 3 The Production Firm's Risk Aversion

The firm's risk aversion towards idiosyncratic risk is a function of the firm's debt-capital ratio, and is given by

$$\frac{V_{BB}(K,B)}{V_B(K,B)} = -\frac{v_{xx}(x)}{v_x(x)}K^{-1}.$$

For $x < p_{BGP}$, the firm is risk averse and the firm's risk aversion towards idiosyncratic risk decreases with x:

$$\frac{d}{dx}\frac{V_{BB}(K,B)}{V_B(K,B)} < 0.$$

Put differently, the firm's risk aversion towards idiosyncratic risk increases with the firm's debt-capital ratio. At $x = p_{BGP}$ the firm's risk aversion towards idiosyncratic risk is zero.

Firms with lower debt-capital ratios, since they are less risk averse towards idiosyncratic risk, have higher *labor-capital ratios and have higher expected output per unit of capital:*

$$\frac{dlkr(x)}{dx} \geq 0$$
$$\frac{\mu_Y(x)}{dx} \geq 0.$$

Proof. *See Appendix I.*

There is no gambling for resurrection since the firm becomes more risk averse towards idiosyncratic risk as its debt-capital ratio rises for the whole range of debt-capital ratios. This is due to the fact that in this model it is impossible for the firm to default on debt, which prevents the firm from shifting wealth from debt holders to equity holders by choosing a more risky cash flow, thus eliminating any incentive for the firm to gamble for resurrection.

4.3 Aggregation

This section shows how to keep track of the heterogeneous firm debt-capital ratios and how to determine the effects of firm heterogeneity on aggregate quantities. We have already seen that along the balanced growth path firms must be willing both to pay dividends and to invest. For this reason, this section will only analyze the case where $p_{BGP} = I_{BGP}$.

Using the firm's optimal choice of the effective labor-capital ratio $lkr(x)$ from Proposition 1, as well as the firm's budget constraint (13), we can see that the firm state variable x evolves according to the diffusion process

$$dx_{it} = \mu_x(x_{it})dt + \sigma_x(x_{it})dW_{it} \tag{22}$$

on the interval $[0, p_{BGP}]$ with an absorbing boundary at $x = 0$ and a reflecting boundary at $x = p_{BGP}$. The drift $\mu_x(x)$ and the instantaneous standard deviation $\sigma_x(x)$ of x are given by

$$\mu_x(x) = \mu_{CF}(lkr(x), 0) + (r_{BGP} + \delta)(x - \Phi_{BGP}) \tag{23}$$
$$\sigma_x(x) = \sigma_{CF}(lkr(x)). \tag{24}$$

In the literature this type of diffusion process is know as a singularly controlled diffusion process.[6]

Now define $f(x,t)$ to be the density of capital in firms with the state variable x. The evolution of the capital density $f(x,t)$ is governed by the Kolmogorov forward equation (KFE) modified to take into account

[6]See, for instance, Harrison (1985).

depreciation. Thus, the evolution of the capital density is given by

$$\partial_t f(x,t) = -\delta f(x,t) + \frac{1}{2}\partial_{xx}[\sigma_x^2(x) f(x,t)] - \partial_x[\mu_x(x) f(x,t)] \tag{25}$$

with the boundary condition, at $x = 0$,

$$f(0,t) = 0$$

and the boundary condition, at $x = p_{BGP}$,

$$I_{Agg,t} = (\mu_x(p_{BGP}) - \frac{1}{2}\partial_x \sigma_x^2(p_{BGP})) f(p_{BGP}, t) - \frac{1}{2}\sigma_x^2(p_{BGP})\partial_x f(p_{BGP}, t). \tag{26}$$

The liquidation rate of installed capital is given by

$$Liq_{Agg,t} = \frac{1}{2}\sigma_x^2(0)\partial_x f(0,t) \tag{27}$$

and the sum of dividend payments and investment is given by

$$D_{Agg,t} + I_{Agg,t} = \frac{1}{2}\sigma_x^2(p_{BGP}) f(p_{BGP}, t). \tag{28}$$

The KFE and its boundary conditions are discussed in more detail in Appendix J. Here I will only provide some of intuition. In the order in which the terms appear in (25), the KFE relates the evolution of the capital density at any one point x to the rate of capital depreciation, the diffusion of capital close to the point x into or away from the point x, as well as the drift of capital across the point x. The boundary condition at $x = 0$ imposes a capital density of zero at $x = 0$ because the capital of liquidated firms is instantaneously removed, with the liquidation rate of capital (27) given by the arrival of capital to the point $x = 0$ from the right due to diffusion. The boundary condition at $x = p_{BGP}$ states that the flow of capital away from $x = p_{BGP}$ to the left is given by the amount of investment at this boundary. Finally, the fact that the sum of aggregate dividend payments and aggregate investment equals $\frac{1}{2}\sigma_x^2(p_{BGP}, t) f(p_{BGP}, t)$ is a property of the singularly controlled diffusion process, and is discussed further, like all other aspects of the KFE, in Appendix J.

Given the capital density $f(x,t)$ we can obtain the aggregate quantities

$$\begin{aligned} Y_{Agg,t} &= \int_0^{p_{BGP}} \mu_Y(x) f(x,t) dx & (29)\\ L_{Agg,t} &= \int_0^{p_{BGP}} A_t^{-1} lkr(x) f(x,t) dx & (30)\\ Liq_{Agg,t} &= \frac{1}{2}\sigma_x^2(0,t)\partial_x f(0) & (31)\\ D_{Agg,t} + I_{Agg,t} &= \frac{1}{2}\sigma_x^2(p_t, t) f(p_t, t). & (32) \end{aligned}$$

The expressions for aggregate output and aggregate labor input follow directly from (7) and the definition of the effective labor-capital ratio, as well as from the definition of the capital density $f(x,t)$. The other relationships follow likewise directly from the previous discussion.

4.4 The Scrapped Capital Converting Firm's Problem

The scrapped capital converting firm purchases old capital from liquidated production firms and then uses this stock of old capital to create new investment goods. Along the balanced growth path the firm's problem is to maximize the net present value of its cash flows subject to its production function and scrapped capital on-hand accumulation function:

$$\begin{aligned}
&E_t m_t^{-1} \int_t^{\infty} e^{-\rho(s-t)} m_s (I_{old,s} - \Phi_{BGP} Liq_{Agg,s}) ds \\
&s.t. \quad I_{old,s} = A_{sc} I_{sc,s}^{\gamma} K_{sc,s}^{1-\gamma} \\
&\qquad\quad dK_{sc,s} = -\delta_{sc} K_{sc,s} ds + Liq_{Agg,s} ds - I_{sc,s} ds.
\end{aligned}$$

Since the firm is a price taker, the price of scrapped capital Φ_t must equal the marginal product of $I_{sc,t}$ in the production of the investment good:

$$\Phi_{BGP} = \frac{dI_{old,t}}{dI_{sc,t}} = \gamma A_{sc} I_{sc,t}^{\gamma-1} K_{sc,t}^{1-\gamma}. \tag{33}$$

Furthermore, the return on holding scrapped capital on-hand must equal the balanced growth path interest rate

$$r_{BGP} = \frac{dI_{old,t}/dK_{sc,t}}{\Phi_{BGP}} - \delta_{sc}.$$

This implies immediately that along the balanced growth path

$$\frac{K_{sc,t}}{I_{sc,t}} = \frac{1-\gamma}{\gamma(r_{BGP} + \delta_{sc})}.$$

The scrapped capital on-hand accumulation constraint

$$dK_{sc,t} = -\delta_{sc} K_{sc,t} dt + Liq_{Agg,t} dt - I_{sc,t} dt$$

then implies that

$$K_{sc,t} = \left(\delta_{sc} + g_{Liq} + \frac{\gamma(r_{BGP} + \delta_{sc})}{1-\gamma} \right)^{-1} Liq_{Agg,t}$$

where g_{Liq} is the BGP growth rate of $Liq_{Agg,t}$. By choosing A_{sc} correctly any desired balanced growth path price of scrapped capital Φ_{BGP} can be calibrated.

4.5 General Equilibrium

This section concludes the analysis of the economy's balanced growth path in the absence of aggregate shocks. The section begins by establishing that the economy's balanced growth rate is given by the growth rate of aggregate technology g_A, as is to be expected in a constant returns to scale economy with labor augmenting aggregate technology. The section then indicates how to derive the economy's balanced growth capital density as well as the economy's output-technology, consumption-output, investment-output, old investment-output, debt-output, and dividend-output ratios.

Along the balanced growth path all aggregate quantities grow at the balanced growth rate $g_{Y,BGP}$, including aggregate consumption. As a result, the representative household's first order conditions for utility maximization (18-20) indicate that the balanced growth interest rate is $r_{BGP} = \rho + g_{Y,BGP}$. We have already observed above that on the balanced growth path firms must be willing both to pay dividends and to invest, so that $p_{BGP} = I_{BGP}$ must hold. Taking the liquidation value of old capital Φ_{BGP} as given for the moment, the wage w_t is the only other price which affects the firm's problem. Furthermore, the wage enters the firm's problem only through its effect on the cost of effective labor (i.e. the wage-technology ratio) in (8). This has two consequences. The first consequence is that $g_{Y,BGP} = g_A$ since balanced growth requires that $\mu_x(x)$ constant over time. The second consequence is that the economy's wage-technology ratio is determined by the requirement that $p_{BGP} = I_{BGP}$. Given the wage-technology ratio, the first order condition for utility maximization by the representative household (18) then yields the economy's consumption-technology ratio.

We can now turn to the behavior of the balanced growth capital density $f_{BGP}(x,t)$. Since firms have a constant returns to scale production technology, the balanced growth capital density must grow at the same rate as the rest of the economy. This means that the evolution of the balanced growth capital density is given by $f(x,t) = K_{Agg,0}\exp(g_A t) f_{BGP}(x)$, where $f_{BGP}(x)$ is defined to integrate to one. The capital density $f_{BGP}(x)$ is determined by the KFE along with the boundary condition at $x = 0$. Given $f_{BGP}(x)$ it is then possible to find the economy's investment-capital ratio from (26). The economy's output-capital ratio is given in turn by (29). This also implies the economy's consumption-output ratio since we already have the economy's investment-output ratio and since $C_{Agg,t} + I_{Agg,t} = Y_{Agg,t} + I_{old,t}$.

4.6 Numerical Results

This section illustrates the balanced growth path results for a series of calibrated economies with different values of Φ_{BGP}. Figures 1-8 display the properties the first calibrated economy.[7] For this economy I have set

$$\Phi_{BGP} = 0.7$$
$$\sigma_Y = 0.5$$
$$\epsilon = 0.05$$

as well as $\rho = 0.05$, $\sigma_Y = 0.5$, $\gamma = 0.5$, $\nu = 1$ and $g_A = 0$. This choice of parameters results in a realistic range of 0 to 0.7 for the firm's debt-capital ratio. The calibration also implies a reasonable instantaneous variance of equity returns for low and medium debt firms of 0.1.[8]

Figure 1 displays the production firm's value of equity per unit of capital as a function of the firm's debt-capital ratio. The value of equity per unit of capital ranges from one at the dividend payment and investment margin to zero at the liquidation margin. Furthermore, the value of equity is decreasing the firm's debt-capital ratio, with the rate of decrease always equal or larger than one and increasing in the debt-capital ratio, as was indicated in Proposition 1. Figure 2 displays the same phenomenon by showing the total value of the firm (consisting of the value of equity and the value of debt) per unit of capital. Obviously, we have the same dividend payment and investment margin and liquidation margin as in Figure 1. Furthermore, we can see that the total value of the firm is decreasing in its debt-capital ratio, at first slowly but then, as the debt-capital ratio rises, more rapidly, as indicated by Proposition 1. At the dividend payment and investment margin the slope of the total value curve is zero, so that the firm cannot change its total value by investing or paying dividends, whereas at any higher debt-capital ratio the firm would be made worse off if it paid dividends or invested, as indicated by Proposition 2.

Figure 3 shows the effect of the firm's debt-capital ratio on the firm's choice of labor input. It can be seen that the firm's labor input is highest for the low debt firm and falls as the firm's debt level rises. The difference between the low debt firm's labor input and the highest debt firm's labor input is substantial, with the lowest debt firm using more than twice the labor input as the highest debt firm. It should be noted that the fall in labor input with the rise in the debt-capital ratio is approximately linear and is not concentrated

[7] This is also Economy 1 in the discussion of the non-balanced growth path dynamics in Section 5.7.

[8] Strictly speaking, this is only true up to a debt-capital ratio of 0.5. For debt-capital ratios above 0.5 the idiosyncratic instantaneous variance of equity prices rises rapidly in this model, mostly because of the combination of leverage with fully secured debt. The observed idiosyncratic variance of US equity prices is around 0.1 (Malkiel and Xu, 1997).

among firms with a high debt-capital ratio.

Figure 4 shows the effect of the firm's debt-capital ratio on the firm's level of expected output. This relationship is similar to the relationship between the debt-capital ratio and labor input, except that the fall off of expected output with increased debt is less drastic than it is for labor input due to the diminishing marginal product of labor. Again, the fall in expected output is approximately linear and is not concentrated among firms with a high debt-capital ratio.

Figure 5 shows the drift of the firm's state variable x (which is of equal size and of the opposite sign as the drift of the firm's debt-capital ratio) as a function of the firm's debt-capital ratio. We can see that the drift in x is decreasing in the firm's debt-capital ratio, so that low debt firms are expected to decrease their debt levels at a higher rate than high debt firms. There are two reasons for this. The first is that high debt-capital ratio firms must spend more of their cash flow in paying interest on their debt, leaving less left over to decrease their debt-capital ratio. The second reason is that, as can be seen in Figure 6, the firm's expected cash flow from operations is decreasing in the firm's debt-capital ratio. The first reason appears to be quantitatively more important since the firm's expected cash flows are not that sensitive to the firm's debt-capital ratio, especially not for low debt firms.

The slow decline of expected firm cash flow from operations despite the rapid decline in expected output is a direct consequence of the optimizing behavior of the lowest debt-capital ratio firm. For the lowest debt-capital firm the optimal choice of labor input maximizes expected cash flow, which means that deviations from this level of labor input will have, at least at first, a negligible effect on expected cash flows from operations. Eventually, as labor input declines further the effect on expected cash flow becomes larger. Figure 6 displays exactly this effect.

Figure 7 shows the instantaneous variance of the firm's debt-capital ratio as a function of the firm's debt-capital ratio. In addition to the firm's expected cash flow from operations, this is the second important variable affected by the firm's choice of labor input. Figure 7 shows that the instantaneous variance of the firm's debt-capital ratio falls rapidly with increasing firm debt, with the high debt firm having about 1/4 the variance as the low debt firm.

Figure 8 displays the firm's risk aversion towards idiosyncratic risk as a function the firm's debt-capital ratio. As indicated in Proposition 3, the lowest debt-capital ratio firm is risk neutral and the firm's risk aversion increases with the firm's debt-capital ratio. Contrary to what one naively might expect, however, the firm's risk aversion does not become infinite at the liquidation margin but instead remains finite. Since we saw in Proposition 3 that the firm's risk aversion explains the firm's trade off of expected cash flow and cash flow volatility, Figure 8 also accounts for the behavior of the expected cash flow of the firm (Figure 6), and the variance of firm cash flows (Figure 7).

Finally, Figure 9 displays this economy's capital density for the production firm. Most of the capital density is concentrated in low debt-capital ratio firms since the drift in the debt-capital ratio is downward for all firms. This effect is slightly ameliorated by the increased volatility of the debt-capital ratio for low debt firms, but not enough for the capital density to increase with the firm debt-capital ratio.

Figures 10-12 display the properties of eight economies with the same calibration as the previous economy, except that the liquidation value of installed capital varies from 0.1 to 0.9 in increments of 0.1. Figure 10 displays the equity value of this set of firms. A firm's equity value falls to zero at the debt-capital ratio which equals the liquidation value of installed capital. Figure 10 also shows that firm equity value is almost linear for economies with a hight debt capacity while becoming quite non-linear for economies with low debt capacity. Figure 11 shows the same phenomenon as Figure 10 by displaying the total (debt and equity) value of the firm.

Finally, Figure 12 shows firm labor input as a function of the debt-capital ratio for these eight economies. We can again see that the liquidation margin depends in a mechanical way on the liquidation value of capital Φ. Furthermore, we can see that the dividend payment and investment margin also depends significantly on the debt capacity. It is noteworthy that the dividend payment and investment margin remains quite far from the liquidation margin even for economies with a high liquidation value of capital.

Figure 12 also shows that the difference between the low debt-capital ratio and the high debt-capital ratio firm's labor input is higher for economies with low debt capacity. Indeed, for the economy with $\Phi = 0.1$ the high debt-capital firm's labor input becomes extremely small. The reason is that this firm needs only a small positive cash flow to service its small outstanding debt while it is very risk averse due to the high cost of liquidation to equity owners. But for firms in the $\Phi = 0.9$ economy, the difference between the labor inputs of low and high debt-capital ratio firms is still substantial, with labor inputs differing by a ratio of 1.8.

5 Dynamics around the Balanced Growth Path

In this section I consider the transition dynamics resulting from an initial capital distribution that differs somewhat from the BGP distribution. I accomplish this by representing the economy's aggregate state as a linear combination of the economy's eigenstates and then using the dynamics of these eigenstates to describe the dynamics of the economy. I start in Section 5.1 by defining what eigenstates are and establishing some notation for the linearized dynamics of the economy. I then proceed in Section 5.2 to analyze the optimal policy of the production firm in this linearized economic environment. Section 5.3 analyses the linearized dynamics of the production firm capital density $f(x,t)$. Section 5.4 analyzes the optimal policy of the scrapped capital converting firm in the linearized economic environment. Section 5.5 then uses the results of the previous sections to construct the linearized general equilibrium dynamics of the economy in the absence of aggregate shocks, while Section 5.6 finishes the analysis by determining the reaction of the linearized economy to aggregate shocks. Section 5.7 provides numerical results for a series of calibrated economies.

5.1 What are Eigenstates?

The linearized dynamics of the economy can be written as

$$\begin{aligned} dA_t &= g_A A_t dt + \sigma_A dW_{At} \\ dy_t &= A y_t dt + B\sigma_A dW_{At} \end{aligned}$$

where, as indicated before, A_t is the level of aggregate technology and y_t is the deviation of the economy from its balanced growth path. y_t can be thought of as n-dimensional vector, A as a n*n dimensional matrix, and B as a n*1 dimensional matrix, with n going to infinity in the limit that describes the economy accurately. The matrix A can be specified in more detail. In particular, it is possible to choose a basis for the vector y_t so that A is diagonal, with the entries $-\eta_j$ along the diagonal. This means that the evolution of each component of the vector y_t is now separated from the evolution of all other components of the vector y_t, and that the evolution of each individual component $y_{j,t}$ is given by

$$dy_{\eta_j,t} = -\eta_j y_{\eta_j,t} dt + b_j \sigma_A dW_{At}. \tag{34}$$

The effects of the state variable y_t on output, consumption, investment in new capital, liquidation of old capital, creation of new investment out of scrapped capital, scrapped capital on-hand, use of scrapped capital to create new investment, wages and the marginal utility of wealth is denoted most easily in terms of the log-deviation of these variables from their respective balanced growth path levels

$$\begin{aligned} \hat{Y}_t &= \frac{A_t^{-1} Y_{Agg,t} - Y_{Agg,BGP}}{Y_{Agg,BGP}} = \sum_{j=1}^{\infty} c_{Y,j} y_{j,t} \\ \hat{C}_t &= \frac{A_t^{-1} C_{Agg,t} - C_{BGP}}{C_{BGP}} = \sum_{j=1}^{\infty} c_{C,j} y_{j,t} \end{aligned}$$

$$
\begin{aligned}
\hat{I}_{Agg,t} &= \frac{A_t^{-1} I_{Agg,t} - I_{Agg,BGP}}{I_{Agg,BGP}} = \sum_{j=1}^{\infty} c_{I,Agg,j} y_{j,t} \\
\widehat{Liq}_{Agg,t} &= \frac{A_t^{-1} Liq_{Agg,t} - Liq_{Agg,BGP}}{Liq_{BGP}} = \sum_{j=1}^{\infty} c_{Liq,j} y_{j,t} \\
\hat{I}_{old,t} &= \frac{A_t^{-1} I_{old,t} - I_{old,BGP}}{I_{old,BGP}} = \sum_{j=1}^{\infty} c_{I,old,j} y_{j,t} \\
\hat{K}_{sc,t} &= \frac{A_t^{-1} K_{sc,t} - K_{sc,BGP}}{K_{sc,BGP}} = \sum_{j=1}^{\infty} c_{K,sc,j} y_{j,t} \\
\hat{I}_{sc,t} &= \frac{A_t^{-1} I_{sc,t} - I_{sc,BGP}}{I_{sc,BGP}} = \sum_{j=1}^{\infty} c_{I,sc,j} y_{j,t} \\
\hat{w}_t &= \frac{A_t^{-1} w_t - w_{BGP}}{w_{BGP}} = \sum_{j=1}^{\infty} c_{w,j} y_{j,t} \\
\hat{m}_t &= \frac{A_t m_t - m_{BGP}}{m_{BGP}} = \sum_{j=1}^{\infty} c_{m,j} y_{j,t}.
\end{aligned}
$$

Similarly, the deviation of the capital density from its balanced growth path can described by

$$
A_t^{-1} f(x,t) - f_{BGP}(x) = \sum_{j=1}^{\infty} e_j(x) y_{j,t}.
$$

The effect of y_t on the liquidation value of installed capital and the trigger level of the firm's state variable x, which leads the firm to pay dividends or to invest, is denoted most easily in terms of the deviation of these two variables from their respective balanced growth path levels (since the balanced growth path of these two variables is not affected by the level of A_t)[9]

$$
\begin{aligned}
\Phi_t &= \Phi_{BGP} + \sum_{j=1}^{\infty} c_{\Phi,j} y_{j,t} \\
p_t &= p_{BGP} + \sum_{j=1}^{\infty} c_{p,j} y_{j,t}.
\end{aligned}
$$

The parameters $c_{Y,j}$, $c_{C,j}$ etc., all still need to be determined. For purposes of normalization we can distinguish two types of eigenstates. The first occurs when the eigenstate does not affect the economic environment (i.e. when $\hat{C}$ and hence also $\hat{m}$, $\hat{r}$ and $\hat{w}$ are all zero, as is the deviation of Φ from its BGP value). The second occurs when one or more parts of the economic environment are affected by the eigenstate.

[9] Here I am still defining p_t in terms of $x = \Phi_{BGP} - B/K$, **not** $x = \Phi_t - B/K$.

In this second case we can, for purposes of normalization, fix $c_{C,j} = 1$.[10] The first order condition for utility maximization by the representative household (18) then indicates that $c_{w,j} = 1$, while the definition of the representative household's marginal utility of wealth implies that $c_{m,j} = -1$.

5.2 Optimal Policy of the Production Firm

Having established the notation for describing the linearized dynamics of the economy, we can now find the production firm's optimal policy in the economic environment created by an eigenstate. For the kind of eigenstate that does not affect the economic environment this is a trivial problem: the firm just follows its balanced growth path policies regarding labor inputs, investment and the payment of dividends. For the kind of eigenstate that does affect the economy economic environment the firm's problem is more interesting and is given by

$$\begin{aligned} V(K_t, B_t, y_{\eta_j,t}) = \max_{\{lkr_s, D_s, I_s\}} \quad m^{-1} \quad & E_t \int_t^\infty e^{-(s-t)(\rho+\eta)} m_s dD_s \\ & s.t. \quad dB_s = [-\mu_{CF}(lkr_z, y_{\eta_j,s}) K_s \\ & \qquad\qquad +h_s p_h(y_{\eta_j,s}) + r_s B_s] ds \\ & \qquad\qquad +\sigma_{CF}(lkr_s) K_s dW_{is} \\ & \qquad\qquad +dD_s + dI_s - h_s \sigma_A dW_{As} \\ & \quad \lim_{s\to\infty} E_t[e^{-(s-t)\rho} m_s B_s] = 0 \\ & \quad dK_s = -\delta K_s ds + dI_s \\ & \quad dy_{\eta_j,s} = -\eta_j y_{\eta_j,s} ds + b_j \sigma_A dW_{As}. \end{aligned}$$

Letting $\sigma_A \to 0$ and thereby imposing certainty equivalence by neglecting the effects of future aggregate shocks, we can find the linear approximation of the firm's value function around $y_{\eta_j,t} = 0$ to be:

$$\begin{aligned} V(K, B, y) &= V(K, B) + \sum_{j=1}^{\infty} [V^{\eta_j}(K, B, y_{\eta_i}) - V(K, B)] \\ &= K[v(x) + \sum_{j=1}^{\infty} (v^{\eta_j}(x, y_{\eta_i}) - v(x))]. \end{aligned}$$

Here $V^\eta(K, B, y_\eta) = K v^\eta(x, y)$ is the value function of the firm in the economic environment created by the

[10] It will turn out that $\hat{C} \neq 0$ for all eigenstates of this type.

eigenstate with the associated eigenvalue η. $V^{\eta}(K, B, y)$ is in turn given by the Bellman equation

$$
\begin{aligned}
(r_{BGP} - \eta y_{\eta} + \epsilon) V^{\eta}(K, B, y_{\eta}) \quad = \quad & -\delta K V_K^{\eta}(K, B, y_{\eta}) \\
& + (-\mu_{CF}(lkr, y_{\eta}) K + (r_{BGP} - \eta y_{\eta}) B) V_B^{\eta}(K, B, y_{\eta}) \\
& + \frac{1}{2} \sigma_{CF}^2 (lkr) K^2 V_{BB}^{\eta}(K, B, y_{\eta}) \\
& - \eta y_{\eta} V_y^{\eta}(K, B, y_{\eta})
\end{aligned}
$$

with the boundary conditions

$$
\begin{array}{lll}
V^{\eta}(K, B, y_{\eta}) = 0 & \text{if} & \Phi_{BGP} + \dfrac{d\Phi(y_{\eta})}{dy_{\eta}} - B/K = 0 \\
V_B^{\eta}(K, B, y_{\eta}) = 1 & \text{if} & V_{BB}^{\eta}(K, B, y_{\eta}) = 0.
\end{array}
$$

The linear approximation of the firm's optimal policy can be obtained by linearizing this Bellman equation around the balanced growth path solution found in Proposition 1. The calculation which lead to this linear approximation are somewhat involved, and are relegated to Appendix K. The result is summarized below in Proposition 4.

Proposition 4 Optimal Policy of the Firm in a Dynamic Environment
When the wage w_t, the marginal utility of wealth m_t, the liquidation value of installed capital Φ_t, the eigenstate $y_{\eta,t}$, and aggregate technology A_t follow the processes

$$
\begin{aligned}
w_t &= (1 + y_{\eta,t}) A_t w_{BGP} \\
m_t &= (1 - y_{\eta,t}) A_t^{-1} m_{BGP} \\
\Phi_t &= \Phi_{BGP} + y_{\eta,t} \frac{d\Phi(y_{\eta})}{dy_{\eta}} \\
dy_{\eta,t} &= -\eta y_{\eta,t} dt \\
dA_t &= g_A A_t dt,
\end{aligned}
$$

the linear approximation of the firm's value function is given by

$$
V^{\eta}(K, B, y_{\eta}) = K[v(x) + y_{\eta} w^{\eta}(x)].
$$

Here $x = \Phi_{BGP} - B/K$ and $w^{\eta}(x)$ is given by the ODE

$$
(\delta + r_{BGP} + \epsilon + \eta) w^{\eta}(x) \quad = \quad \eta v(x)
$$

$$
\begin{aligned}
&+\frac{\partial \mu_{CF}(lkr(x,0),0)}{\partial lkr}\frac{dlkr}{dy_\eta}(x)v_x(x)\\
&+\left[\frac{\partial \mu_{CF}(lkr(x,0),0)}{\partial y}+\eta(\Phi_{BGP}-x)\right]v_x(x)\\
&+\left[\mu_{CF}(lkr(x,0),0)-(\delta+r_{BGP})(\Phi_{BGP}-x)\right]w_x^\eta(x)\\
&+\frac{1}{2}\frac{\partial \sigma_{CF}^2(lkr(x,0))}{\partial lkr}\frac{dlkr}{dy}(x)v_{xx}(x)\\
&+\frac{1}{2}\sigma_{CF}^2(lkr(x,0))w_{xx}^\eta(x)
\end{aligned}
$$

with the boundary conditions

$$
\begin{aligned}
w^\eta(0) &= v_x(0)\frac{d\Phi(y_\eta)}{dy_\eta}\\
w_x^\eta(p_{BGP}) &= 0.
\end{aligned}
$$

The firm picks its optimal effective labor-capital ratio

$$
lkr(x,y_\eta)=lkr(x)+y_\eta\frac{\partial lkr(x,y_\eta)}{\partial y_\eta}
$$

according to the FOC for maximizing the firm's value

$$
\begin{aligned}
&-\frac{\partial^2 \mu_{CF}(lkr(x),y_\eta)}{\partial lkr\ \partial y_\eta}v_x(x)-\frac{\partial \mu_{CF}(lkr(x),y_\eta)}{\partial lkr}w_x(x)=\\
&=\left[\frac{\partial^2 \mu_{CF}(lkr(x),y_\eta)}{\partial lkr^2}v_x(x)+\frac{1}{2}\frac{\partial^2 \sigma_{CF}^2(lkr)}{\partial lkr^2}v_x x(x)\right]\frac{dlkr(x,0)}{dy_\eta}\\
&\qquad+\frac{1}{2}\frac{\partial \sigma_{CF}^2(lkr(x))}{\partial lkr}w_{xx}(x).
\end{aligned}
$$

Furthermore, the firm pays out dividends and invests when

$$
x=p_{BGP}+y_\eta\frac{dp(y_\eta)}{dy_\eta}
$$

where

$$
\frac{dp}{dy_{\eta,t}}=-\frac{w_{xx}(p_{BGP})}{v_{xxx}}.
$$

Finally, if

$$
w(p_{BGP})=0,
$$

the firm is indifferent between investing and paying dividends.

Proof. *See Appendix K.*

5.3 Eigenfunctions of the Capital Density

This section analyzes the dynamics of the capital density $f(x,t)$ when firms follow their optimal policies in the economic environment created by an eigenstate which is not further perturbed by aggregate shocks.

Again, we can use the firm's budget constraint (13) to find the evolution of the firm state variable x according to the diffusion process

$$dx_{it} = \mu_x(x_{it}, y_t)dt + \sigma_x(x_{it})dW_{it} \tag{35}$$

with an absorbing boundary at $x = \Phi_{BGP} - \Phi_t$ and and a reflecting boundary at $x = p_t$. Here the drift $\mu_x(x,y)$ and the instantaneous standard deviation $\sigma_x(x)$ of x are given by

$$\mu_x(x,t) = \mu_{CF}(lkr(x), y) + (r_{BGP} - \eta y + \delta)(x - \Phi_{BGP}) \tag{36}$$

$$\sigma_x(x) = \sigma_{CF}(lkr(x)). \tag{37}$$

The capital density $f(x,t)$ is similarly still governed by the Kolmogorov Forward Equation[11]

$$\partial_t f(x,t) = -\delta f(x,t) + \frac{1}{2}\partial_{xx}[\sigma_x^2(x,t)f(x,t)] - \partial_x[\mu_x(x,t)f(x,t)] \tag{38}$$

this time with the time varying boundary conditions at the liquidation margin and the investment margin

$$0 = f(\Phi_t - \Phi_{BGP}) \tag{39}$$

$$I_{Agg,t} = \mu_x(p_t,t)f(p_t,t) - \frac{1}{2}\sigma_x^2(p_t,t)\partial_x f(p_t,t) + \frac{dp_t}{dt}f(p_t,t). \tag{40}$$

The liquidation rate of capital given by

$$Liq_{Agg,t} = \frac{1}{2}\sigma_x^2(\Phi_t - \Phi_{BGP})\partial_x f(\Phi_t - \Phi_{BGP}) \tag{41}$$

and the sum of dividend payments and investment is given by

$$D_{Agg,t} + I_{Agg,t} = \frac{1}{2}\sigma_x^2(p_t,t)f(p_t,t). \tag{42}$$

We can now look at the evolution of the capital density $f(x,t)$ in the economic environment created by an eigenstate y_η. As mentioned above, there are two possible types of eigenstates is this economy. The first type does not affect the economic environment faced by the firm (i.e., $\hat{C}$, $\hat{m}$ and $\hat{w}$ are all zero, as is the deviation of Φ from its BGP value). In this case the firm's behavior concerning labor input, investment

[11] Again see Appendix J for details.

and dividend payments is no different than on the BGP. The second type of eigenstate does affect the firm's economic environment and thus affects the firm's behavior concerning its labor input, investment and dividend payments. Proposition 5 below describes the behavior of the capital density for both types of eigenstates.

Proposition 5 Eigenfunctions of the Capital Density

In the linearization of the Kolmogorov Forward Equation and its boundary conditions, the production firm capital density associated with the eigenstate y_η evolves according to

$$f(x,t) = A_t e^{-\eta t} e_\eta(x).$$

Here $e_\eta(x)$ is given by the ODE

$$\begin{aligned}
0 \quad = \quad & (\eta - \delta + \frac{1}{2}\partial_{xx}\sigma^2(x) - \partial_x \mu_x(x))e_\eta(x) \\
& +(\partial_x \sigma_x^2(x) - \mu_x(x))\partial_x e_\eta(x) \\
& +\frac{1}{2}\sigma_x^2(x)\partial_{xx} e_\eta(x) \\
& + \left(\frac{1}{2}\frac{d\{\partial_{xx}\sigma_x^2(x,y_\eta)\}}{dy_\eta} - \frac{d\{\partial_x \mu_x(x,y_\eta)\}}{dy_\eta}\right) f_{BGP}(x) \\
& + \left(\frac{1}{2}\frac{d\{\partial_{xx}\sigma_x^2(x,y_\eta)\}}{dy_\eta} - \frac{d\{\partial_x \mu_x(x,y_\eta)\}}{dy_\eta}\right) \partial_x f_{BGP}(x) \\
& +\frac{1}{2}\frac{d\sigma_x^2(x,y_\eta)}{dy_\eta}\partial_{xx} f_{BGP}(x).
\end{aligned}$$

with the boundary condition at the liquidation margin $x = 0$

$$e_\eta(0) = \frac{d\Phi(y_\eta)}{dy_\eta}\partial_x f_{BGP}(0)$$

and the boundary condition at the investment and dividend payment margin $x = p_{BGP}$

$$\begin{aligned}
A_t^{-1}\frac{dI_{Agg,t}}{dy_\eta} \quad = \quad & -\left(\partial_x \mu(p_{BGP})\frac{dp}{dy_\eta} + \frac{d\mu_x(x,y_\eta)}{dy_\eta}\right) f_{BGP}(p_{BGP}) \\
& +\frac{1}{2}\left(\partial_{xx}\sigma_x^2(p_{BGP})\frac{dp}{dy_\eta} + \frac{d\{\partial_x \sigma_x^2(p_{BGP})\}}{dy_\eta}\right) f_{BGP}(p_{BGP}) \\
& -\left(\mu_x(p_{BGP}) - \frac{1}{2}\partial_x \sigma_x^2(p_{BGP})\right)\left(\partial_x f_{BGP}(p_{BGP})\frac{dp}{dy_\eta} + e_\eta(p_{BGP})\right) \\
& +\frac{1}{2}\left(\sigma_x^2(p_{BGP})\frac{dp}{dy_\eta} + \frac{d\sigma_x^2(p_{BGP})}{dy_\eta}\right)\partial_x f_{BGP}(p_{BGP}) \\
& +\frac{1}{2}\sigma_x^2(p_{BGP})\left(\partial_{xx} f_{BGP}(p_{BGP})\frac{dp}{dy_\eta} + \partial_x e_\eta(p_{BGP})\right) \\
& -\eta\frac{dp}{dy_\eta} f_{BGP}(p_{BGP}).
\end{aligned}$$

The boundary condition at the liquidation margin $x = 0$ also implies that

$$A_t^{-1}\frac{dLiq_{Agg,t}}{dy_\eta} = \frac{1}{2}\frac{d\sigma_x^2(0)}{dy}\partial_x f_{BGP}(x) + \frac{1}{2}\sigma_{x,BGP}^2(0)\partial_x e_\eta(0) + \frac{1}{2}\sigma_x^2(0)\partial_{xx} f_{BGP}(0)\frac{d\Phi}{dy_\eta} + \frac{1}{2}\partial_x\sigma_x^2(0)\frac{d\Phi}{dy_\eta}\partial_x f_{BGP}(0).$$

Proof. *See Appendix L.*

5.4 Optimal Policy of the Scrapped Capital Converting Firm

This section describes the scrapped capital converting firm's optimal policy in the economic environment created by an eigenstate. I will again consider two types of eigenstates. In the first type the level of consumption and the interest rate are at their balanced growth path values (or $\hat{C} = \hat{r} = 0$). In this economic environment it is necessary for the price of old capital to be at its balanced growth path level Φ_{BGP} in order to assure that production firms are willing both to invest and to pay dividends (or $\hat{\Phi} = 0$). Thus it is necessary that

$$\hat{\Phi} = (1-\gamma)\hat{K}_{sc} - (1-\gamma)\hat{I}_{sc} = 0$$

or

$$\hat{K}_{sc} = \hat{I}_{sc},$$

from log-linearizing the equation for the marginal product of using scrapped capital in producing the new investment good (33). The scrapped capital converting firm's production function then implies that

$$\hat{I}_{old} = \hat{K}_{sc}.$$

The log-linearized scrapped capital accumulation equation (16)

$$\frac{d\hat{K}_{sc}}{dt} = -(\delta_{sc} + g_A) + \frac{Liq_{BGP}}{K_{sc,BGP}}\widehat{Liq} - \frac{I_{sc,BGP}}{K_{sc,BPG}}\hat{I}_{sc}$$

then implies that, for this first type of eigenstate,

$$\widehat{Liq} = \frac{K_{sc,BGP}}{Liq_{BGP}}\left(\frac{I_{sc,BGP}}{K_{sc,BGP}} - \eta + \delta_{sc} + g_A\right)\hat{K}_{sc}.$$

The second type of eigenstate is associated with the deviation of consumption and the interest rate from their balanced growth path levels. In this case the required return for holding scrapped capital K_{sc} is given by

$$\hat{r} = \left(\frac{r_{BGP} + \delta_{sc}}{r_{BGP}}\right)(\hat{I}_{sc} - \hat{K}_{sc}) + r_{BGP}^{-1}(1-\gamma)\left(\frac{d\hat{K}_{sc}}{dt} - \frac{d\hat{I}_{sc}}{dt}\right)$$

from log-linearizing the return to holding scrapped capital

$$r_t = \frac{MP_K}{\Phi} + \frac{d\Phi/dt}{\Phi} - \delta_{sc}.$$

Here $MP_K = dI_{old}/dK_{sc}$ denotes the marginal production of scrapped capital in place. Using $d\hat{K}_{sc}/dt = -\eta\hat{K}_{sc}$ and $d\hat{I}_{sc}/dt = -\eta\hat{I}_{sc}$ and setting, as a normalization, $\hat{C}_0 = 1$, we then have that

$$\hat{I}_{sc,0} - \hat{K}_{sc,0} = -\frac{\eta}{r_{BGP} + \delta_{sc} + \eta(1-\gamma)}$$

and, from the log-linearized scrapped capital accumulation equation,

$$\frac{Liq_{BGP}}{K_{sc,BGP}}\widehat{Liq}_0 = (\delta_{sc} + g_A - \eta + \frac{I_{sc,BGP}}{K_{sc,BGP}})\hat{K}_{sc,0} - \frac{I_{sc,BGP}}{K_{sc,BGP}}\frac{\eta}{r_{BGP} + \delta_{sc} + \eta(1-\gamma)}.$$

Finally, this yields

$$\hat{\Phi}_0 = \frac{\eta(1-\gamma)}{r_{BGP} + \delta_{sc} + \eta(1-\gamma)}.$$

5.5 Eigenstates

This section uses the aggregate resource constraint to pick out which of the potential eigenstates described above are actual eigenstates of the economy. Recall from Section 5.1 that any candidate eigenstate has the log deviations of output, consumption, investment, liquidation of old capital, scrapped capital in place, investment of scrapped capital in new investment, and output of new capital from scrapped capital given by

$$\begin{aligned}
\hat{Y}_t &= \frac{A_t^{-1}Y_{Agg,t} - Y_{Agg,BGP}}{Y_{Agg,BGP}} = \sum_{j=1}^{\infty} c_{Y,j}y_{j,t} \\
\hat{C}_t &= \frac{A_t^{-1}C_{Agg,t} - C_{BGP}}{C_{BGP}} = \sum_{j=1}^{\infty} c_{C,j}y_{j,t} \\
\hat{I}_{Agg,t} &= \frac{A_t^{-1}I_{Agg,t} - I_{Agg,BGP}}{I_{Agg,BGP}} = \sum_{j=1}^{\infty} c_{I,Agg,j}y_{j,t} \\
\widehat{Liq}_{Agg,t} &= \frac{A_t^{-1}Liq_{Agg,t} - Liq_{Agg,BGP}}{Liq_{BGP}} = \sum_{j=1}^{\infty} c_{Liq,j}y_{j,t} \\
\hat{I}_{old,t} &= \frac{A_t^{-1}I_{old,t} - I_{old,BGP}}{I_{old,BGP}} = \sum_{j=1}^{\infty} c_{I,old,j}y_{j,t} \\
\hat{K}_{sc,t} &= \frac{A_t^{-1}K_{sc,t} - K_{sc,BGP}}{K_{sc,BGP}} = \sum_{j=1}^{\infty} c_{K,sc,j}y_{j,t} \\
\hat{I}_{sc,t} &= \frac{A_t^{-1}I_{sc,t} - I_{sc,BGP}}{I_{sc,BGP}} = \sum_{j=1}^{\infty} c_{I,sc,j}y_{j,t}
\end{aligned}$$

as well as deviation of the capital density from its balanced growth path

$$A_t^{-1} f(x,t) - f_{BGP}(x) = \sum_{j=1}^{\infty} e_j(x) y_{j,t}.$$

with changing boundary margins

$$\begin{aligned}
\Phi_t &= \Phi_{BGP} + \sum_{j=1}^{\infty} c_{\Phi,j} y_{j,t} \\
p_t &= p_{BGP} + \sum_{j=1}^{\infty} c_{p,j} y_{j,t}.
\end{aligned}$$

We can now use what we learned about the optimal policies of the production firms, the scrapped capital converting firms and the behavior of their capital stocks in the economic environment created by an eigenstate to derive the parameters $c_{Y,j}$, $c_{C,j}$, etc., for any particular η and to check if the economy's aggregate resource constraint is satisfied.

5.5.1 Eigenstates with Balanced Growth Path Prices

As indicated above, there are two types of eigenstates in this economy. In the first type, all prices are at their balanced growth path levels and production firms and the representative household follow their balanced growth path policies. An eigenstate of this type can be constructed starting with the behavior of the scrapped capital converting firm. Since we require that $\hat{\Phi}_0 = 0$ it is necessary that, as discussed in Section 5.4, $\hat{I}_{old,0} = \hat{I}_{sc,0} = \hat{K}_{sc,0}$ and

$$\widehat{Liq}_0 = \frac{K_{sc,BGP}}{Liq_{BGP}} \left(\frac{I_{sc,BGP}}{K_{sc,BGP}} - \eta + \delta_{sc} + g_A \right) \hat{K}_{sc,0}.$$

Using the results in Proposition 5 we can now find $e_\eta(x)$ and $\hat{I}_{Agg,0}$ and from (29) that

$$\hat{Y}_{Agg,0} = Y_{Agg,BGP}^{-1} \int_0^{p_{BGP}} \mu_Y(x) e_\eta(x) dx.$$

Having found $\hat{I}_{old,0}$, $\hat{I}_{Agg,0}$ and $\hat{Y}_{Agg,0}$ we can now check if this potential eigenstate is indeed an actual eigenstate by determining if it satisfies the aggregate resource constraint

$$\hat{Y}_{Agg,0} Y_{Agg,BGP} + \hat{I}_{old,0} I_{old,BGP} = \hat{I}_{Agg,0} I_{Agg,BGP}.$$

5.5.2 An Eigenstate with Non-Balanced Growth Path Prices

The second possible type of eigenstate involves non-balanced growth path prices. In this case, we can normalize $\hat{C}_0 = 1$ and let $\hat{C}_t = e^{-\eta t}$ and $\hat{r}_t = -\eta r_{BGP}^{-1}$. In this kind of economic environment $\hat{\Phi}_0$ is determined by both the behavior of the scrapped capital converting firm and the production firm. Both ways of determining $\hat{\Phi}_0$ must agree for an eigenstate to exist.

First, from Section 5.4, $\hat{\Phi}_0$ can be determined by the behavior of the scrapped capital converting firm since

$$\hat{I}_{sc,0} - \hat{K}_{sc,0} = -\frac{\eta}{r_{BGP} + \delta_{sc} + \eta(1-\gamma)}$$

and hence

$$\hat{\Phi}_0 = \frac{\eta(1-\gamma)}{r_{BGP} + \delta_{sc} + \eta(1-\gamma)}. \tag{43}$$

Secondly, for the production firm Proposition 4 implies a $\hat{\Phi}_0$ for any η which leaves the production firm indifferent between paying dividends or investing. Since it can be observed that (43) is increasing in η while, as it turns out, $\hat{\Phi}_0$ for the production firm is decreasing in η, there is one unique eigenstate in which prices deviate from their balanced growth path values.

Using the results in Proposition 5 we can now find $e_\eta(x)$ and $\hat{I}_{Agg,0}$ and from (29) that

$$\hat{Y}_{Agg,0} = Y_{Agg,BGP}^{-1}\left(\int_0^{p_0} \mu_Y(x) e_\eta(x)dx + \int_0^{p_0} \frac{\partial \mu_Y(x)}{dy_\eta} f_{BGP}(x)dx\right)$$

Having found $\hat{I}_{old,0}$, $\hat{I}_{Agg,0}$ and $\hat{Y}_{Agg,0}$ we can now check if this potential eigenstate is indeed an actual eigenstate by determining if it satisfies the aggregate resource constraint

$$\hat{Y}_{Agg,0} Y_{Agg,BGP} + \hat{I}_{old,0} I_{old,BGP} = \hat{C}_0 C_{BGP} + \hat{I}_{Agg,0} I_{Agg,BGP}.$$

5.6 Perturbation of the Eigenstates by Aggregate Shocks

The economy's aggregate state variable is given by the level of aggregate technology A_t and the eigenstates $\{y_{\eta_n}\}_{n=1}^{\infty}$. When there are no shocks to aggregate technology, we know from the above analysis that the economy's aggregate state variable evolves according to

$$\begin{aligned} dA_t &= g_A A_t dt \\ dy_{\eta_j,t} &= -\eta_j y_{\eta_j,t} dt \quad \forall j \geq 1. \end{aligned}$$

When there are aggregate technology shocks, the economy's aggregate state variable evolves according to

$$\begin{aligned} dA_t &= g_A A_t dt + \sigma_A A_t dW_{At}. \\ dy_{\eta_j,t} &= -\eta_j y_{\eta_j,t} dt + b_j \sigma_A dW_{At} \quad \forall n \geq 1. \end{aligned}$$

where the additional b_j terms need to be determined.

5.6.1 Optimal Hedging of Aggregate Shocks by the Production Firm

We can find the b_j's by projecting the non-balanced growth path capital density after the arrival of a shock to aggregate technology on to the eigenstates of the economy. To do this, however, we first need to understand how the capital density across production firms is affected by optimal hedging of shocks to the economy's eigenstates.

Proposition 6 Optimal Hedging of Aggregate Shocks by Production Firms
The optimal hedge of a production firm of a shock to the eigenstate y_{η_j} is given by

$$h_\eta(x) = -\frac{b_j w^{\eta_j}(x)}{v_{xx}(x)}$$

if this eigenstate affects prices. Shocks to eigenstates that do not affect prices are not hedged by production firms.
Proof. *See Appendix M.*

5.6.2 Finding the Effect of Aggregate Shocks on the Economy's Eigenstates

Using our representation of the aggregate state variable, we know that the production firm's capital density is given by

$$f(x,t) = A_t \left(f_{BGP}(x) + \sum_{j=1}^{\infty} e_j(x) y_{j,t} \right)$$

with the boundaries

$$\begin{aligned} \Phi_t - \Phi_{BGP} &= \sum_{j=1}^{\infty} c_{\Phi,j} y_{j,t} \\ p_t &= p_{BGP} + \sum_{j=1}^{\infty} c_{p,j} y_{j,t}. \end{aligned}$$

and that the scrapped capital converting firm's capital stock is given by

$$K_{sc,t} = A_t \left(K_{sc,BGP} + \sum_{j=1}^{\infty} c_{K,sc,j} y_{j,t} \right).$$

When the economy is perturbed by an aggregate technology shock, several things happen. First, technology A_t jumps by $A_t dW_{A,t}$, which means that the balanced growth path production firm's capital density jumps by $f_{BGP}(x) A_t dW_{A,t}$ and the balanced growth path scrapped capital converting firm's capital jumps by $K_{sc,BGP} A_t dW_{A,t}$. What happens to the actual production firm's capital density $f(x,t)$ and scrapped capital converting firm's capital stock $K_{sc,t}$? The capital density is perturbed by the fact that production firms hedge aggregate shocks, with the production firm capital density moving according to

$$df(x,t) = \sum_{j=1}^{\infty} dy_j [h'_j(x) f(x) + h_j(x) f'(x,t)]$$

The actual level of $K_{sc,t}$ is not affected by the technology shock. We thus have that the b_j's are given by

$$\begin{aligned} f_{BGP}(x) &= \sum_{j=1}^{\infty} b_j e_j(x) - \sum_{j=1}^{\infty} b_j [h'_j(x) f_{BGP}(x) + h_j(x) f'_{BGP}(x)] \\ K_{sc,BGP} &= \sum_{j=1}^{\infty} b_j \frac{dK_{sc}}{dy_j} \end{aligned}$$

Numerically the b_j's can be found by running a OLS regression using the first N eigenstates for a sufficiently large N.

5.7 Numerical Results

This section provides numerical results on the dynamic behavior of four economies. Economy 1 is the same economy for which we discussed the balanced growth path properties in Section 4.6. Economies 2-4 are the same as Economy 1 except for deviations along a single dimension of parameter space. Specifically, in Economy 2 the parameter γ has been changed to 0.999 and in Economy 3 to 0.001, while in Economy 4 Φ_{BGP} has been raised to 0.95. All other parameters are the same as in Section 4.6 (i.e. $\epsilon = 0.05$, $\rho = 0.05$, $\sigma_Y = 0.5$, $\gamma = 0.5$ and $g_A = 0$). Table 1 summarizes the parameterization of the four economies.

The solid lines in Figures 13-20 display the impulse response functions of Economy 1 for a permanent 1% aggregate technology shock, while the dashed lines display the equivalent impulse response functions for a benchmark economy without any financial contracting problems for comparison.

Table 1: Parameterization of Economies 1-4

Parameter	Ec. 1	Ec. 2	Ec. 3	Ec. 4
Φ_{BGP}	0.700	0.700	0.700	0.950
γ	0.500	0.999	0.001	0.500

Figure 13 shows the impulse response of output. The first thing to notice is that the debt constrained economy's output response is hump shaped, a phenomenon not found in the economy without financial constraints. The qualitative properties of the hump-shaped response of output are broadly consistent with the VAR evidence, with the output response building over the first two years and reaching its peak in the third year.[12] This contrasts with the larger immediate jump in output followed by exponential reversion to the balanced growth path in the model with no financial frictions. However, the quantitative importance of the hump-shaped output response for this calibration is not particularly large, with the maximum difference in the output responses for the two economies 0.06% for a 1% shock. Figure 14 shows that the impulse response of output is associated with changes in the Solow residual which are not accounted for by changes in aggregate technology. We can see that the hump-shaped output response is due to an initial lag in the Solow residual behind the level of aggregate technology, with firms waiting for their financial position to improve to produce more output.

Figure 15 shows the impulse response of consumption. The initial jump in consumption is almost identical for the economies with and without financial constraints, but the consumption level of the financially constrained economy grows towards the new balanced growth level of consumption at a slightly faster rate. Figure 16 shows the impulse response of new investment, with investment jumping slightly less in the financially constrained economy and decaying at about the same rate towards its new balanced growth level.

Figure 17 shows the impulse response of liquidation. We can see that liquidation is initially reduced with the impact of the aggregate technology shock then jumps back up quickly, but to less than its new balanced growth path level. After two quarters the liquidation rate starts to fall again and dips to its initial level after two years, after which it recovers slowly to its new balanced growth path level. The reduction of liquidation below its new balanced growth path level explains how the capital stock of the financially constrained economy can grow to its new balanced growth path level at a faster rate than in the economy without financial constraints,

[12] See, for instance, King, Plosser, Stock and Watson (1991).

even though new investment is lower.

Figure 18 shows the impulse response function for the creation of new investment from the stock of scrapped capital. We can see that the general pattern from the impulse response function of liquidation carries through to this impulse response, with an initial slight increase followed by a decrease from the end of the first year until the end of year 3. After year 3 the creation of new investment out of scrapped capital increases toward its new balanced growth path level. Figure 19 shows that the impulse response function for the stock of scrapped capital is almost exactly the same as the impulse response function for the creation of new investment out of scrapped capital, except that the initial level of K_{sc} is predetermined and hence starts out a t a lower level than I_{old}.

Figures 20-23 show impulse response functions for financial variables and creation and liquidation margins. Figure 20 shows the evolution of firm debt levels, and we can see that aggregate debt falls strongly in response to a positive technology shock. This effect is quite large, with the face value of outstanding debt falling by 1.7% (or 0.6% of output) in response to the 1% technology shock. After the initial fall, aggregate debt levels then rise to their new balanced growth path levels, but quite slowly, only reaching their pre-shock levels after about five years.

Figure 21 shows that this movement in the aggregate debt level is in part due to change in the debt capacity of firms, with debt capacity falling by 0.17% in response to the 1% technology shock. However, Figure 22 shows that the movement of the investment and dividend payment margin is more important in explaining the fall in overall debt since the movement of this margin is about five times as large as the movement of the liquidation margin.

We can see what is going on here in more detail in Figure 23, which displays the optimal hedge the production firm. This figure shows the inflow of financial wealth in response to the positive technology shock. It can be seen that the resulting decrease in a firm's debt-capital ratio depends on the firm's leverage. For the lowest debt firm, the 1% technology shock decreases the debt-capital ratio by somewhat more than 0.5%, while for the highest debt firm the effect is substantially lower, with a decrease in the debt-capital ratio of 0.14%.

Figures 24-34 and Figures 35-43 show the same impulse responses and the optimal firm hedging policy for Economy 2 and Economy 3, respectively, both with the same $\Phi_{BGP} = 0.7$ value but with different production functions for the scrapped capital converting firm. In general these figures are very similar to Figures 13-23 and show that variation in γ is not very important for the behavior of the economy, at least not for this value of Φ_{BGP}.

Figure 24 shows the impulse response function of output for Economy 2, i.e. for an economy with almost constant debt capacity and very little scrapped capital on-hand. The hump-shaped impulse response function of output is again apparent, albeit with a slightly larger initial response of output and with the output response building for a shorter time period, somewhat over one year. Figure 25 shows the impulse response function for the component of the Solow residual not accounted for by the permanent technology shock. Again, this impulse response function for Economy 2 is similar to the one found for Economy 1, except that the size of the component of the Solow residual not accounted for by aggregate technology is slightly smaller.

The consumption impulse response displayed in Figure 26 again looks very similar to that of the previous economy, with consumption jumping the same amount as in the economy without financial constraints and then converging faster to its new balanced growth path level. The impulse response for investment, seen in Figure 27, is slightly different. We can see that the impulse response function for investment in the economy with almost constant debt capacity is more like the impulse response function for investment in the economy without financial constraints, except for a slight hump-shape and a more rapid decline of investment in the financially constrained economy. The impulse response function for liquidation, seen in Figure 28, is also somewhat different, especially in its initial response, with liquidation spiking up, not down, in response to the permanent technology shock. After this initial spike liquidation declines for about three years to a level below its new balanced growth path level and then rises slowly to its new balanced growth path level. In Figure 29 we can see the same pattern in the impulse response for the creation of new investment from the stock of scrapped capital, except that the spike up is delayed somewhat. Figure 30 shows the same behavior for the stock of scrapped capital on-hand.

Figures 31-34 show the impulse responses for debt, liquidation and investment margins as well as the production firm's optimal hedge for Economy 2. Figure 31 displays the impulse response function of debt. Again, the behavior of debt here looks very similar to the behavior of debt in Economy 1, with the fall in debt in response to the permanent technology shock slightly larger in the economy with an almost constant debt capacity. Figure 32 shows that debt capacity is indeed almost constant in Economy 2 and Figure 33 show that the movement of the investment margin is much larger. Figure 34 shows that the optimal firm hedge is again broadly similar to that of the previous economy, except that high debt-capital firms now take on aggregate risk, paying out financial resources to the representative household in the event of a positive aggregate technology shock. This leads to the upward spike in liquidation in response to a positive technology shock seen in Figure 28.

Figure 35 shows the impulse response function of output for Economy 3, that is for an economy in which debt capacity is sensitive to both investment of scrapped capital I_{sc} and scrapped capital on-hand K_{sc}. The hump-shaped impulse response function of output is again apparent, this time with a slightly smaller initial response of output and with the output response building for a longer time period, about three years. Figure 36 shows that the impulse response function for the component of the Solow residual is not accounted for by the permanent technology shock. Again, this impulse response function for Economy 3 is similar to the one found for Economy 1, except that the size of the component of the Solow residual not accounted for by aggregate technology is slightly larger, but not significantly so.

The consumption impulse response displayed in Figure 37 again looks very similar to that of Economy 1 and Economy 2, with consumption jumping the same amount as in the economy without financial constraints and then converging faster to its new balanced growth path level. The impulse response for investment, seen in Figure 38, is also very similar, with investment jumping somewhat less initially than in Economy 1. The impulse response function for liquidation, seen in Figure 39, is again similar, except that the initial downward spike in liquidation is considerably larger and the subsequent level of liquidation is lower. In Figures 40 and 41 that the lower level of liquidation also shows up in lower levels of new investment from the stock of scrapped capital a reduced level of scrapped capital on-hand.

Figures 42-45 show the impulse responses for debt, liquidation and investment margins as well as the production firm's optimal hedge for Economy 3. Figure 42 shows that the impulse response of debt is large, with debt falling by 2% in response to a 1% aggregate technology shock, a slightly larger response than in Economy 1. Figure 43 shows that debt capacity moves significantly in Economy 3, as is to be expected with $\gamma = 0.001$, with debt capacity falling in response to the positive aggregate technology shock due to the increase in liquidation and its impact on the stock of scrapped capital on-hand. Figure 44 shows that the movement of the investment margin nevertheless is still much larger, contributing more to the fall in aggregate debt outstanding. Figure 45 shows that the optimal firm hedge is again similar to that of Economy 1, except that high debt-capital firms now are even more financially exposed to aggregate technology shocks, receiving financial resources from the representative household in the event of a positive aggregate technology shock. This leads to the downward spike in liquidation seen in Figure 39.

Finally, Economy 4 deviates from Economy 1 by having a higher balanced growth path debt capacity $\Phi_{BGP} = 0.95$. As a consequence of the lower cost of liquidation in this economy, Economy 4 has a much higher balanced growth path rate of liquidation and a much higher balanced growth path level of scrapped capital on-hand than Economy 1. The importance of this difference for impulse response functions will become apparent below.

Figure 46 shows the impulse response function of output for Economy 4. The hump-shaped appearance of the impulse response function is much stronger than for Economies 1-3. The initial jump in output is smaller than in the previous three economies and the rise in output that give rise to the hump-shape is considerably larger, to 1.14% above its pre-shock level, not the 1.04% we have seen in the previous three economies. The time it takes for the hump to peak is similar to that of Economy 1, around three years. Figure 47, however, shows the impulse response function for the component of the Solow residual not accounted for by the permanent technology shock is still similar to that displayed in Economies 1-3.

The consumption impulse response displayed in Figure 48 again looks very similar to that of the previous three economies, except that this time consumption jumps somewhat more. The impulse response for investment, seen in Figure 49, is considerably different than any impulse response from Economies 1-3. We can see that investment in Economy 4 is far less responsive to a technology shock, with investment rising less than 1% in response to an aggregate technology shock of 1%, and with investment falling after its initial jump before rising to its new balanced growth path level. Figure 50 shows that main force behind capital accumulation in response to an aggregate technology shock in economy 4 is a large reduction in the level at which capital is liquidated due to financial distress, with an initial spike downward in liquidation that is followed by a very slow rise in liquidation to its new balanced growth path level. Figures 51 and 52 show the same general behavior for the impulse responses of I_{old} and K_{sc}, with these variables at first falling and then only slowly rising to their new balanced growth path levels.

Figures 53-55 show the impulse response functions for debt, debt capacity, and the dividend payment margin. We can see a very large -5% fall in debt in response to the 1% aggregate technology shock, and again this change in the debt level is more due to the movement of the dividend payment margin than movement of firm debt capacity. Finally, Figure 56 shows that the optimal hedge of the production firm.

What do we learn from the general equilibrium response of these four economies to an aggregate technology shock? All four economies display a hump-shaped impulse response for output, a feature that also arises for other parameterizations of the model. Other authors, among others Kyotaki and Moore (1997), Carlstrom and Fuerst (1997), Bernanke, Gertler and Gilchrist (1998) and Cooley and Quadrini (1998), have also observed hump-shaped output responses for models with financial constraints. For these papers the intuition behind hump shaped output responses is that financially better off firms produce more output, but that it takes time for positive aggregate shocks to improve the financial position of firms.

This is not quite the correct intuition for this model, although it does capture a significant part of the intuition behind the hump shaped impulse response of output. As we saw for all four economies, a component of the Solow residual is not explained by the movement of aggregate technology. Instead, this component of the Solow residual is due to the interaction of time-varying firm policies and the time-varying capital densities of firms. Nevertheless, this component of the Solow residual still affects the firm's decision about how much output to produce. In particular, the initial undershooting of the Solow residual below its new balanced growth path level means that firms, given the prices they face, choose to produce less.

However, this is not all that is going on here. Another aspect is that an economy's ability to intertemporally smooth production and consumption is in part determined by its stock of scrapped capital on hand. With a small scrapped capital stock on-hand an economy is not able to smooth production and consumption by changing this capital stock much, while with a large scrapped capital stock on-hand an economy can do exactly this. We can see this phenomenon especially clearly in the behavior of Economy 4: here movements in the stock of scrapped capital on-hand played an important part in the dynamics of the production firm capital density, with the increase in the level of the production firm capital stock taking place mostly through a reduction in liquidation. Without this large stock of scrapped capital on-hand prices move to offset many of forces that give rise to the hump-shaped output response.

In addition, there is no hump-shaped behavior in the level of outstanding aggregate debt, such as in Carlstrom and Fuerst's paper. Instead, in this model the hump-shaped impulse response of output is due to the effects of general equilibrium price movements on firm policies (and the interaction of these firm policies with the evolution of the capital density across firms) and is not reflected in the evolution of the level of aggregate debt.

Finally, it should be noted that giving firms access to fairly priced hedging opportunities against aggregate shocks does not eliminate the importance of financial constraints for business cycle dynamics. In particular, firms do not insure away the affect of aggregate shocks on their financial position. Instead, because aggregate shocks change the relative value of firms with different debt-capital ratios, firms exploit the opportunity to hedge aggregate shocks in a manner that has an effect on business cycle dynamics. For instance, as we were able to see in the dynamics of Economies 3 and 4, firms may choose to hedge aggregate shock

6 Conclusion

This paper analyzes the general equilibrium dynamics of an economy with debt constrained firms exposed to idiosyncratic production uncertainty in a general equilibrium setting. I find that firm financial constraints affect the dynamics of the economy's response to aggregate technology shocks by creating a component of the

Solow residual that is not accounted for by aggregate technology. This time-varying productivity movement induces intertemporal substitution of labor supply and gives rise to a hump-shaped impulse response for output.

A useful avenue for research would be the incorporation of sticky prices and sticky wages into the model. This would allow for potentially larger effects of firm financial constraints on firm output since firms would no longer be at least in part insured again aggregate technology shocks by offsetting movements in wages.

Proof of Proposition 1 and 2

$V(K,B) = Kv(x)$ results from the fact that both the production technology and the debt capacity of the firm are linear in capital. Given the same sequence of idiosyncratic shocks W_{xt}, any firm with capital K can reproduce K times the dividends of a firm with capital 1 if both firms start with the same state variable x and follow the same policies as a function of x. Since $V(K,B)$ is linear in dividends, we have that $V(K,B) = Kv(x)$. It is useful to note that $V(K,B) = Kv(\Phi_{BGP} - B/K)$ implies

$$\begin{aligned} V_K(K,B,t) &= v(x) + (\Phi_{BGP} - x)v_x(x) \\ V_B(K,B,t) &= -v_x(x) \\ V_{BB}(K,B,t) &= K^{-1}v_{xx}(x). \end{aligned}$$

i.) To derive the ODE in v(x), write the value function as

$$\begin{aligned} V(K_t,B_t) &= m_t^{-1}E_t\int_t^T e^{-(\rho+\epsilon)s}m_s dD_s \\ &= \lim_{\Delta\to 0} E_t(D_{t+\Delta} - D_t) + e^{-(\rho+\epsilon+g_A)\Delta}E_tV(K_{t+\Delta},B_{t+\Delta}). \end{aligned}$$

Differentiating on both sides w.r.t. Δ, letting $\Delta \to 0$ and using Ito's Lemma, we have that

$$\begin{aligned} 0 = {} & dD_t \\ & -(\rho+\epsilon+g_A)V(K_t,B_t)dt \\ & +(-\delta K_t dt + dI_t)V_K(K_t,B_t) \\ & +(-\mu_{CF}(lkr)K_t dt + dD_t + dI_t + r_{BGP}B_t dt)V_B(K_t,B_t) \\ & +(\frac{1}{2}\sigma_{CF}^2(lkr)K_t^2 dt)V_{BB}(K_t,B_t). \end{aligned}$$

Note that the first order conditions for dividend payment and investment are:

$$1 = -V_B(K,B)$$

$$V_K(K,B) = -V_B(K,B),$$

and that the first order condition for the maximization of the appreciation of $V(K,B)$ by choice of the labor-capital ratio is:

$$\frac{\partial\mu_{CF}(lkr)/\partial lkr}{\partial\sigma^2_{CF}(lkr)/\partial lkr}=\frac{1}{2}K\frac{V_{BB}(K,B)}{V_B(K,B)}.$$

Rewriting the equation for the value function using $V(K,B)=Kv(x)$, as well as using the derivatives this functional form implies, and noting that $r_{BGP}=\rho+g_A$, we have the ODE:

$$(r_{BGP}+\epsilon)v(x)=[\mu_{CF}(lkr)-(r_{BGP}+\delta)(\Phi_{BGP}-x)]v_x(x)+\frac{1}{2}\sigma^2_{CF}(lkr)v_{xx}(x). \qquad (44)$$

Similarly, the first order conditions for dividend payment and investment can be rewritten as:

$$\begin{aligned} v_x(p_{BGP}) &= 1 \\ v(I_{BGP}) &= (1-\Phi_{BGP}+I_{BGP})v_x(I_{BGP}) \end{aligned}$$

as can the first order condition for the choice of lkr:

$$\frac{\partial\mu_{CF}(lkr)/\partial lkr}{\partial\sigma^2_{CF}(lkr)/\partial lkr}=-\frac{1}{2}\frac{v_{xx}(x)}{v_x(x)}.$$

ii.) As shown in i.), one of the boundary conditions of $v(x)$ is that $v_x(p_{BGP})=1$ (this is also known at the "value matching condition"). The boundary condition $v(0)=0$ follows directly from the fact that firms become worthless to equity owners when $B=\Phi_{BGP}K$. The boundary condition $v_{xx}(p_{BGP})=0$ is a requirement for the optimal choice of p_{BGP} (this boundary condition is also known as the "smooth pasting condition"). Note first that the ODE for $v(x)$ is linear in $v(x)$, so that if $w(x)$ fulfills the ODE, then any multiple of $w(x)$ fulfills the ODE also. The boundary condition $v_x(p_{BGP})=1$ for the payment of dividends determines $v(x)$. The boundary condition $v_x(p_{BGP})=1$ does not, however, determine the optimal p_{BGP}. Instead, the optimal p_{BGP} is determined by $v_{xx}(x)=p_{BGP}$: from the family of all possible value functions $w^p(x)$, one for each possible p, including the non-optimal ones, the actual $v(x)=w^p(x)$ is the one the maximal one, i.e. $v(x)=\max_p w^p(x)$, where the condition $w^p(p)=1$ is imposed. Then note that if $w^p_{xx}(p)>0$, lowering p will raise $w^p(x)$ for all $x<p$ (using the fact that if $w^p(x)$ solves the ODE, then $qw^p(x)$ solves the ODE also). Conversely, if $w^p_{xx}(x)<0$, then raising p will raise $w^p(x)$ for all $x<p$. So $w^p_{xx}(p)=0$ is the condition for optimal p, and we have that $v_{xx}(p_{BGP})=0$.

Properties of $v(x)$:

First note that the scrap value of capital is strictly less than the discounted value of the firm's cash flows from that capital without any agency problems:

$$\Phi_{BGP}<\int_t^\infty e^{-(r_{BGP}+\delta)s}\bar{\mu}_{CF}ds,$$

(where $\bar{\mu}_{CF} = \bar{\mu}_Y^{1/(1-\alpha)} w_t^{-\alpha/(1-\alpha)} \alpha^{\alpha/(1-\alpha)} A_t^{\alpha/(1-\alpha)}$ is the upper bound on the possible cash flow per unit capital) since a firm with contracting and agency problems can not utilize capital more efficiently that a firm without agency problems. Hence, we have that $[\bar{\mu}_{CF} - \Phi_{BGP}(r_{BGP} + \delta)] > 0$.

Property i.) $v(x)$ is twice differentiable since $v(x)$ can be constructed by finding $v_{xx}(x)$ using the ODE from $v(x)$ and $v_x(x)$, starting out from $v(0) = 0$ and $v_x(0)$.

Property ii) $v(x) \geq 1$ for $0 \leq x \leq p$ follows from the ODE, the conditions for the choice of lkr, and the fact that $[\bar{\mu}_{CF} - \Phi_{BGP}(r_{BGP} + \delta)] > 0$. Recall that the ODE is

$$(r_{BGP} + \delta + \epsilon)v(x) = [\mu_{CF}(lkr(x)) + (r_{BGP} + \delta)(x - \Phi_{BGP})]v_x(x) + \frac{1}{2}\sigma_{CF}^2(lkr(x))v_{xx}(x). \tag{45}$$

Differentiating the ODE w.r.t. x we have that

$$\epsilon v_x(x) = [\mu_{CF}(lkr(x)) + (r_{BGP} + \delta)(x - \Phi_{BGP})]v_{xx})(x) + \frac{1}{2}\sigma_{CF}^2(lkr(x))v_{xxx}(x). \tag{46}$$

Evaluating the ODE at $x = 0$, we have, since $v(0) = 0$, that

$$v_{xx}(0) = -\frac{2}{\sigma_{CF}^2(lkr(0))}[\mu_{CF}(lkr(0)) - (r_{BGP} + \delta)\Phi_{BGP}]v_x(0). \tag{47}$$

The case of $v(0) = v_x(0) = v_{xx}(0)$ is of no interest, since the ODE then implies that $v(x) = 0$ for all x, and $v(x)$ could never fulfill the boundary condition $v_x(p_{BGP}) = 1$. Since due to limited liability $v(x) \geq 0$, we hence need that $v_x(0) > 0$. Since $[\bar{\mu}_{CF} - (r_{BGP} + \delta)\Phi_{BGP}] > 0$, it is possible to choose lkr so that $[\mu_{CF}(lkr(0)) - (r_{BGP} + \delta)\Phi_{BGP}] > 0$, and hence it is possible that $v_{xx}(0) < 0$. If the firm chooses a lkr so that $[\mu_{CF}(lkr(0) - \Phi(r_{BGP} + \delta)] \leq 0$, then, on the other hand, from (47) we know that $v_{xx}(0) \geq 0$. But the firm will never choose such a lkr: with $v_{xx}(0) \geq 0$ the firm is risk neutral or risk seeking, and the firm will always choose the highest expected return technology it can, so that in this case $\mu_{CF}(lkr(0)) = \bar{\mu}_{CF}$. But, since $[\bar{\mu}_{CF} - (r_{BGP} + \delta)\Phi_{BGP}] > 0$, using this technology the firm would not be risk neutral or risk seeking. Therefore, by contradiction, we know that $v_{xx}(0) < 0$.

By evaluating (46), the ODE differentiated w.r.t. x, at $x = 0$, we have that

$$\frac{1}{2}\sigma_{CF}^2(lkr(0))v_{xxx}(0) = \epsilon v_x(0) - [\mu_{CF}(lkr(0)) - (r_{BGP} + \delta)\Phi_{BGP}]v_{xx}(0). \tag{48}$$

Since $v_x(0) > 0$, $[\mu_{CF}(lkr(0)) - (r_{BGP} + \delta)\Phi_{BGP}] > 0$, and $v_{xx}(0) < 0$, we have that $v_{xxx}(0) > 0$. More generally, we know from the ODE (45) that

$$\frac{1}{2}\sigma_{CF}^2(lkr(x))v_{xx} = -[\mu_{CF}(lkr(x)) + (r_{BGP} + \delta)(x - \Phi_{BGP})]v_x(x) + (r_{BGP} + \delta + \epsilon)v(x) \tag{49}$$

and from (46) that

$$\frac{1}{2}\sigma_{CF}^2(lkr(x))v_{xxx}(x) = \epsilon v_x(x) - [\mu_{CF}(lkr(x)) + (r_{BGP}+\delta)(x-\Phi_{BGP})]v_{xx}(x). \tag{50}$$

We know that $v_{xx}(x) \leq 0$ for $0 \leq x \leq p_{BGP}$, since p_{BGP} is set so that $v_{xx}(x) > 0$ does not occur. From this and from $v(0) = 0$ and $v_x(0) > 0$ we can conclude that $v(x) \geq 0$, $v_x(x) > 0$, for $0 \leq x \leq p$, and, from the argument above about the choice of $\mu_{CF}(lkr)$, that $[\mu_{CF}(lkr(x)) + (r_{BGP}+\delta)(x-\Phi_{BGP})] > 0$. Therefore, we can conclude from (50) that $v_{xxx}(x) > 0$. Hence $v_{xx}(x)$ rises from a negative value at $x = 0$ and $v(x)$ is concave for some interval above zero. Furthermore, we know that $v_x(0) > 1$ from $v_x(p) = 1$: since $v_{xx}(0) < 0$, we know that the optimal p is greater than zero since the value of the value function could be increased by raising p above zero. Since $v_x(p_{BGP}) = 1$, we have from $v_{xx}(x) < 0$ for $0 \leq x < p$ that $v_x(0) > 1$.

There exists a $p_{BGP} > 0$ such that $v_{xx}(p_{BGP}) = 0$: if there were no $p_{BGP} > 0$ then we would have $v_{xx}(x) < 0$ for all $x \geq 0$. But then, from differentiating

$$(r_{BGP}+\delta+\epsilon)v(x) = [\mu_{CF}(lkr(x)) + (x-\Phi)(r_{BGP}+\delta)]v_x(x) + \frac{1}{2}\sigma_x^2(lkr(x))v_{xx}(x),$$

w.r.t x, we have that

$$\epsilon v_x(x) = [\mu_{CF}(lkr(x)) + (x-\Phi)(r_{BGP}+\delta)]v_{xx}(x) + \frac{1}{2}\sigma_x^2(lkr(x))v_{xxx}(x),$$

and since $[\mu_{CF}(lkr(x)) + (x-\Phi)(r_{BGP}+\delta)] > 0$ and $v_{xx}(x) < 0$ for $x < p_{BGP}$, we have that

$$\frac{1}{2}\sigma_{CF}^2(lkr(x))v_{xxx}(x) > \epsilon v_x(x).$$

Knowing that $v_x(x) \geq 1$, we see that

$$v_{xxx}(x) > \frac{2\epsilon}{\bar{\sigma}_x^2},$$

where $\bar{\sigma}_x^2$ is the maximum possible σ_x^2 which occurs when the firm acts in a risk neutral manner. Hence, from $v_{xx}(x) = v_{xx}(0) + \int_0^x v_{xxx}(s)ds$, that $v_{xx}(M) \geq 0$ for $M \geq \frac{v_{xx}(0)\epsilon}{2\bar{\sigma}^2}$. But this contradicts the assumption that $v_{xx}(x) < 0$ for all $x \geq 0$. Hence there is an inflection point in $v(x)$ if $\epsilon > 0$.

Proof of Proposition 3

The firm's risk aversion declines with x or, put differently, $d[-v_{xx}(x)/v_x(x)]/dx < 0$ for $0 \leq x < p_{BGP}$. Since

$$\frac{d[-v_{xx}(x)/v_x(x)]}{dx} = \frac{-v_{xxx}(x)}{v_x(x)} + \left(\frac{-v_{xx}(x)}{v_x(x)}\right)^2,$$

we need to show that $v_{xxx}(x)/v_x(x) > [-v_{xx}(x)/v_x(x)]^2$ for the firm's risk aversion to fall with x. This is the same as showing that $v_{xxx}(x)v_x(x) > [v_{xx}(x)]^2$. Now, from (49), we have that

$$v_{xx}(x) = \frac{2}{\sigma^2_{CF}(lkr(x))}[(r_{BGP} + \delta + \epsilon)v(x) + (\mu_{CF}(lkr(x)) + (x - \Phi)(r_{BGP} + \delta))v_x x(x)]$$

and from (50) that

$$v_{xxx}(x) = \frac{2}{\sigma^2_{CF}(lkr(x))}[\epsilon v_x(x) - (\mu_{CF}(lkr(x)) + (x - \Phi)(r_{BGP} + \delta))v_{xx}(x)],$$

so

$$\begin{aligned}
[v_{xx}(x)]^2 &= \frac{2}{\sigma^2_{CF}(lkr(x))}(r_{BGP} + \delta + \epsilon)v(x)v_{xx}(x) \\
&\quad - \frac{2}{\sigma^2_{CF}(lkr(x))}[\mu_{CF}(lkr(x)) + (x - \Phi)(r_{BGP} + \delta)]v_x(x)v_{xx}(x)
\end{aligned}$$

and

$$\begin{aligned}
v_{xxx}(x)v_x(x) &= \frac{2\epsilon}{\sigma^2_{CF}(lkr(x))}[v_x(x)]^2 \\
&\quad - \frac{2}{\sigma^2_{CF}(lkr(x))}[\mu_{CF}(lkr(x)) + (x - \Phi)(r_{BGP} + \delta)]v_{xx}(x)v_x(x).
\end{aligned}$$

So what is $(v_{xxx}(x)v_x(x))/(v_{xx}(x))^2$? We can see that

$$\frac{v_{xxx}(x)v_x(x)}{v_{xx}(x)^2} = \frac{A + B}{C + B},$$

where

$$\begin{aligned}
A &= \epsilon v_x(x)^2 \\
B &= -(\mu_{CF}(lkr(x)) + (x - \Phi)(r_{BGP} + \delta))v_{xx}(x)v_x(x) \\
C &= r_{BGP} + \delta + \epsilon)v(x)v_{xx}(x).
\end{aligned}$$

Since $A > 0, B > 0, C < 0$ and $C + B > 0$, we can see that $(A + B)/(C + B) > 1$. Thus we have shown that $d[-v_{xx}(x)/v_x(x)]/dx < 0$.

$dlkr/dx \geq 0$ follows directly from the FOC for the choice of lkr and the observation that the firm's risk aversion falls with x.

Kolmogorov Forward Equation

The Kolmogorov forward equation is well-known (see for instance Karlin and Taylor (1975), Section 15.5), and is traditionally written as

$$\partial_t f(x,t) = \frac{1}{2}\partial_{xx}[\sigma_x^2(x,t)f(x,t)] - \partial_x[\mu_x(x,t)f(x,t)],$$

where $\mu_x(x,t)$ is the drift of x, denoted in the rest of the paper as $g_x(t)$, and $\sigma_x^2(x,t)$ is the instantaneous variance of x. The modified KFE used in this paper is obtained by adding the term $-\delta f(x,t)$ to take depreciation into account.

One way to understand the KFE is to observe that the flow across any point x is given by

$$\text{Flow}(x,t) = (\mu(x,t) - \frac{1}{2}\partial_x \sigma_x^2(x,t))f(x,t) - \frac{1}{2}\sigma_x^2(x,t)\partial_x f(x,t),$$

while the evolution of $f(x,t)$ is governed by

$$\partial_t f(x,t) = -\partial_x \text{Flow}(x,t) - \delta f(x,t).$$

Together these two relationships imply the KFE.

It remains to show that $I_t = \frac{1}{2}\sigma_x^2 f(0,t)$ and $S_t = \frac{1}{2}\sigma_x^2 f(b_{BGP},t)$. This result can be derived by working with the transition probability of the Brownian Motion with drift

$$p_\mu(x,y,t) = (2\pi t\sigma^2)^{-\frac{1}{2}} e^{-\frac{(x-y+\mu t)^2}{2t\sigma^2}}.$$

Imagine starting out with the density

$$f(x,0) = \begin{cases} 1 & \text{if } x \geq 0 \\ 0 & \text{if } x < 0. \end{cases}$$

and letting this density evolve until time t, driven by a diffusion process with no drift. At time t, how much work must be done to push all mass that diffused into the region $x < 0$ back to the region $x \geq 0$? The answer is given by

$$\begin{aligned}
\int_{-\infty}^{0}\int_0^\infty (2\pi t\sigma^2)^{-\frac{1}{2}}\; x\; e^{-\frac{(x-y)^2}{2t\sigma^2}}\; dy\; dx &= \int_0^\infty \int_0^\infty (2\pi t\sigma^2)^{-\frac{1}{2}}\; x\; e^{-\frac{(x+y)^2}{2t\sigma^2}}\; dx\; dy \\
&= 2\int_0^\infty \int_0^u (2\pi t\sigma^2)^{-\frac{1}{2}}\; (u-y)\; e^{-\frac{u^2}{2t\sigma^2}}\; dy\; du \\
&= \int_0^\infty (2\pi t\sigma^2)^{-\frac{1}{2}}\; u^2\; e^{-\frac{u^2}{2t\sigma^2}}\; du \\
&= \frac{1}{2}t\sigma^2.
\end{aligned}$$

By taking $t \to 0$, we see that the rate at which a singular control at $x = 0$ does work is $\frac{1}{2}\sigma^2$ for this particular density. Note now that as $t \to 0$, $p(x, y, t) \to 0$ for all $x \neq y$, so only the value of f(0) is relevant for the rate of work done by the singular control at $t = 0$. Thus, the rate of work done by a singular control at $x = 0$ is given by $\frac{1}{2}\sigma^2 f(0)$ for a Brownian Motion without drift, for any density $f(x)$. Similarly, note that as $t \to 0$, we have that $p_\mu(x, y, t)/p_{\mu=0}(x, y, t) \to 1$. As a consequence, the rate of work done by a singular control is independent of the drift term of the Brownian Motion, and the above expression holds for all Brownian motions and all densities $f(x)$.

The steady-state KFE is modified for the non-steady state environment, with the term $g_{x,BGP}$ in the steady state KFE replaced by the equivalent term $g_{x,t}$ in the modified KFE, and with $g_{x,BGP}$ and b_{BGP} replaced by the equivalent terms $g_{x,t}$ and b_t in the boundary conditions for the non-steady state KFE. Finally, there is the additional term $g_{b,t}$ in the boundary condition at $x = b_t$ in the non-steady state KFE which results from additional drift of the boundary b_t (as can be seen from the change of the variables $x_{it} \to x_{it} - b_t$, followed by the same reasoning about this boundary condition as above).

Linearization of the Production Firm's Optimal Policy in a Dynamic Economic Environment

Recall that $x = \Phi_{BGP} - B/K$. We therefore have that $dx/dy = 0$, $dx/dB = -K^{-1}$, and $dx/dK = BK^{-2}$. Hence

$$
\begin{aligned}
V_K(K, B, y) &= v(x, y) && (51)\\
V_B(K, B, y) &= -v_x(x, y) && (52)\\
V_{BB}(K, B, y) &= v_{xx}(x, y)K^{-1} && (53)\\
V_y(K, B, y) &= Kv_y(x, y). && (54)
\end{aligned}
$$

Substituting these expressions into the Bellman equation for the dynamic economic environment we obtain

$$
\begin{aligned}
(\delta + r_{BGP} - \eta y + \epsilon)v(x, y) &= [\mu_{CF}(lkr, y) - (\delta + r_{BGP} - \eta y)(\Phi_{BGP} - x)]v_x(x, y)\\
&\quad + \frac{1}{2}\sigma^2_{CF}(lkr)v_{xx}(x, y) - \eta y v_y(x, y).
\end{aligned}
$$

Using the linear approximation

$$v(x, y) = v(x) + yw(x)$$

of the firm's value function the above Bellman equation yields

$$(\delta + r_{BGP} + \epsilon + \eta)w(x) = \eta v(x)$$

$$
\begin{aligned}
&+\frac{\partial \mu_{CF}(lkr(x),0)}{\partial lkr}\frac{dlkr(x)}{dy}v_x(x)\\
&+\left[\frac{\partial \mu_{CF}(lkr(x),0)}{\partial y}+\eta(\Phi_{BGP}-x)\right]v_x(x)\\
&+\left[\mu_{CF}(lkr(x),0)-(\delta+r_{BGP})(\Phi_{BGP}-x)\right]w_x(x)\\
&+\frac{1}{2}\frac{\partial \sigma^2_{CF}(lkr(x),0)}{\partial lkr}\frac{dlkr(x)}{dy}v_{xx}(x)\\
&+\frac{1}{2}\sigma^2_{CF}(lkr(x))w_{xx}(x).
\end{aligned}
$$

The boundary condition $w(0)=v_x(0)\frac{d\Phi(y)}{dy}$ stems from by the fact that the firm becomes worthless to equity owners when the firm's debt level reaches the firm's debt capacity. From $v(x,y)=0$ if $x=-\frac{d\Phi(y)}{dy}y$, we have that, differentiating w.r.t. y,

$$
\frac{d}{dy}v\left(-\frac{d\Phi(y)}{dy}y,y\right)=0=-v_x(0,0)\frac{d\Phi(y)}{dy}+v_y(0,0)=-v_x(0)\frac{d\Phi(y)}{dy}+w(0).
$$

The boundary condition $w_x(p_{BGP})=0$ stems from the need to have a linear approximation of the firm's dividend trigger $p(y)$. For a firm paying dividends we know that

$$
\begin{aligned}
V_B(K,B,y) &= -1\\
V_{BB}(K,B,y) &= 0\\
\Phi_{BGP}-B/K &= p(y).
\end{aligned}
$$

Changing notation and linearizing we have

$$
v_x(p(y),y)=1=v_x(p(y))+yw_x(p(y))
$$

$$
v_{xx}(p(y),y)=0=v_{xx}(p(y))+yw_{xx}(p(y))
$$

which, differentiating w.r.t. y at $y=0$, yields

$$
\begin{aligned}
0 &= v_{xx}(p_{BGP})\frac{dp(y)}{dy}+w_x(p_{BGP})\\
0 &= v_{xxx}(p_{BGP})\frac{dp(y)}{dy}+w_{xx}(p_{BGP}).
\end{aligned}
$$

Since $v_{xx}(p_{BGP})=0$, this implies the boundary condition $w_x(p_{BGP})=0$ as well as the linear approximation of the firm's trigger policy for paying dividends

$$
\frac{dp(y)}{dy}=-\frac{w_{xx}(p_{BGP})}{v_{xxx}(p_{BGP})}.
$$

Next, I derive the requirement that $w(I_{BGP}) = 0$ must hold for a linear approximation of the firm's investment trigger $I(y)$ to exist. Investment occurs at the firm's investment trigger $I(y)$ which satisfies

$$\frac{v(I(y),y)}{v_x(I(y),y)} + \Phi_{BGP} - I(y) = 1. \tag{55}$$

Rearranging and differentiating w.r.t. y yields

$$\begin{aligned} v_{xx}(I(y),y)I(y) + v_{xy}(I(y),y) \quad = \quad & v_x(I(y),y)I_y(y) + v_y(I(y),y) \\ & +\Phi(y)[v_{xx}(I(y),y)I_y(y) + v_{xy}(I(y),y)] \\ & -I(y)v_x(I(y),y) \\ & -I(y)[v_{xx}(I(y),y)I(y) + v_{xy}(I(y),y)]. \end{aligned} \tag{56}$$

At $y = 0$, using the fact that $v_{xx}(I_{BGP},0) = 0$, $v_{xy}(I_{BGP},0) = 0$ and $v_x(I_{BGP},0) = 1$, we have that

$$w(I_{BGP}) = 0.$$

Any other value of $w(I_{BGP})$ would imply that $I_y(y)$ is not well defined. We now have to verify that if $w(I_{BGP}) = 0$ the linear approximation of the firm's trigger for investment coincides with the linear approximation of the firm's trigger for paying dividends. To see that this is indeed the case, differentiate (56) w.r.t. x:

$$\begin{aligned} v_{xxx}(I(y),y)I_y(y) + v_{xxy}(I(y),y) \quad = \quad & v_{xx}(I(y),y)I_y(y) + v_{xy}(I(y),y) \\ & +\Phi(y)[v_{xxx}(I(y),y)I_y(y) + v_{xxy}(I(y),y)] \\ & -I_y(y)v_{xx}(I(y),y) \\ & -I(y)[v_{xxx}(I(y),y)I_y(y) + v_{xxy}(I(y),y)]. \end{aligned}$$

Using $v_{xy}(I_{BGP},0) = 0$, $v_{xx}(I_{BGP},0) = 0$, we have that at $y = 0$

$$I_y(0) = -\frac{v_{xxy}(I_{BGP},0)}{v_{xxx}(I_{BGP},0)} = -\frac{w_{xx}(I_{BGP})}{v_{xxx}(I_{BGP})} = p_y(0).$$

The firm chooses its labor input $lkr(x,y)$ to maximize its expected appreciation:

$$\max_{\{lkr(x,y)\}} \mu_{CF}(lkr(x,y),y)[v_x(x) + yw_x(x)] + \frac{1}{2}\sigma^2_{CF}(lkr(x,y),y)[v_{xx}(x) + yw_{xx}(x)]. \tag{57}$$

We already have the solution for this problem when $y = 0$. Now we need to find $dlkr(x,y)/dy$. The FOC for maximization of (57) is

$$\frac{d}{dlkr}\left(\mu(lkr(x,y),y)[v_x(x) + yw_x(x)] + \frac{1}{2}\sigma^2_{CF}(lkr(x,y))[v_{xx}(x) + yw_{xx}(x)]\right) = 0$$

or

$$\frac{\partial \mu(lkr(x,y),y)}{\partial lkr}[v_x(x)+yw_x(x)]+\frac{1}{2}\frac{\partial \sigma^2_{CF}(lkr(x,y))}{\partial lkr}[v_{xx}(x)+yw_{xx}(x)]=0.$$

We already know this FOC holds for $y=0$. It must also hold around $y=0$, so that

$$\frac{d}{dy}\left(\frac{\partial \mu(lkr(x,y),y)}{\partial lkr}[v_x(x)+yw_x(x)]+\frac{1}{2}\frac{\partial \sigma^2_{CF}(lkr(x,y))}{\partial lkr}[v_{xx}(x)+yw_{xx}(x)]\right)=0$$

or

$$\begin{aligned}\left(\frac{\partial^2 \mu_{CF}(lkr(x,0),0)}{\partial lkr^2}\frac{dlkr(x,0)}{dy}+\frac{\partial^2 \mu_{CF}(lkr(x,0),0)}{\partial lkry}\right)v_x(x)+&\\ \frac{\partial \mu_{CF}(lkr(x,0),0)}{\partial lkr}w_x(x)+&\\ +\frac{1}{2}\frac{\partial^2 \sigma^2_{CF}(lkr(x,0),0)}{\partial lkr^2}\frac{dlkr(x,0)}{dy}v_{xx}(x)+&\\ +\frac{1}{2}\frac{\partial \sigma^2_{CF}(lkr(x,0))}{\partial lkr}w_{xx}(x) &= 0.\end{aligned}$$

This is the FOC condition found in the proposition.

Eigenfunctions of the Kolmogorov Forward Equation in the Economic Environment Created by an Eigenstate

We are looking for the eigenfunction $e_\eta(x)$ so that the capital density

$$f(x,t)=e^{g_A t}(f_{BGP}(x)+y_{eta,t}e_\eta(x))$$

with the boundary condition

$$f(-y_\eta\frac{d\Phi_\eta}{dy_\eta},y_\eta)=0$$

evolves (in the linearization of the KFE around the BGP) as indicated by the law of motion for an eigenstate of the economy without further aggregate shocks $dy_{eta,t}=-\eta y_{eta,t}dt$.

Looking at the left boundary condition, we can see that it implies in the linear approximation that

$$e_\eta(0)=-\partial_x f_{BGP}(0)\frac{d\Phi}{dy_\eta}.$$

$\partial_x e_\eta(0)$ remains a free parameter that is not determined by any boundary condition at $x=0$. Furthermore, linearizing the liquidation rate of installed capital

$$Liq_{Agg,t}=\frac{1}{2}\sigma_x^2(y_\eta\frac{d\Phi}{dy_\eta},y_\eta)\partial_x f(y_\eta\frac{d\Phi}{dy_\eta},y_\eta)$$

yields

$$
\begin{aligned}
\frac{dLiq_{Agg,t}}{dy} &= \frac{1}{2}\frac{d\sigma_x^2(0)}{dy}\partial_x f_{BGP}(x) + \frac{1}{2}\sigma_{x,BGP}^2(0)\partial_x e_\eta(0) \\
&\quad + \frac{1}{2}\sigma_x^2(0)\partial_{xx} f_{BGP}(0)\frac{d\Phi}{dy_\eta} + \frac{1}{2}\partial_x\sigma_x^2(0)\frac{d\Phi}{dy_\eta}\partial_x f_{BGP}(0).
\end{aligned}
$$

Similarly, linearizing the boundary condition at the right boundary

$$
\begin{aligned}
I_{Agg,t} &= -\left(\mu(p_{BGP} + y_\eta\frac{dp}{dy_\eta}, y_\eta) - \frac{1}{2}\partial_x\sigma_x^2(p_{BGP} + y_\eta\frac{dp}{dy}, y_\eta)\right) f(p_{BGP} + y_\eta\frac{dp}{dy}, y_\eta) \\
&\quad + \frac{1}{2}\sigma_x^2(p_{BGP} + y_\eta\frac{dp}{dy}, y_\eta)\partial_x f(p_{BGP} + y_\eta\frac{dp}{dy}, y_\eta) - \eta\frac{dp}{dy} f(p_{BGP} + y_\eta\frac{dp}{dy}, y_\eta)
\end{aligned}
$$

yields

$$
\begin{aligned}
\frac{dI_{Agg,t}}{dy} &= -\left(\partial_x\mu_{BGP}\frac{dp}{dy} + \frac{d\mu_x(p_{BGP})}{dy}\right) f_{BGP}(p_{BGP}) \\
&\quad + \frac{1}{2}\left(\partial_{xx}\sigma_{x,BGP}(p_{BGP})\frac{dp}{dy} + \frac{d\{\partial_x\sigma_x^2(p_{BGP})\}}{dy}\right) f_{BGP}(x) \\
&\quad - \left(\mu_{BGP}(p_{BGP}) - \frac{1}{2}\partial_x\sigma_{BGP}^2(p_{BGP})\right)\left(\partial_x f_{BGP}(p_{BGP})\frac{dp}{dy} + e(p_{BGP})\right) \\
&\quad + \frac{1}{2}\left(\partial_x\sigma_{BGP}^2(p_{BGP})\frac{dp}{dy} + \frac{d\sigma_x^2(p_{BGP})}{dy}\right)\partial_x f_{BGP}(p_{BGP}) \\
&\quad + \frac{1}{2}\sigma_{x,BGP}^2(p_{BGP})\left(\partial_{BGP} f_{BGP}(p_{BGP})\frac{dp}{dy} + \partial_x e(p_{BGP})\right) \\
&\quad - \eta\frac{dp}{dy} f_{BGP}(p_{BGP}).
\end{aligned}
$$

Taking the capital density $f(x, y_\eta) = f_{BGP}(x) + y_\eta e_\eta(x)$ and substituting it into the KFE

$$
\begin{aligned}
\partial_t f(x, y_\eta) = -\eta y_\eta e_\eta(x) &= -\delta(f_{BGP}(x) + y_\eta e_\eta(x)) \\
&\quad + \frac{1}{2}\partial_{xx}\sigma_{BGP}(x, y_\eta)(f_{BGP}(x) + y_\eta e_\eta(x)) \\
&\quad + \partial_x\sigma_x^2(x, y_\eta)(\partial_x f_{BGP}(x) + y_\eta\partial_x e_\eta(x)) \\
&\quad + \frac{1}{2}\sigma_x^2(x, y_\eta)(\partial_{xx} f_{BGP}(x) + y_\eta\partial_{xx} e_\eta(x)) \\
&\quad - \partial_x\mu(x, y_\eta)(f_{BGP}(x) + y_\eta e_\eta(x)) \\
&\quad - \mu(x, y_\eta)(\partial_x f_{BGP}(x) + y_\eta\partial_x e_\eta(x))
\end{aligned}
$$

which yields, keeping only terms in y_η,

$$
0 = (\eta - \delta + \frac{1}{2}\partial_{xx}\sigma^2(x) - \partial_x\mu_x(x))e_\eta(x)
$$

$$
\begin{aligned}
&+(\partial_x \sigma_x^2(x) - \mu_x(x))\partial_x e_\eta(x) \\
&+\frac{1}{2}\sigma_x^2(x)\partial_{xx} e_\eta(x) \\
&+\left(\frac{1}{2}\frac{d\{\partial_{xx}\sigma_x^2(x,y_\eta)\}}{dy_\eta} - \frac{d\{\partial_x \mu_x(x,y_\eta)\}}{dy_\eta}\right) f_{BGP}(x) \\
&+\left(\frac{1}{2}\frac{d\{\partial_{xx}\sigma_x^2(x,y_\eta)\}}{dy_\eta} - \frac{d\{\partial_x \mu_x(x,y_\eta)\}}{dy_\eta}\right) \partial_x f_{BGP}(x) \\
&+\frac{1}{2}\frac{d\sigma_x^2(x,y_\eta)}{dy_\eta}\partial_{xx} f_{BGP}(x).
\end{aligned}
$$

Optimal Hedging Policy of the Production Firm

Let technology A_t and the eigenstate y_t evolve according to

$$
\begin{aligned}
dA_t &= \sigma_A A_t dW_{A,t} \\
dy_t &= -\eta y_t + b\sigma_A dW_t
\end{aligned}
$$

and let the marginal utility of wealth be given by $m_t = A_t^{-1} m(y_t)$ so that

$$
\frac{dm(y_t)/dy_t}{m(y_t)} = -1
$$

(this is the normalization that $\hat{C}_0 = 1$ for an eigenstate that affects consumption). Then, neglecting terms that are quadratic or higher in σ_A,

$$
\frac{dm_t}{m_t} = -\eta y_t dt - \sigma_A(b+1)dW_{A,t}.
$$

When the representative household enters a hedging contract it pays out the riskless payment hp_h at the utility cost $hp_h m_t$ and receives the risky payment $h\sigma_A dW_{A,t}$ with the expected utility stream

$$
\begin{aligned}
E_t[(dm_t)(h\sigma_A dW_{A,t})] &= E_t[-m_t\sigma_A(b+1)h\sigma_A dW_{A,t}^2] \\
&= m_t h\sigma_A^2(b+1)dt.
\end{aligned}
$$

Since in equilibrium the representative household must be indifferent about holding a marginal hedging contract, we can equate the utility cost and the utility benefit of hedging to get the price of a hedge

$$
p_h = -\sigma_A^2(b+1).
$$

The production firm enters the hedging position $h(x)$, which maximizes its value to the representative household. The marginal impact of $h(x)$ on the firm's state variable x is given by $dx_t = -hp_h dt + h\sigma_A dW_{A,t}$.

The firm's value is then affected by hedging according to (using Ito's Lemma)

$$\begin{aligned} dv(x,y) &= -hp_h v_x(x,y)dt \\ &\quad + \frac{1}{2}h^2\sigma_A^2 v_{xx}(x,y)dt \\ &\quad + [h\sigma_a v_x(x,y) + \sigma_A b v_y(x,y)]dW_{A,t} \\ &\quad + h\sigma_A v_{xy}(x,y)b\sigma_a dt \end{aligned}$$

which has the impact on expected utility

$$m_t(-hp_h v_x(x,y) + \frac{1}{2}h^2\sigma_A^2 v_{xx}(x,y) + h\sigma_A^2 v_{xy}(x,y)b)dt$$

$$-m_t(b+1)\sigma_A(h\sigma_A v_x(x,y) + \sigma_A v_y(x,y)b)dt.$$

Dividing by m_t and substituting in $p_h = -\sigma_A^2(b+1)$ we have

$$\sigma_A^2(\frac{1}{2}h^2 v_{xx}(x,y) + hbv_{xy}(x,y) - (1+b)bv_y(x,y))dt.$$

Differentiating this expression w.r.t. h we can find the optimal hedge h which maximizes the value of the firm to the representative household:

$$h = -\frac{bv_{xy}(x,y)}{v_{xx}(x,y)} = -\frac{b_\eta w^\eta(x)}{v_{xx}(x)}.$$

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